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AN

ADVANCED ARITHMETIC

THEORETICAL AND PRACTICAL

BY

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PREFACE.

THE scope of this book will be most readily comprehended by examining the Table of Contents on the following pages. It will be seen (1) that the work embraces about all the subjects usually contained in Higher Arithmetics, and (2) that an attempt has been made to arrange the whole upon a systematic plan, and a progressive order of development.

In preparing the book, the author has endeavored to secure the following objects:

1. **Scientific Accuracy and Development**;—clear and concise definitions; full and rigid analyses; and a graded and logical arrangement and development of subjects.

2. **Practical Utility**;—data for problems taken from the most recent statistics; elimination of all obsolete terms, measures and processes; methods of calculation which the experience of business men has proved to be most practical; and an immense number of practical problems which furnish copious illustrations of commercial terms, usages, values and transactions.

The method of induction by means of parallel problems, which has contributed so largely to the popularity of the other books of this series, is also a characteristic feature of the present treatise. A practical union of oral and written arithmetic, of synthesis and analysis, is thus secured, which helps the student to advance with increasing facility, interest and power.

The author feels assured that the simple and thorough manner in which the subjects of Fractions, Percentage and Proportion are treated will meet the approval of teachers, as a thorough knowledge of these subjects is essential in all the most important applications of Arithmetic. The subjects of Averages, Square and Cube Roots, and Denominate Numbers, including the Metric System, have also received that broad and thorough consideration commensurate with their importance.

In addition to the subjects usually presented under the Applications of Percentage, such as Insurance, Commission, etc., Quantitative Chemical Analysis has been introduced. This innovation supplies a

long-felt want, a want that has been intensified in late years by the establishment of Agricultural Colleges and Experiment Stations where the course of study consists largely of chemical analyses. In all the problems under this head the given per cents of constituents are taken from analyses made at one or the other of the Stations of the Louisiana State University and Agricultural and Mechanical College.

Special attention is called to the rational treatment of Arithmetical Signs. The theory of combining numbers connected by the signs $\times$ and $\div$, as presented in this book, is original with the author, who feels confident that it has never before appeared in any Arithmetic. From a want of knowledge of this theory, numbers so connected are differently combined by different authors, and erroneous results appear in many text-books that are extensively used.

The Appendix, to be used at the discretion of the teacher, in addition to Short Methods and Miscellaneous Tables and Problems, contains a fair presentation of Circulating Decimals, Foreign Exchange, Arithmetical and Geometrical Progressions, Annuities, Building Associations, and Horner's excellent method of extracting roots.

If the problems in this book are thought to be more numerous than is always necessary, the teacher may select such as will best meet the individual wants of the pupils.

In conclusion, the author acknowledges his obligations and returns his thanks to the many persons whose printed works or verbal suggestions have assisted him in the preparation of this book, and respectfully solicits any suggestions for its further improvement.

J. W. N.

BATON ROUGE, LA., June, 1889.

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ARITHMETIC.

CHAPTER I.

INTEGERS.

PRELIMINARY DEFINITIONS.

Art. 1. A **unit** is a single thing.

(1) One, one day, one apple, one ten, are units.

2. Units are **like** when they are the same, and **unlike** when they are of different kinds.

(1) One dollar and one dollar are like units.

(2) One day and one gallon are unlike units.

3. A **number** is a unit or a collection of like units.

(1) Six, five men, ten balls, four days, are numbers.

4. The **unit of a number** is one of the units of which the number is formed.

(1) The unit of six is one; of twelve days, one day; of thirty-four pounds, one pound.

5. Like numbers are those which have like units, and **unlike numbers** are those which have unlike units.

(1) Two days and ten days are like numbers.

(2) Three days and five boys are unlike numbers.

6. An **abstract number** is one that is not applied to a particular object, and a **concrete number** is one that is applied to some object.

(1) Nine, six, two, etc., are abstract numbers.

(2) *Nine pens, six miles, etc.,* are concrete numbers.

An abstract number expresses *how many* times one quantity is contained in another, and a concrete number expresses *how much* one quantity is in terms of another.

7. An **integer**, or **whole number**, is a number that is composed of whole or entire units, and a **fraction, or fractional number**, is a number that is composed of equal parts of a whole or entire unit.

(1) Six, ten pints, five days, etc., are integers.

(2) Two-thirds of a pint, three-fourths of a bushel, etc., are fractions.

8. **Arithmetic** is the science and art of numbers.

9. **Science** is classified knowledge.

10. **Art** is the practical application of science.

11. **Principles** are the primary truths of science.

12. A **theorem** is a truth to be proved.

13. A **problem** is a construction or question to be solved.

14. An **example** is a problem or theorem used to illustrate a principle or process.

15. A **rule** is a general description of a process.

16. A **formula** is the expression, by symbols, of a rule or principle.

17. In arithmetic, **exercises** are problems or theorems.

In written exercises the successive steps of the work are to be written; and in mental or oral exercises the work is not to be written.

18. **Parallel problems** are those involving the same principles, and which are solved in a similar manner.

NOTE.—In this treatise, the exercises often embrace parallel problems, in which case such as are designed for oral work are designated in their numbers by a different style of type; and are generally intended to illustrate the principles involved in the succeeding *problems.*

NOTATION AND NUMERATION.

19. **Notation** is the writing of numbers.

20. **Numeration** is the reading of numbers.

21. Two methods of notation are in general use: the **Arabic** and the **Roman.**

ARABIC NOTATION.

22. The **Arabic notation** was introduced into Europe from Arabia in the tenth century. In it ten characters, called figures, are used to express numbers, viz.:

Figures:	0,	1,	2,	3,	4.
Names:	Naught,	One,	Two,	Three,	Four.
Figures:	5,	6,	7,	8,	9.
Names:	Five,	Six,	Seven,	Eight,	Nine.

The figure 0, also called cipher or zero, denotes *none* or *nothing*. The other nine figures are called *digits* or *significant* figures.

23. The **value** of a figure is the number it represents.

24. By these ten figures all abstract integers may be expressed by combining them according to the following

PRINCIPLES.—When two figures are written side by side—

1°. *The unit of the one on the left is understood to be ten times the unit of the one on the right.*

2°. *The value of the first is understood to be increased by the value of the second.*

Thus, in 64 the unit of 4 is *one;* hence, the unit of 6 is ten times one, or ten. Hence, 64 represents 6 tens and 4 ones, or sixty-four.

25. In any integer the **places** of the figures are numbered from the right.

In 63,429, 9 is in the first place, 2 is in the second, 4 in the third, etc.

26. The figure in the *first place* denotes **ones**, called units of the **first order**.

27. The figure in the *second place* denotes *ten* ones, or tens, called units of the **second order**.

28. The figure in the *third place* denotes *ten* tens, or hundreds, called units of the **third order**.

(1) 13 denotes 1 ten, 3 ones: read, thirteen.

(2) 34 denotes 3 tens, 4 ones: read, thirty-four.

(3) 90 denotes 9 tens, 0 ones: read, ninety.

(4) 270 denotes 2 hundreds, 7 tens, 0 ones: read, two hundred seventy.

(5) 405 denotes 4 hundreds, 0 tens, 5 ones: read, four hundred five.

EXERCISE I.

Read the following numbers:

1. 21.	2. 81.	3. 137.	4. 256.	5. 2.
6. 50.	7. 70.	8. 568.	9. 607.	10. 37.
11. 42.	12. 18.	13. 797.	14. 111.	15. 521.
16. 37.	17. 92.	18. 590.	19. 888.	20. 9.
21. 68.	22. 30.	23. 371.	24. 700.	25. 99.
26. 77.	27. 29.	28. 333.	29. 414.	30. 999.

Express by figures:

31. Seventy.	32. Five hundred.
33. Forty-one.	34. Six hundred five.
35. Fifty-five.	36. Three hundred twenty-six.
37. Eighty-three.	38. Four hundred seven.
39. Thirteen.	40. Nine hundred nine.
41. Sixty-nine.	42. Two hundred twenty-two.
43. Twenty-seven.	44. Eight hundred eight.
45. Seventeen.	46. One hundred eleven.
47. *Ninety-nine.*	48. Nine hundred ninety-nine.

29. ***Ten hundreds are one thousand.***

Hence, the figures in *fourth*, *fifth* and *sixth places* denote respectively **one-thousand, ten-thousands, hundred thousands,** called units of the **fourth, fifth** and **sixth orders.**

30. Every three figures in an integer, counting from the right, are a **period.** Periods of figures are separated by commas.

The second period denotes ones, tens and hundreds of thousands.

(1) 18,325 denotes 18 thousands, 325 ones, or eighteen thousand three hundred twenty-five.

(2) 407,500 denotes 407 thousands, 500 ones, or four hundred seven thousand five hundred.

Read the following numbers:

49. 3,407.	**50.** 24,536.	**51.** 124,376.
52. 6,040.	**53.** 80,271.	**54.** 308,550.
55. 7,005.	**56.** 92,065.	**57.** 999,999.

Express by figures:

58. Four thousand seven hundred twenty-seven.

59. Seventy-four thousand three hundred fifty-four.

60. Fifty-five thousand six hundred twelve.

61. Eight thousand one hundred forty.

62. Nine thousand nine hundred ninety-nine.

63. One thousand six. **64.** Six thousand fifteen.

65. Two thousand ten. **66.** Three thousand fifty.

67. Five thousand eleven. **68.** Seven thousand one.

69. One hundred twenty-two thousand five hundred nine

70. Six hundred nine thousand.

71. Seven hundred forty thousand fifteen.

72. Nine hundred twenty-five thousand five hundred forty.

31. ***Ten hundred-thousands are one million.***

Hence, the figures in the *seventh*, *eighth* and *ninth places* denote respectively, **millions, ten-millions, hundred-millions**, called units of the **seventh, eighth** and **ninth orders.**

Consequently the third period consists of ones, tens and hundreds of millions.

(1) 33,500,006 denotes 33 millions, 500 thousands, 6 ones, or thirty-three million five hundred thousand six.

(2) 105,000,379 denotes 105 millions, 0 thousands, 379 ones, or one hundred five million three hundred seventy-nine.

Express in figures and read the following numbers:

73. Two million twenty thousand three hundred nine.
74. Forty-one million five hundred thousand sixty-nine.
75. One hundred fifty-three million sixty-seven thousand four hundred seventy-seven.
76. Six units of the 8th order, nine of the 6th, five of the 3d, and seven of the 2d.
77. Nine units of the 9th order, eight of the 7th, seven of the 5th, and five of the 1st.
78. Thirty-two units of the 7th order, sixty-four of the 5th, and 125 of the 1st.

32. A **scale in arithmetic** is the relation between the successive orders of units.

In the Arabic system of notation, the scale is ten; that is, the value of the unit in any order is ten times as great as the unit in the next lower order; hence, it is called the Decimal Scale, from the Latin *decem*, meaning ten.

33. The value of a digit depends on the place it occupies

The value of 3 in 37 is 3 tens or thirty; of 3 in 321 is 3 hundreds, *or three hundred.*

The Common or French Method of Numeration.

34. The **French method** of writing and reading large numbers is shown in the following

NUMERATION TABLE.

Places of units	15th. Hundreds.	14th. Tens.	13th. Ones.	12th. Hundreds.	11th. Tens.	10th. Ones.	9th. Hundreds.	8th. Tens.	7th. Ones.	6th. Hundreds.	5th. Tens.	4th. Ones.	3d. Hundreds.	2d. Tens.	1st. Ones.
Figures	3	6	[illegible],	5	0	4,	7	2	5,	9	1	3,	8	7	6.
Periods: Numbers	Fifth.			Fourth.			Third.			Second.			First.		
Periods: Names	Trillions.			Billions.			Millions.			Thousands.			Ones.		

To express larger numbers, other periods are formed in like manner, called **quadrillions, quintillions, sextillions, septillions, octillions, nonillions, decillions, etc., etc.**

RULE FOR NOTATION.—*Begin at the left, and write the figures of each period in their proper orders, filling all vacant orders and periods with ciphers.*

Express by figures:

79. Four hundred forty thousand five hundred one.

80. Six million five thousand forty-seven.

81. Nine million forty-nine thousand thirty-one.

82. Twenty-nine million six hundred eleven thousand fifteen.

83. One hundred seven million eleven thousand one hundred sixty-seven.

84. Seven hundred thirty million six hundred nine thousand three hundred ninety-two.

85. Thirteen billion thirteen million thirteen thousand thirteen.

86. Two hundred forty-five billion one hundred seven million fifty-nine thousand eight hundred seventy.

87. Twenty-five trillion two hundred seven million thirty seven thousand three hundred one.

88. One hundred forty-three trillion six hundred twenty seven billion two hundred twenty-five million three hundred forty-six thousand eleven.

89. Sixteen quadrillion three trillion seventeen billion five million thirty-four thousand two.

RULE FOR NUMERATION.—1. *Begin at the right, and separate the number into periods of three figures each.*

2. *Begin at the left, and read each period containing one or more figures as if it stood alone, adding its name.*

Read the following numbers:

90.	63754.	**91.**	56481304.
92.	7532642.	**93.**	801603709.
94.	196003205.	**95.**	4321780651.
96.	13423005.	**97.**	105433456789.
98.	7492030400.	**99.**	6073456654321.
100.	82123457183.	**101.**	900405060708090.

English Method of Numeration.

35. In the English method of numeration six figures form a period. The first period denotes *ones*, the second *millions*, the third *billions*, etc.

(1) 30075,807250 denotes 30075 millions, 807250 ones; read, thirty thousand seventy-five million eight hundred seven thousand two hundred fifty.

(2) 50,000320,400062 denotes 50 billions, 320 millions, 400062 ones; read, fifty billion three hundred twenty million four hundred thousand sixty-two.

THE ROMAN NOTATION.

36. In the **Roman notation** seven capital letters are employed to denote numbers, viz.:

Letters:	I,	V,	X,	L,	C,	D,	M,
Values:	one,	5 ones,	1 ten,	5 tens,	1 hund.,	5 hunds.,	1 thousand.

Placing a dash over a letter increases its value 1000 *times.* Thus:

$\overline{V}$	$\overline{X}$	$\overline{L}$	$\overline{C}$	$\overline{D}$	$\overline{M}$
5 thous.	10 thous.	5 ten-thous.	1 hund. thous.	5 hund. thous.	1 million.

Hence, I, X, C, M, $\overline{X}$, $\overline{C}$, $\overline{M}$, etc., represent 1 unit of the first, second, third, etc., order, called *unit letters;* and V, L, D, $\overline{V}$, $\overline{L}$, $\overline{D}$, etc., represent 5 units of first, second, third, etc., order, called 5-*unit* letters.

To express other numbers the letters are combined thus:

1°. *Writing the unit letter* 1, 2 *or* 3 *times, expresses* 1, 2 *or* 3 *units of that order.*

Thus, III denotes 3 ones or 3; XX, 2 tens or 20; CCC, 3 hundreds or 300.

2°. *Writing the unit letter before the* 5-*unit letter expresses* 4 *units of that order.*

Thus, IV denotes 4 ones or 4; XL, 4 tens or 40; CD, 4 hundreds or 400.

3°. *Writing the unit letter* 1, 2 *or* 3 *times after the* 5-*unit letter, expresses* 6, 7 *or* 8 *units of that order.*

Thus, VI denotes 6 ones or 6; LXX, 7 tens or 70; DCCC, 8 hundreds or 800.

4°. *Writing a unit letter before the unit letter of the next higher order, expresses* 9 *units of the lower order.*

Thus, IX denotes 9 ones or 9; M$\overline{X}$, 9 thousand or 9000.

5°. *Writing in order the letters representing the lo values of the digits in any number, expresses that num in the Roman notation.*

Thus, 874 is denoted by DCCCLXXIV.

EXERCISE II.

Express by Roman notation:

1.	10.	100.	1000.	13.
2.	20.	200.	2000.	24.
3.	30.	300.	3000.	38.
4.	40.	400.	4000.	69.
5.	50.	500.	5000.	250.
6.	60.	600.	6000.	541.
7.	70.	700.	7000.	875.
8.	80.	800.	8000.	2027.
9.	90.	900.	9000.	6304.
10.	100.	1000.	10000.	9999.

Express by Arabic notation:

LXXIV.	XXXVIII.	MDCCCLXXXV.
CCXIX.	LXXXI.	$\overline{\text{X}}$CXLIX.
DCXVI.	$\overline{\text{V}}$CCCLV.	$\overline{\text{L}}$DCCXVIII.
MDCXVI.	$\overline{\text{X}}$CDLX.	$\overline{\text{MCDXX}}$CI.

NOTATION OF DOLLARS AND CENTS.

37. The **sign of dollars** is $, which is called the dol mark.

A period (.), called the **decimal point**, is placed bef a number **expressing** cents.

(1) $3 is 3 dollars. (4) $2.75 is 2 dollars 75 cents.
(2) $.15 is 15 cents. (5) $31.09 is 31 dollars 9 cents.
(3) $.04 is 4 cents. (6) $125.18 is 125 dollars 18 cen

EXERCISE III.

Read the following numbers:

1. $ 18.	2. $ 2.37.	3. $ 64.05.
4. $ 134.	5. $ 101.10.	6. $ 75.09.
7. $8375.	8. $7803.31.	9. $864.07.

In expressing dollars and cents, by figures, *place $ before the number, the decimal point before the cents, and a cipher between the decimal point and any number of cents less than* 10.

Express by figures:

10. Five cents.
11. Nine dollars seventy-three cents.
12. Twelve cents.
13. Twenty dollars eleven cents.
14. Sixty-one cents.
15. Forty-one dollars twenty cents.
16. Three cents.
17. Seventy dollars six cents.
18. Fifty-nine cents.
19. Five dollars sixty-five cents.
20. Four hundred seventy-eight dollars forty-two cents.

QUESTIONS.

What is a unit? A number? The unit of a number? Like numbers? Unlike numbers? An abstract number? A concrete number? An integer?

What is arithmetic? Science? Art? Problems? Parallel problems? In this treatise how are oral problems often designated, and what are they intended to illustrate?

What is notation? Nnmeration? How many characters are used in Arabic notation? On what principles are they combined so as to express any number? Give an example.

What is the value of a number? In 5384 what is the value of 3? 4? 5? 8? What does a figure in the fifth place denote? In the sixth? In the seventh? What is the difference between the French and English methods of numeration? Which method is in common use?

What letters are used in the Roman notation? What values do they represent? How combined so as to express other numbers?

What is the rule for expressing dollars and cents by figures?

ADDITION.

38. (1) There are 8 marbles in one sack, 6 marbles in another and 4 marbles in a box; how many marbles are there in all?

Uniting 8 marbles and 6 marbles I have 14 marbles; now uniting 14 marbles and 4 marbles I have 18 marbles. Hence, there are 18 marbles in all.

39. Addition is uniting two or more numbers into *one.*

40. The **parts** are the numbers added, and the **sum** or **amount** is the result of addition.

(1) 8 hats and 7 hats are 15 hats. Here, 8 hats and 7 hats are the parts, and 15 hats the sum.

41. A **sign** is a symbol used to denote an operation or relation.

42. The **sign of addition** is $+$, which is called *plus*, meaning *more.* When $+$ stands between two numbers, it indicates that they are to be added.

43. The **sign of equality** is $=$, which is read equals, or *is equal to.*

Thus, $7 + 4 = 11$, is read: 7 plus 4 equals 11. It may also be read: 7 and 4 are 11.

44. PRINCIPLES.—1°. *Only like numbers can be added.*

For, a number is a collection of like units; hence, numbers of unlike units cannot be collected into one number.

(1) 5 days, 8 days and 2 days can be added.

(2) 5 days, 8 pints and 2 men cannot be added.

(3) Numbers composed of *ones, tens, hundreds,* etc., may be added by adding units of like order. Thus, $43 + 25 =$ 4 tens 3 ones $+$ 2 tens 5 ones $=$ 6 tens 8 ones $= 68$.

2°. *The sum is the same in whatever order the parts are added.*

(1) $8 + 5 + 7 + 3 = 13 + 7 + 3 = 20 + 3 = 23$.

(2) $8 + 5 + 7 + 3 = 8 + 5 + 10 = 8 + 15 = 23$.

ADDITION TABLE.

	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

EXPLANATION.—The results of adding any figure at the top, as 3, to each of the figures in the first column on the left, are found in the column of numbers under 3. Thus, 3 and 0 are 3, 3 and 1 are 4, etc.

Oral Drill in Matter and Method.

45. (1) Add by 2's from 0 to 20.

Two, four, six, eight......................eighteen, twenty.

In like manner add:

2. By 2's from 1 to 21.
3. By 3's from 0 to 30.
4. By 3's from 1 to 31.
5. By 3's from 2 to 32.
6. By 4's from 0 to 40.
7. By 4's from 1 to 41.
8. By 4's from 2 to 42.
9. By 4's from 3 to 43.

10. Add the other digits 5, 6, 7, 8, 9, in the same manner.

11. How many are 8 + 2? 18 + 2? 28 + 2? 38 + 2? 48 + 2? 58 + 2? 68 + 2? 78 + 2? 88 + 2? 98 + 2?

12. How many are 9 + 3? 19 + 3? etc., to 99 + 3.

13. How many are 7 + 4? 17 + 4? etc., to 97 + 4.

14. How many are 8 + 5? 18 + 5? etc., to 98 + 5.

15. How many are 9 + 6? 19 + 6? etc., to 99 + 6.

16. How many are 5 + 7? 15 + 7? etc., to 95 + 7.

17. How many are 6 + 8? 16 + 8? etc., to 96 + 8.

What is the sum:

18. When 3, 5 and 6 are the parts?

19. When 4, 7 and 9 are the parts?

20. When 5, 6, 8 and 4 are the parts?

21. James spends 8 cents for oranges and 7 cents for apples; how much does he spend?

22. Lucy spent 6 cents for candy and had 7 cents left; how much had she at first?

23. John gave his father 8 apples, his mother 9 apples, his sister 6 apples, and had 3 apples left; how many apples had he at first?

24. What is the sum of 37 and 28?

ANALYSIS: 37 = 3 tens 7 units; 28 = 2 tens 8 units; 2 tens and 3 tens are 5 tens; 8 units and 7 units are 15 units = 1 ten 5 units; 5 tens and 1 ten 5 units = 6 tens 5 units = 65.

Or, 37 + 28 = 37 + 20 + 8 = 57 + 8 = 65.

25. 43 + 16 = ?	**26.** 46 + 39 = ?	**27.** 48 + 73 = ?
28. 54 + 27 = ?	**29.** 83 + 28 = ?	**30.** 91 + 68 = ?
31. 65 + 46 = ?	**32.** 79 + 56 = ?	**33.** 39 + 85 = ?

34. James gave 37 cents for a slate, and 42 cents for a book; what did both cost?

35. A pole is 64 feet in the air, 9 feet in the earth, and 18 feet in the water; how long is the pole?

36. A lad, having spent 55 cents, finds he has 45 cents left; how much had he at first?

37. Mary has 24 apples, and Susan 28 more than Mary; how many has Susan?

38. Charles read 64 pages one day, 23 pages the next day, and 45 pages the next; how many pages did he read in all?

46. ***To find the sum of two or more integers.***

(1) What is the sum of 465, 842 and 96?

Explanation.—Since none but units of the same name can be added, for convenience I write figures of the same order in the same column.	**Operation.** 465 842 96 1403

Adding the ones, I say 6, 8, 13 ones, which are 1 ten 3 ones. I write the 3 ones beneath and carry the 1 ten to the next column.

Now, adding the tens, I say 1, 10, 14, 20 tens, which are 2 hundreds 0 tens. I write the 0 tens beneath and carry the 2 hundreds to the next column.

Next, adding the hundreds, I say 2, 10, 14 hundreds, which I write beneath.

The result, 1403, is the required sum.

RULE I. *Write the numbers so that units of the same order may be in the same column.*

II. *Add the units of the lowest order, write the ones of the sum in the result, and add the tens of the sum with the units of the next order.*

III. *So proceed with the units of each order, and write the entire sum of the units of the highest order.*

METHODS OF PROOF.—1°. Add the figures *downwards* instead of *upwards.*

2°. Separate the numbers into two or more divisions; find the sum of the numbers in each division, and then add these sums together, as in the next example.

(2) Find the sum of 3425, 2831, 694, 86 and 5437.

In proving the work, the first two numbers are added, which gives 6256; then the last three, which gives 6217; now the sum of these is 12478, which being the same as the former result, the work is probably correct.

3425		
2831	—	6256
694		
86		
5437	—	6217
12478	=	12478

EXERCISE IV.

Add and prove the following:

3.	**4.**	**5.**	**6.**	**7.**	**8.**
86	48	67	38	63	47
97	73	88	98	34	84
94	26	52	79	80	67
35	59	76	89	63	28
48	79	81	67	94	57

9.	**10.**	**11.**	**12.**	**13.**
839	816	728	608593	7488
927	707	695	587406	43705
706	694	581	46967	546629
3964	582	435	993879	37530
147	7316	69	304	964485
4382	39	9	75275	17292

14. 87, 25 and 47.

15. 375, 280, 564, 119 and 75.

16. 971, 7430, 97476 and 76734.

17. 76767, 7654, 50121 and 775.

18. 403, 5025, 60007 and 89190.

19. 45364, 8965, 786, 9374 and 47.

Find the sum of the consecutive integers:

20. From 81 to 95, inclusive.

21. From 374 to 391, inclusive

22. From 493 to 510, inclusive.

23. From 5397 to 5416, inclusive.

24. From 31989 to 32028, inclusive.

47. When numbers composed of dollars and cents are to be added, dollars are written under dollars, and cents under *cents, so that* the points stand in the same *vertical* line.

25. *$36.25* + $44.09 + $187. + $6.83 + $.75 =?

26 $50.04 + $961.20 + $8.75 + $1465.83 + $.09 =?

27. What is the sum of $87.20, $971.65, $84.07, $3281.64, $5984.30?

28. A farmer paid $45.60 for flour, $86.75 for bacon, $135 for a mule, and $12.75 for a saddle; what did he pay for all?

48. There is a practical advantage in adding two columns at one operation.

(29)

43
74
65
84
266

To the number at the bottom I add the tens, then the ones of the next number above it. Thus, 84 + 60 = 144, + 5 = 149, + 70 = 219, + 4 = 223, + 40 = 263, + 3 = 266.

49. Numbers may be rapidly added, by adding the sums of two or three figures taken at a time.

(30)

8
9
6
7
4
5
8
47

Thus, 13 (8 + 5) and 11 (4 + 7) are 24, and 15 (9 + 6) are 39, and 8 are 47.
Or, naming results only, I say 13, 24, 39, 47

Let these methods be employed in doing the following:

31.	32.	33.	34.	35.	36.
43	25	37	54	45	4894
64	82	43	65	13	7384
75	93	65	76	69	5291
87	54	58	87	99	6387
36	62	77	98	78	239
94	58	88	88	86	641
31	53	92	7[illegible]	53	3073
82	66	24	5[illegible]	65	5405

37. A man invested \$675.20 in hogs, \$964 in sheep, \$3387 in horses, and had \$358.95 left; how much had he at first?

38. A speculator bought a drove of horses for \$4729, a drove of sheep for \$846.75, a drove of cattle for \$3289, and sold them at a profit of \$599.65; what did he receive for all?

39. It is 1684 miles from A to B, 737 miles farther from B to C than from A to B, and 1386 miles farther from C to D than from B to C; how far is it from A to D through B and C?

40. North America contains 8,593,000 square miles, South America 7,362,000 square miles, Europe 3,825,000 square miles, Asia 17,300,000 square miles, and Africa 11,557,000 square miles. How many square miles are there in these five continents?

41. In 1881 the population of England was 24,613,926; of Wales, 1,360,513; of Scotland, 3,735,573; of Ireland, 5,174,836. What was the total population of the United Kingdom of Great Britain and Ireland?

42. The territory of the United States has been acquired as follows:

	SQ. MILES.
Territory ceded by England, 1783	866391
Louisiana, acquired from France, 1803	862922
Florida, acquired from Spain, 1821	58680
Texas, admitted to the Union, 1845	365573
Oregon, settled by treaty, 1846	298804
California, etc., conquered from Mexico, 1847	515764
Arizona, acquired from Mexico by treaty, 1854	57466
Alaska, acquired from Russia by purchase, 1867	577390

What is the total area of the United States?

QUESTIONS.

What is addition? The parts? The sum? Make the sign of addition. Of equality.

What kind of numbers can be added? Cannot be added? Give *examples. Give* the general rule.

SUBTRACTION.

50. On a tree were 11 apples, but John picked 6 of them. How many apples were left on the tree?

Taking 6 apples from 11 apples, 5 apples are left; because 5 and 6 are 11. Hence, there were 5 apples left.

51. **Subtraction** is taking one number from another, in order to find their difference.

52. The **subtrahend** is the number to be subtracted; the **minuend** is the number from which the subtrahend is to be taken; and the **difference** or **remainder** is the result of subtraction.

53. The **sign** of subtraction is —, which is called *minus*, meaning *less*.

When — stands between two numbers, it indicates that the one *after* it is to be taken from the one before it.

(1) $11 - 8 = 3$ is read, 11 minus 8 equals 3, or 11 less 8 is 3.

54. PRINCIPLES.—1°. *The minuend is the sum of the subtrahend and difference.*

(1) $12 - 5 = 7$; $12 = 5 + 7$.

Hence, subtraction is the *reverse* of addition. The one *unites* numbers, the other *separates* them.

2°. *Only like numbers and like orders of units can be subtracted, one from the other.*

(1) 6 pints can be subtracted from 13 pints.

(2) 6 pints cannot be subtracted from 13 yards.

(3) $48 - 23 =$ 4 tens 8 ones — 2 tens 3 ones = 2 tens 5 ones = 25.

3°. *If two numbers are equally increased, their difference is not altered.*

(1) 8 less 5 = 10 less 7 = 13 less 10 = 3.

(2) To subtract 27 from 52, I add 10 ones to 52 and 1 ten to 27, and then subtract units of like orders.

52 + 10 ones	= 5 tens	12 ones
27 + 1 ten	= 3 tens	7 ones
	2 tens	5 ones = 25

Oral Drill in Matter and Method.

55. (1) Subtract by 2's from 20 to 0.

Twenty, eighteen, sixteen ....two, none.

In like manner, subtract:

2. By 2's from 21 to 1.
3. By 3's from 30 to 0.
4. By 3's from 31 to 1.
5. By 3's from 32 to 2.
6. By 4's from 40 to 0.
7. By 4's from 41 to 1.
8. By 4's from 42 to 2.
9. By 4's from 43 to 3.
10. Subtract the other digits, 5, 6, 7, 8, 9 in the same manner.

11. Name every tenth number from 95 to 5; from 97 to 7; 91 to 1.

12. From every tenth number from 95 to 5 take 1.

13. From every tenth number from 91 to 11 subtract 2.

From every tenth number:

14. From 90 to 10 take 3.
15. From 92 to 12 take 4.
16. From 94 to 14 take 5.
17. From 93 to 13 take 6.
18. From 101 to 11 take 7.
19. From 105 to 15 take 8.
20. From 104 to 14 take 9.
21. From 109 to 19 take 11.

22. The sum of two numbers is 18, and one of them is 7; what is the other?

23. The greater of two numbers is 21 and their difference is 6; what is the less?

24. James had 25 cents and spent 9 cents for apples; how much did he have left.

25. John had 30 cents, and after buying a pencil had 22 cents left; what did the pencil cost?

26. The minuend is 17 and the subtrahend 8; what is the remainder?

27. The minuend is 14 and the remainder 9; what is the subtrahend?

28. Thomas lacks $8 of having $23; how much has he?

29. Henry has 18 marbles and John has 12 marbles; how many more has Henry than John?

30. What number must be added to 16 to produce 24?

31. What is the difference between 43 and 28?

43 = 3 tens 13 ones; 28 = 2 tens 8 ones; 3 tens 13 ones — 2 tens 8 ones = 1 ten 5 ones = 15. Or,

$$28 = 20 + 8;\ 43 - 20 = 23, - 8 = 15.$$

How many are:

32. 64 — 32?	**33.** 83 — 45?	**34.** 75 — 47?
35. 73 — 58?	**36.** 91 — 17?	**37.** 96 — 78?
38. 67 — 29?	**39.** 57 — 36?	**40.** 133 — 67?

41. Henry had 53 cents, but he paid 26 cents for some paper; how many cents had he left?

42. 47 gallons of syrup have been drawn from a tank that contained 93 gallons; how many gallons are left?

43. James has caught 65 fishes and Moses 48; how many more fishes must Moses catch to have as many as James?

56. ***To find the difference between any two integers.***

(1) From 4637 take 1374.

Explanation.—Since none but units of the same name can be subtracted from each other, for convenience I write figures of the same order in the same column.

Operation.

$$\begin{array}{r} 4637 \\ \underline{1374} \\ 3263 \end{array}$$

Beginning at the right, I say, 4 ones from 7 ones leave 3 ones, which I write beneath. Then since 7 tens cannot be taken from 3 tens, I add 10 tens to 3 tens, which make 13 tens, and 7 tens from 13 tens leave 6 tens, which I write beneath. To compensate for the 10 tens added to the minuend I now add 1 hundred (= 10 tens) to the 3 hundreds of the subtrahend, and say, 4 hundreds from 6 hundreds leave 2 hundreds, which I write beneath. Finally, I say 1 thousand from 4 thousands leave 3 thousands, which I write beneath.

RULE.—I. *Write the less number under the greater, placing units under units, tens under tens, etc., and begin at the right to subtract.*

II. *Subtract each figure in the lower line from the one above it, and set the remainder below.*

III. *If any figure in the lower line is greater than the one above it, add* 10 *to the upper figure before subtracting, and consider the next left-hand figure of the subtrahend one more, or the next left-hand figure of the minuend one less, and proceed as before.*

Proof.—*Add the remainder to the subtrahend: the sum should be equal to the minuend.*

EXERCISE V.

Subtract and prove:

2.	3.	4.	5.	6.	7.	8.
37	43	65	70	81	93	100
24	18	47	54	42	76	69

9.	10.	11.	12.	13.	14.
384	405	208	540	671	394
153	243	186	378	583	208

15.	16.	17.	18.	19.
4250	3027	6284	5371	8001
1854	2308	4709	568	925

20.	21.	22.	23.	24.
1000	2050	6173	5037	9763
481	827	4285	2308	6878

25.	26.	27.	28.
43251	64937	84275	506826
38709	8496	68739	70398

29.	30.	31.	32.
$325.15	$403.60	$371.10	$4037.80
84.64	329.35	186.75	609.28

33.	34.	35.	36.
$8004.00	$3000.	$1011.	$10000.
907.25	2527.75	903.25	.85

37. From 8543 take 329.
38. From 9050 take 1643.
39. From 10000 take 989.
40. From 34207 take 23728.
41. From 94360 take 85692.
42. From 2034325 take 586437.
43. From $560.20 take $483.76.
44. From $409 take $48.35.
45. From $3243 take $1819.80.
46. From $615.35 take $587.

Perform the following indicated operations:

47. 8071 — 6495.
48. $430.25 — $64.37.
49. 6387 — 5809.
50. $8040.35 — $3286.
51. 3705 — 1837.
52. $5721. — $4348.25.
53. 6003 — 3002.
54. $9007. — $5008.75.

Find the difference:

55. Of 4210 and 3184.
56. Of $241.35 and $85.
57. Of 56371 and 45625.
58. Of $100. and $74.65.

59. How many years from the discovery of America in 1492 to the Declaration in 1776 ?

60. How many years from the settlement of St. Augustine in 1565 to the settlement of New York in 1613 ?

61. Washington died in the year 1799, aged 67 years. In what year was he born ?

62. Henry Clay died in the year 1852, at the age of 75. In what year was he born?

63. A horse cost $154 and a cow $56.25 ; how much more did the horse cost than the cow?

64. A man paid \$10246 for a farm and sold it for \$793.50 less than cost. How much did he receive for it?

65. A man lacks \$864 of having \$2013.20; how much has he?

66. If you start from New Orleans and travel east 521 miles, and then west 276 miles, how far will you be from New Orleans?

67. What number is that, to which if 6956 be added, the sum will be one million?

68. How many times can 327 be subtracted from 1308?

69. If 429 be subtracted from 3500 eight times, what will be the last remainder.

70. If 273 be subtracted, first from 3000, then from the remainder, and so on until the remainder is less than 273, what will the final remainder be?

71. A merchant gained \$2035.15 by selling a lot of goods for \$10317; how much did they cost?

72. The highest mountain in the world is Mt. Everest, in Asia, 29002 feet high; the highest in America is Mt. St. Elias, 17900 feet; how much higher is Mt. Everest?

ADDITION AND SUBTRACTION.

57. When several numbers are connected by the signs $+$ and $-$, the indicated operations may be performed in the order of the signs, commencing at the left (see also Art. 89).

(1) To find the value of $6 + 7 - 4 + 5$, add 6 and 7, subtract 4 from the sum, and add 5 to the remainder, which gives 14.

58. The parenthesis ( ), is used to include several numbers to be treated as one.

(1) $(7 + 5) - (6 + 3)$ denotes the difference between the sum of 7 and 5 and the sum of 6 and 3. That is, $(7 + 5) - (6 + 3) = 12 - 9 = 3$.

EXERCISE VI.

Note.—Oral or mental exercises are designated by the kind of type used in numbering them, as 1, 3, 5, etc. See Art. 18, Note.

Find the values of:

1. 16 — 5 — 4.	**2.** 2063 — 874 — 629.
3. 14 — 9 + 6.	**4.** 3027 — 2599 + 1826.
5. 12 — (18 — 11).	**6.** 2411 — (2700 — 509).
7. 16 — 7 — 5 + 2.	**8.** 381 — 37 — 186 + 153.
9. 13 + 6 — 7 — 8.	**10.** 423 + 84 — 97 — 128.
11. 25 — 6 — 4 — 11.	**12.** 584 — 125 — 200 — 86.
13. 10 — 4 + 7 — 8.	**14.** 407 — 98 + 23 — 163.
15. 17 — (11 — 7) + 5.	**16.** 542 — (640 — 185) + 78.
17. 16 — 8 — (6 — 1).	**18.** 361 — 93 — (214 — 67).
19. 20 — (12 — 10 + 3).	**20.** 564 — (473 — 86 + 47).
21. 5 + (13 — 6 + 2).	**22.** 91 + (387 — 258 + 63).
23. (15 + 4) — (15 — 4).	**24.** (317 + 169) — (317 — 169).

25. John had 40 cents, of which he spent 12 cents for candy and 15 cents for apples. How much had he left?

26. A man owed $5463; at one time he paid $1864.40, and at another time $1697.35; how much did he then owe?

27. A lady purchased a hat for $12 and a shawl for $15; how much change should she get from a fifty-dollar bill?

28. A farmer took in exchange for a horse worth $175, a wagon at $60.30, and a cow at $45.55, and the balance in cash. How much cash did he receive?

29. James, John and Henry have respectively 20, 30 and 40 apples; how many will each have if James give John 12, John give Henry 16 and Henry give James 24?

30. In three fields there are, respectively, 637, 421 and 525 sheep. How many will be left in each field if 168 sheep go

from the first field into the second, 219 from the second into the third, and 98 from the third into the first?

31. If William had $55 more than he has he could buy a pony worth $75 and have $15 left. How much money has he?

32. If the distance around the moon were 23214 miles greater than it is, it would be 5101 miles greater than the distance around the world, which is 24899 miles. What is the distance around the moon?

33. Which of the numbers, 27 and 42, is more nearly equal to 35?

34. Which of the numbers, 36279 and 52812, is more nearly equal to 44613?

35. A boy bought a pony for $53; at one time he paid $12, at another $13, and at another $15. How much did he then owe?

36. A man died and left property to the amount of $27563, of which his wife received $9188, each of his two daughters $4594, and his son the balance. What did the son receive?

37. A boy bought a lot of turkeys for $23, and paid $3.50 to have them carried to market; for how much must he sell them to gain $7.50?

38. A merchant bought a store for $11875 and merchandise for $17835; after selling goods to the amount of $8568, for how much must he sell the store and remnant of goods so as to gain $2833?

QUESTIONS.

What is subtraction? The minuend? The subtrahend? The remainder? Sign of subtraction? What called? For what is the parenthesis used? What kind of numbers can be subtracted? Give the rule? How is subtraction proved?

MULTIPLICATION.

59. (1) One horse has 4 feet; how many feet have 3 horses?

I write 4 feet as many times as there are horses, and add them together. This is *adding equal numbers*.	4 feet 4 feet 4 feet 12 feet
Instead of writing 4 feet *three* times, I write it *once*; then under it place 3, the *number* of *times* it is to be taken, and say, 3 *times* 4 feet are 12 feet. This is *multiplication*.	4 feet 3 12 feet

60. **Multiplication** is a short method of adding equal numbers; or the process of taking one number as many times as there are units in another.

61. The number to be taken is called the **multiplicand**; the number that shows how many times it is to be taken, the **multiplier**; and the result, the **product**.

The *multiplicand* and *multiplier* are called the **factors** of the product.

62. The **sign of multiplication** is $\times$, which is read: *times, or multiplied by*. When $\times$ stands between two numbers, it indicates that one of them is to be multiplied by the other.

63. PRINCIPLES.—1°. *The multiplicand and product are like numbers.*

(1) 7 days $\times$ 5 $=$ 35 days; here 7 days and 35 days are like numbers.

2°. *One concrete number cannot be multiplied by another concrete number.*

(1) 3 days cannot be multiplied by 5 yards, nor 5 cents by 4 cents.

3°. *The multiplier is always an abstract number.*

4°. *The product is the same in whatever order the factors are taken.*

$$4 \times 5 \times 6 \times 2 = 20 \times 6 \times 2 = 120 \times 2 = 240.$$
$$4 \times 5 \times 6 \times 2 = 4 \times 5 \times 12 = 4 \times 60 = 240.$$

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Oral Drill in Matter and Method.

64. Multiply the odd numbers from 1 to 11:

1. By 2. **4.** By 8. **7.** By 4. **10.** By 7.

2. By 5. **5.** By 10. **8.** By 11. **11.** By 3.

3. By 1. **6.** By 6. **9.** By 9. **12.** By 12.

Multiply the even numbers from 2 to 12:

13. By 3.	**16.** By 9.	**19.** By 2.	**22.** By 6.
14. By 7.	**17.** By 4.	**20.** By 5.	**23.** By 0.
15. By 10.	**18.** By 11.	**21.** By 12.	**24.** By 8.

25. If one saddle cost $9, how much will 5 saddles cost?

Solution: Five saddles will cost 5 times as much as 1 saddle, and 5 times $9 are $45.

26. If 8 bushels of apples will make a barrel of cider, how many bushels will make 7 barrels?

27. What cost 7 dozen eggs, at 12 cents a dozen?

28. If a boy walk 11 miles a day, how far will he walk in 6 days?

29. If there are 5 school days in a week, how many school days are there in 12 weeks?

30. If 4 men can do a certain work in 5 days, how long will it take 1 man to do the work?

Solution: It will take one man 4 times as long to do the work as 4 men, and 4 times 5 days are 20 days.

31. If 9 men can build a wall in 8 days, how long will it take 1 man to build it?

32. If a quantity of corn supply 7 sheep for 6 days, how long will it supply 1 sheep?

65. ***To find the product of two integers when the multiplier does not exceed 12.***

(1) Multiply 534 by 6.

Operation.

$$\begin{array}{r} 534 \\ 6 \\ \hline 3204 \end{array}$$

EXPLANATION.—Beginning at the right, I say, 6 times 4 ones are 24 ones (= 2 tens 4 ones); write the 4 in the unit's place and carry the 2 tens. Then, 6 times 3 tens are 18 tens, and 2 tens are 20 tens (2 hundreds, 0 tens); write the 0 in the ten's place and carry the 2 hundreds. Finally, 6 times 5 hundreds are 30 hundreds, and 2 hundreds are 32 hundreds, which I write beneath.

EXERCISE VII.

Find the following products:

2. 605 × 5.	3. 986 × 7.	4. 58765 × 8.
5. 487 × 6.	6. 4576 × 4.	7. 64382 × 7.
8. 325 × 9.	9. 7586 × 8.	10. 98374 × 6.
11. 792 × 8.	12. 3869 × 9.	13. 837941 × 9.
14. 647 × 6.	15. 8765 × 5.	16. 407649 × 11.

17. A train of 8 cars has 94 passengers in each car; how many passengers are on the train?

18. If 284 pounds of cheese are made from the milk of one cow, how many pounds can be made from the milk of 7 cows?

19. What is the cost of 476 sheep at $4 a head?

20. How many pounds of cotton can 9 men pick in 1 day, if each pick on an average 238 pounds?

66. ***To find the product of two integers when the multiplier is greater than 12.***

(21) Multiply 637 by 425.

Operation.

$$\begin{array}{r} 637 \\ \underline{425} \\ 3185 \\ 1274 \\ \underline{2548} \\ 270725 \end{array}$$

Explanation.—I first multiply by 5 (units), and place the first figure of the product, 3185, under the 5 (units). Next, I multiply by 2 (tens), and place the first figure of the product, 1274, under the 2 (tens). Lastly, I multiply by 4 (hundreds), and place the first figure of the product, 2548, under the 4 (hundreds). I then add these several products, called partial products, for the entire product.

ANALYSIS.

$$\begin{array}{rcl} 5 \text{ ones} \times 637 &=& 3185 \text{ ones} = 3185 \\ 2 \text{ tens} \times 637 &=& 1274 \text{ tens} = 12740 \\ \underline{4 \text{ hundreds}} \times 637 &=& 2548 \text{ hundreds} = \underline{254800} \\ 4 \text{ hundreds, } 2 \text{ tens } 5 \text{ ones} \times 637 &=& 270725 \end{array}$$

RULE I. Write the multiplier under the multiplicand.

II. *Begin at the right, multiply the multiplicand by the ones, tens, hundreds, etc., of the multiplier, placing the right hand figure of each partial product under the figure of the multiplier used to obtain it, and add the partial products.*

PROOF.—*Multiply the multiplier by the multiplicand, and if the product is the same as before, the work is probably correct.*

Multiply and prove:

22. 84 by 15.	**23.** 240 by 36.	**24.** 1375 by 124.
25. 68 by 24.	**26.** 308 by 47.	**27.** 4608 by 225.
28. 92 by 68.	**29.** 575 by 91.	**30.** 3089 by 504.
31. 79 by 37.	**32.** 999 by 60.	**33.** 6437 by 873.

Find the following products:

34. 365 × 73.	**35.** 3812345 × 31243.
36. 569 × 37.	**37.** 67853 × 8765.
38. 169 × 144.	**39.** 987648 × 481007.
40. 376 × 13.	**41.** 7060504 × 30204.
42. 1128 × 53.	**43.** 204265 × 562402.

Find the cost:

44. Of 85 watches, at $64 each.

45. Of 318 turkeys, at 87 cents each.

46. Of 146 wagons, at $73 each.

47. Of 7198 hats, at $2.16 each.

48. Of 8862 cows, at $18.90 each.

49. Of 7575 mules, at $75.75 each.

50. A milkman sells 219 quarts a day, at $.05 a quart. How much do his sales amount to in a year?

51. A grocer sold 48 hams, each weighing 14 pounds, at $0.16 a pound. What did he receive for all?

52. What will be the cost of 13 bales of cotton, each weighing 452 pounds, at 8 cents a pound?

53. If a wheel revolves 246 times in going a mile, how many times will it revolve in going 235 miles?

54. How much money would it take to supply 573 men with $264 each?

55. Light travels at the rate of 185172 miles per second, and in reaching the earth from the sun requires 493 seconds; how far away is the sun?

SPECIAL METHODS OF MULTIPLICATION.

67. ***To multiply by a number which can be separated into factors.***

(56) Multiply 164 by 24.

Explanation.—The factors of 24 are 6 and 4. I first multiply 164 by 6, and then multiply the product by 4.

Operation.

$$\begin{array}{r} 164 \\ \underline{6} \\ 984 \\ \underline{4} \\ 3936 \end{array}$$

Analysis: The product is $164 \times (6 \times 4)$. Now, as the product is the same in whatever order the factors are taken, I multiply 164 by 6, and that product by 4.

In like manner multiply:

57. 347 by 18, or by 6×3.

58. 518 by 36.

59. 582 by 49, or by 7×7.

60. 675 by 42.

61. 759 by 63, or by $3 \times 3 \times 7$.

62. 852 by 108.

63. If a miller grinds 143 barrels of flour in a day, how many barrels will he grind in 56 days?

64. If one barrel weighs 167 pounds, what is the weight of 35 barrels?

65. If 37 men can build a bridge in 45 days, how many days will it take one man to build it?

68. ***To multiply by 10, 100, 1000, etc.***

(66) Multiply 237 by 100.

Explanation.—I annex as many ciphers to the multiplicand as there are ciphers in the multiplier.

Operation
23700

Analysis: 237 × 100 = 237 hundreds = 23700.

Find the following products:

67. 47 × 100. **68.** 863 × 1000.

69. 504 × 10. **70.** 9025 × 10000.

71. There are 100 cigars in a box; how many are there in 13 boxes?

72. One cubic foot of water weighs 1000 ounces; what is the weight of 137 cubic feet?

73. Ten mills make a cent, and 100 cents make a dollar; how many mills are equal to $17.

69. ***To multiply when ciphers are on the right in one or both factors.***

(74) Multiply 7600 by 30.

Operation.

7600
30
228000

Explanation.—I multiply as if there were no ciphers on the right in the numbers; then annex to the product as many ciphers as there are on the right in both factors.

ANALYSIS: 7600 × 30 = 76 hundreds × 3 tens = 228 thousands = 228000.

75. Multiply 2100 by 400.

76. Multiply 3650 by 1200.

77. Multiply 402000 by 350000.

78. The salary of the President of the United States is $50,000 a year; how much will it amount to in 120 years?

79. Sound travels about 1140 feet per second; how far does it travel in 5 minutes?

80. If a man make 2100 steps in walking a mile, how many steps will he make in *walking* a distance of 24900 miles?

70. ***To multiply by an integer which is a little less than 10, 100, 1000, etc.***

Multiply 376 by 97.

Explanation.—97 is 3 less than 100. I first multiply 376 by 100, which is done by annexing two ciphers. In multiplying 376 by 100, I take it 3 times two much; hence I multiply 376 by 3 and subtract the product, 1128, from 37600 to obtain the true result.	**Operation.** 37600 1128 36472

Multiply:

82. 377 by 9. **83.** 4375 by 997.

84. 593 by 98. **85.** 64013 by 9999.

86. If an engine travel at an average speed of 34 miles an hour, how far can it travel in 99 hours?

87. 24 sheets of paper are a quire, and 20 quires are a ream. How many sheets are there in 998 reams?

88. Find the value of 16642 $\times$ 996 $\times$ 99.

71. ***To multiply two numbers when one part taken as units, in the multiplier, is a factor of another part so taken.***

(89) Multiply 570372 by 120324.

Explanation.—In the multiplier I select a part, expressed by one or more figures, which is a factor of another part, represented by one or more figures. Thus, I select 3, 24 and 120, since 8 $\times$ 3 = 24, and 5 $\times$ 24 = 120.	**Operation.** 570372 120324 1711116 13688928 68444640 68629440528

I now multiply 570372 by 3, and obtain the first partial product, 1711116; then multiply this product by 8, (= 570372 $\times$ 24), writing this right hand figure under the 4. Next, I multiply the second partial product (13688928), by 5, (= 570372 $\times$ 120), and write the right hand figure of this product under the 0 of the multiplier; and, finally, add to obtain the total product.

Find the following products:

90. 8862 × 189. **91.** 80900 × 5040.

92. 657 × 408. **93.** 267388 × 14982.

94. 9008 × 784. **95.** 102735 × 273162.

72. ***To multiply by an integer more than 10 and less than 20.***

(96) Multiply 7638 by 16.

Explanation.—I consider a cipher to be placed before the multiplicand; then multiply by the unit's figure of the multiplier, and in addition to carrying as usual, I also carry the figure last multiplied.

Operation.

$$\begin{array}{r} 07638 \\ \hline 122208 \end{array} \quad \text{Ans.}$$

Thus: 6 × 8 = 48, write 8 and carry 4 and 8, or 12; 6 × 3 = 18, and 12 = 30, write 0 and carry 3 and 3, or 6; 6 × 6 = 36, and 6 are 42, write 2 and carry 4 and 6, or 10; 6 × 7 = 42, and 10 = 52, write 2 and carry 5 and 7, or 12; 6 × 0 = 0, and 12 = 12.

Find the following products:

97. 456 × 16. **98.** 5086 × 17.

99. 345 × 15. **100.** 7348 × 13.

101. A man buys 14 horses, for which he pays on an average $175 each; how much do they all cost?

102. A farmer plants on each of 19 acres 8945 hills of corn; how many does he plant in all?

For other contractions, see Appendix.

QUESTIONS.

What is multiplication? The multiplicand? The multiplier? The product? Sign of multiplication? Factors? Can a concrete number be multiplied by a concrete number? Give the general rule? How is multiplication proved?

How do you multiply when the multiplier is some power of 10? When it has 0s on the right? When it is a little less than some power of 10? When it is between 12 and 20?

DIVISION.

73. (1) I have 12 apples which I wish to divide equally among 4 boys; how many apples can I give to each?

If I give each boy one apple, it will require 4 apples, and 8 apples will be left. If I give each of them another apple, 4 apples will be left. Again, if I give each of them an apple a third time, there will be none left. Hence, I can give each boy one apple three times, or one apple taken as many times as 4 is contained in 12, which is three apples.

$$\begin{array}{r} 12 \\ \underline{4} \\ 8 \\ \underline{4} \\ 4 \\ \underline{4} \\ 0 \end{array}$$

Again, since 12 apples are to be divided into 4 equal parts, each part will be *one-fourth* of 12 apples, which is 3 apples. This is Division.

$$\begin{array}{r|l} 4 & \underline{12} \\ & 3 \end{array}$$

74. **Division** is a short method of finding how many times one number is contained in another of the same kind; or the process of finding one of the equal parts of a number.

75. The number to be divided is the **dividend**; the number by which it is divided, the **divisor**; and the result, the **quotient.**

76. When a part of the dividend is left after the division is performed, it is the **remainder,** and must always be less than the divisor. When there is no remainder, the division is said to be exact.

77. The **sign of division** is ÷, which is read *divided by*, or *contains*, and indicates that the number *before* it is to be divided by the number after it.

NOTES.—1. When the dividend is a concrete number and the divisor an abstract number, ÷ should be read *divided by*.

(1) 24 days ÷ 6 = 4 days, is read, 24 days divided by 6 equals 4 days.

2. When the dividend and divisor are both concrete numbers, ÷ should be read *contains;* for, really, there is no such thing as dividing by a concrete number, except as a *measure*.

(2) 16 pints ÷ 8 pints = 2, is read 16 pints contains 8 pints 2 times. In such cases it is better to use the colon (:). See Ratio.

3. When the dividend and divisor are both abstract numbers, ÷ may be read *divided by*, or *contains*.

4. Division is also denoted by writing the divisor under the dividend, with a line between them, or by placing the divisor on the left of the dividend, with a curved line between them.

5. Division is the reverse of multiplication; the latter *unites* equal parts into one number, and the former *separates* a number into *equal parts*.

78. PRINCIPLES.—1°. *When the dividend and divisor are like numbers, the quotient is an abstract number.*

2°. *When the divisor is an abstract number, the dividend and quotient are like numbers.*

3°. *The dividend is equal to the product of the divisor and quotient, plus the remainder.*

Oral Drill in Matter and Method.

79. Name, in order, the numbers exactly divisible

1. By 2 from 2 to 20.
2. By 6 from 6 to 60.
3. By 3 from 3 to 30.
4. By 7 from 7 to 70.
5. By 4 from 4 to 40.
6. By 8 from 8 to 80.
7. By 5 from 5 to 50.
8. By 9 from 9 to 90.

1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.
0	12	26	34	48	53	67	71	89
5	10	22	36	44	58	63	77	81
9	15	20	32	46	54	68	73	87
1	19	25	30	42	56	64	78	83
7	11	29	35	40	52	66	74	88
3	17	21	39	45	50	62	76	84
8	13	27	31	49	55	60	72	86
4	18	23	37	41	59	65	70	82
6	14	28	33	47	51	69	75	80
2	16	24	38	43	57	61	79	85

(9) Divide the numbers in the first and second columns by 2, giving the quotients and remainders.

2 in 0, 0 quo., 0 rem.; 2 in 12, 6 quo., 0 rem.; 2 in 5, 2 quo., 1 rem.; 2 in 10, 5 quo., 0 rem., etc., etc.

In a similar manner divide the numbers:

10. In the 1st, 2d, and 3d columns, by 3.

11. In the 1st to 4th columns, by 4.

12. In the 1st to 5th columns, by 5.

13. In the 1st to 6th columns, by 6.

14. Proceed in a similar manner with 7, 8 and 9 as divisors.

80. Equal parts. When a number is divided: Into *two* equal parts, one of the parts is *one-half* of the number;

Into *three* equal parts, one of the parts is *one-third* of the number;

Into *four* equal parts, one of the parts is *one-fourth* of the number;

Into *five* equal parts, one of the parts is *one-fifth* of the number, and so on.

The parts one-half, one-third, one-fourth, one-fifth, etc., are written: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, etc.

To find 1-*half* of a number: Divide the number by 2.

To find 1-*third* of a number: Divide the number by 3.

To find 1-*fourth* of a number: Divide the number by 4.

To find 1-*fifth* of a number: Divide the number by 5.

(15) If 5 hats cost $20, how much will 1 hat cost?

Solution: 1 hat costs 1-fifth as much as 5 hats, and 1-fifth of $20 is $4.

16. I paid $8 for 4 barrels of potatoes; what was the price per barrel?

17. If a horse travel 18 miles in 3 hours, what is his rate per hour?

18. If I divide 40 nuts equally among 8 children, how many nuts shall I give to each child?

19. At 5 cents a piece, how many oranges can I buy for 40 cents?

Solution: At 5 cents a piece I can buy as many oranges for 40 cents as the times 5 cents are contained in 40 cents, which is 8. Or,

At 1 cent a piece, for 40 cents I can buy 40 oranges, and at 5 cents a piece I can buy 1-fifth of 40 oranges, which is 8 oranges.

20. How many days will 63 pounds of flour last a family that uses 7 pounds a day?

21. If I drive my horse 6 miles an hour, how many hours will it take me to drive him 48 miles?

22. A teacher having 54 pupils, formed them into classes of 9 each, how many classes did she have?

23. If 1 man can do a work in 48 hours, how long will it take 6 men to do the work?

Solution: Six men can do the work in 1-sixth of the time it takes 1 man to do it, and 1-sixth of 48 hours is 8 hours.

24. If 1 person consumes a barrel of flour in 42 weeks, how long will it supply a family of 7 persons?

25. In how many days can 8 horses do as much work as 1 horse in 56 days?

81. There are two methods of division in general use, viz.:

Short division is a process in which the result only is written, the work being performed in the mind.

Long division is a process in which the work is written more or less *in full.*

82. ***To perform short division.***

(1) Divide 13628 by 4.

Operation.

$$\begin{array}{r|l} 4 & \underline{13628} \\ & \ \ 3407 \end{array}$$

Explanation.—I write the numbers as in the margin; then say, 4 in 13, 3 times and 1 remainder; write the 3 below and prefix the 1 to 6, making 16. Then I say, 4 in 16, 4 times; write the 4 below.

Next, I say, 4 in 2, 0 times and 2 remainder; write the 0 below and prefix the 2 to 8, making 28.

Finally, I say, 4 in 28, 7 times, which I write below. The quotient is 3407.

Analysis: 13 thous. 6 hunds. 2 tens 8 ones =
12 thous. 16 hunds. 0 tens 28 ones.
Divide by 4, 3 thous. 4 hunds. 0 tens 7 ones = 3407.

NOTE.—When there is a final remainder, it must be written over the divisor and annexed to the quotient.

EXERCISE VIII.

Divide:

2. 3754 by 3.	**3.** 8102 by 5.	**4.** 14678 by 6.
5. 7872 by 2.	**6.** 3402 by 7.	**7.** 75613 by 7.
8. 5508 by 4.	**9.** 50024 by 8.	**10** 95328 by 8.
11. 3730 by 5.	**12.** 92167 by 3.	**13.** 18903 by 9.

14. $2643 \div 3 = ?$ **15.** $\frac{4572}{7} = ?$

16. $6235 \div 5 = ?$ **17.** $\frac{5878}{8} = ?$

18 How many yards of cloth will it take, at 5 cents a yard, to amount to 37290 cents?

19. A merchant spent $33229 for hats, paying on an average $7 apiece; how many hats did he buy?

20. A farmer received $2752 for a lot of land which he sold at the rate of $8 per acre; how many acres did he sell?

21. The product of two numbers is 22575, and one of the numbers is 7; what is the other number?

22. A factory worth $64192 is owned in equal shares by 8 men. What is the value of a share?

23. Five boys gathered 14375 chestnuts, which they shared equally; how many had each boy?

24. When rice is 7 cents a pound, how many pounds can be bought for 2653 cents?

25. A man owed a debt of \$5220; if he pay $\frac{1}{3}$ of it, then $\frac{1}{4}$ of it, then $\frac{1}{5}$ of it, and then $\frac{1}{12}$ of it, how much would he then owe?

83. *To perform long division.*

(26) Divide 151353 by 24.

Operation.

```
24)151353(6306 9/24
   144
   ---
     73
     72
     ---
      153
      144
      ---
        9
```

Explanation.—Since the least number of left-hand figures that will contain the divisor is three, I take 151 for the first partial dividend.

Now I say, 24 in 151, 6 times with a remainder. I place the 6 on the right, multiply it by the divisor (24), subtract the product (144) from 151, and to the remainder (7) annex the next figure of the dividend (3) for the second partial dividend.

I now say, 24 in 73, 3 times with a remainder. I place the 3 on the right, multiply it by the divisor (24), subtract the product (72) from 73, and to the remainder (1) annex the next figure of the dividend (5) for a third partial dividend.

I next say, 24 in 15, 0 times with a remainder. I place the 0 on the right, and annex the next figure of the dividend (3) to the remainder (15) for a fourth partial dividend.

Finally, I say, 24 in 153, 6 times with a remainder. I place the 6 on the right, and, proceeding as before, I obtain a final remainder of 9, which I place over the divisor, and annex to the quotient.

Analysis: 151 thous. 3 hunds. 5 tens 3 ones =
144 thous. 73 hunds. 5 tens 3 ones =
144 thous. 72 hunds. 15 tens 3 ones =
144 thous. 72 hunds. 0 tens 153 ones.
Divide by 24, 6 thous. 4 hunds. 0 tens $6\frac{9}{24}$ ones = $6406\frac{9}{24}$.

RULE I. *Write the divisor on the left of the dividend, with a line between them, and draw a line on the right.*

II. *Find how many times the divisor is contained in the least number of the left hand figures of the dividend that will contain it, and place the quotient on the right.*

III. *Multiply the divisor by this quotient figure, subtract the product from the figures of the dividend used, and to the remainder annex the next figure of the dividend.*

IV. *Divide as before, and continue the operation until all the figures of the dividend have been brought down.*

V. *When one of the partial dividends is less than the divisor, write* 0 *for the next figure of the quotient, and bring down the next figure of the dividend.*

PROOF.—*Multiply the quotient by the divisor, and to the product add the remainder, if any; the result should be equal to the dividend.*

Divide:

27. 875 by 25.

28. 37728 by 48.

29. 5481 by 63.

30. 268056 by 657.

31. 10875 by 29.

32. 2879253 by 5679.

33. 34905 by 65.

34. 7062272 by 784.

35. 29495 by 85.

36. 457216 by 608.

37. Divide 534195 by 27.

38. Divide 48395046 by 49.

39. Divide 7967848 by 52.

40. Divide 1167861 by 135.

41. Divide 87635163 by 387.

42. Divide 300521761 by 225.

43. Divide 245379633477 by 1263.

44. What number multiplied by 4502 will produce 4326422?

45. A farmer raises 1786 bushels of corn on 47 acres of land; find the yield per acre.

46. It is 24899 miles around the world; how long would it take a man to make the trip traveling at the average daily rate of 39 miles?

47. The president of the United States receives $50000 annual salary; how much is that per day, counting 365 days in the year?

48. An army contractor paid $39865 for 2345 barrels of beef; how much did the beef cost him per barrel?

49. How many kettles, each weighing 348 pounds can be made from 20000 pounds of iron?

50. The earth moves around the sun at the rate of about 68000 miles per hour; what is its rate per minute?

51. How many bales will 281765 pounds of cotton make, allowing 517 pounds to the bale?

52. The Bible contains 31173 verses; how many must be read each day, that the book may be read through in a year of 365 days?

53. The annual cost of keeping a plank road in repair is $9264.92, at the rate of $28.42 per mile; how many miles long is the road?

54. If a pipe discharges 67 gallons in a minute, in what time will it empty a vat of 5484888 gallons?

55. How many hours will it take a person to count $212,492,745 at the rate of $1035 per hour.

SPECIAL METHODS OF DIVISION.

84. Principle.—*Multiplying or dividing both dividend and divisor by the same integer, does not change the quotient.* See Art. 124.

85. ***To divide by a divisor which can be separated into factors.***

56. Divide 315 by 35.

Explanation.—The factors of 35 are 5 and 7. I divide the dividend by one of the factors of the divisor, then divide the quotient by the other factor.

Operation.

$$\begin{array}{r|l} 5 & 315 \\ \hline 7 & 63 \\ \hline & 9 \text{ Ans.} \end{array}$$

Analysis: Dividing both dividend and divisor by the same number does not change the quotient. Hence,

$$315 \div 35 = \underline{5|315 \div 35} = 63 \div 7 = 9.$$

57. $5184 \div 36 = ?$ **58.** $9765 \div 45 = ?$

59. $9792 \div 24 = ?$ **60.** $18144 \div 81 = ?$

(61) Divide 893 by 30, using the factors 2, 3 and 5.

Explanation.—Dividing by 2, I obtain 446 *twos* and 1 remainder; dividing 446 *twos* by 3, I have 148 *sixes* and 2 *twos*, or 4, remainder; again dividing 148 *sixes* by 5, I have 29 *thirties* and 3 *sixes*, or 18, remainder; hence, 18 + 4 + 1, or 23, is the true remainder.

Operation.

$$\begin{array}{r|lr} 2 & 893 & \\ \hline 3 & 446 & 1 \\ \hline 5 & 148, \quad 2 \textit{ twos} = & 4 \\ \hline & 29, \quad 3 \textit{ sixes} = & 18 \\ & \text{True remainder,} & \overline{23} \end{array}$$

RULE.—*Find the product of each remainder by all the divisors preceding the one that produced it, and the sum of the products, with the first remainder, if any, will be the true remainder.*

62. $4642 \div 63 = ?$ **63.** $6573 \div 32 = ?$

64. $3971 \div 108 = ?$ **65.** $9364 \div 75 = ?$

86. *To divide by 10, 100, 1000, etc.*

(66) Divide 576 by 10.

Explanation.—I cut off as many figures from the right of the dividend as there are 0s in the divisor, for the remainder (6); the remaining figures form the quotient (57).

Operation.

57,6

Analysis: 576 = 5 hunds. 7 tens 6 ones.

ten|5 hunds. 7 tens 6 ones.

5 tens 7, and 6 ones remainder.

67. 4735 ÷ 100 = ?   **68.** 89123 ÷ 10000 = ?

69. 6887 ÷ 1000 = ?   **70.** 87034 ÷ 1000 = ?

87. ***To divide by any multiple of 10, 100, 1000, etc.***

(71) Divide 3943 by 40.

Explanation.—I cut off the ciphers at the right of the divisor, and as many figures from the right of the dividend. I then divide the remaining part of the dividend (394) by the remaining part of the divisor (4), and annex to the remainder (2) the figures cut off (3), and thus obtain the true remainder (23).

Operation.

$$\begin{array}{l|l} 4{,}0 & 394{,}3 \\ \hline & 98 \text{ quo. } 23 \text{ rem.} \end{array}$$

Analysis: $40 = 4 \times 10$, $3943 \div 10 = 394$ quo. 3 rem.
Now dividing by 4 (Art. 85), we get 98 quo. 23 rem.

Divide:

72. 576 by 80.   **73.** 71831 by 6400.

74. 63242 by 3500.   **75.** 893741 by 17000.

76. If 40 barrels of molasses cost $480, what is the price of 1 barrel?

77. A farmer sold 600 acres of land for $7800; how much was that per acre?

78. A merchant sold 800 yards of silk for 184,000 cents; how much was that per yard?

88. ***To divide by a divisor, which, by a simple multiplication, can be changed into a number with 0s on the right.***

(79) Divide 1341 by 225.

Explanation. — I multiply both dividend and divisor by 4, which does not change the quotient; I then divide the new dividend (5364) by the new divisor (900), by Art. 87, and obtain 5 quo. and 864 rem.

Operation.

$$\begin{array}{l|l} 225 & 1341 \\ \hline 9{,}00 & 53{,}64 \\ \hline & 5, \quad 864 \text{ rem.} \end{array}$$

$$\begin{array}{l|l} 4 & 864 \\ \hline & 216 \text{ true rem.} \end{array}$$

The remainder being a part of the dividend has been made too large by the multiplication by 4, hence I divide it by the multiplier (4), to get the true remainder *(216)*.

NOTE.—Multiplying a divisor, which ends:

With 5, by 2, will give one 0 on the right;

With 25 or 75, by 4, will give two 0s on the right.

With 125, 375, 625 or 875, by 8, will give three 0s on the right.

A divisor, whose last significant figure is an even number, may be multiplied by 5

Find the following quotients:

80. 379 ÷ 5.	**81.** 4125)17213(.
82. 373 ÷ 25.	**83.** 5625)431576(.
84. 917 ÷ 75.	**85.** 9375)561007(.
86. 824 ÷ 125.	**87.** 13875)5683201(.
88. 731 ÷ 16.	**89.** 320)8702003(.

90. At 75 cents each, how many books can I buy for $14.25?

91. A lady bought a sewing machine for $36, and paid for it in monthly payments of $2.25 each. In how many months did she pay the debt?

92. A sugar planter packed 31500 pounds of sugar in hogsheads, putting 1125 pounds in each. How many hogsheads did he fill?

93. If the directors of a railroad company appropriate $402875 for the purchase of passenger cars, at $2750 each, how many cars can be bought with the appropriation?

QUESTIONS.

What is division? The dividend? The divisor? The quotient? Sign of division? Give the general rule. How is division proved? How do you divide when the divisor can be separated into factors? When the divisor is 10, 100, etc.? When the divisor has 0s on the right? Mention the different ways of indicating division? What relation does division sustain to multiplication?

ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION.

Relation of the Four Signs.

89. ***To combine numbers connected by the signs + and —.***

The signs + and — denote opposite operations, and when several numbers are connected by these signs, as

$$24 - 6 - 12 - 3 + 18,$$

the indicated operations may be performed in any order whatever, provided that any two of the numbers be added when their signs are alike (both + or both —), subtracted when their signs are unlike (one + and the other —), and the sign of the greater, in each case, prefixed to the result.

In this case, where no sign is written, + is understood.

The above expression may be simplified thus:

1°. $+24 - 6 = +18, -12 = +6, -3 = +3, +18 = +21 = 21.$

2°. $+18 - 3 = +15, -12 = +3, -6 = -3, +24 = +21 = 21.$

3°. $+24 + 18 = +42;\ -6 - 12 - 3 = -21;\ +42 - 21 = 21.$

FORMULA: $8 - 6 + 9 - 4 = 8 + 9 - 6 - 4 = (8 + 9) - (6 + 4).$

A parenthesis, preceded by the sign —, may be removed by changing the signs (+ to — and — to +) of all the terms within it. Conversely, when — is placed before a parenthesis, the signs of all the enclosed terms must be changed.

EXERCISE IX.

Find the value of:

1. $9 + 3 - 6 + 4.$	2. $18 + 9 - (6 + 8).$
3. $12 - 14 + 3 + 2.$	4. $12 - 18 + 9 - 2.$
5. $15 - (6 + 7 - 12).$	6. $-10 - 4 + 12 + 6.$
7. $18 - 6 + 7 - 2.$	8. $-(12 - 2) + (15 - 3).$

9. $24 - (12 - 4) + (16 - 10 - 3) - (17 - 12 - 13 + 2).$

90. ***To combine numbers connected by the signs × and ÷.***

The signs × and ÷ also indicate opposite operations. Hence, when several numbers are connected by these signs, as

$$24 \div 6 \div 12 \div 3 \times 18,$$

the indicated operations may be performed in any order whatever, provided that any two of the numbers be *multiplied* when their signs are alike (both $\times$ or both $\div$), *divided* when their signs are unlike (one $\times$ and the other $\div$), and the sign of the greater, in each case, prefixed to the result.

In this case, where no sign is written, $\times$ is understood.

The preceding expression may be simplified thus:

1°. $\times 24 \div 6 = \times 4, \div 12 = \div 3, \div 3 = \div 9, \times 18 = \times 2 = 2.$
2°. $\times 18 \div 3 = \times 6, \div 12 = \div 2, \div 6 = \div 12, \times 24 = \times 2 = 2.$
3°. $\times 24 \times 18 = \times 432; \div 6 \div 12 \div 3 = \div 216; \times 432 \div 216 = \times 2 = 2.$

FORMULA: $24 \div 6 \times 8 \div 4 = 24 \times 8 \div 6 \div 4 = (24 \times 8) \div (6 \times 4).$

A parenthesis, preceded by the sign $\div$, may be removed by changing the signs ($\times$ to $\div$ and $\div$ to $\times$) of all the terms within it. Conversely, when $\div$ is placed before a parenthesis the signs of all the enclosed terms must be changed.

NOTE.—Let it be observed that this process differs from that of Art. 89, only in these particulars, viz.: $\times$ takes the place of $+$; *multiply*, of *add*; $\div$, of $-$; *divide*, of *subtract*. That is, the *principle* of combination is the same, which it evidently ought to be, since $\times$ and $\div$ express reverse operations, just as $+$ and $-$ do.

In short, every expression like $8 + 5 - 6 - 3$ is an algebraic sum; and every expression like $8 \times 5 \div 6 \div 3$ is an algebraic product. See the application of this principle to fractions, Arts. 131 to 134.

Simplify:

10. $18 \div 6 \times 10 \div 5.$
11. $10 \div 30 \times 3 \times 2.$
12. $36 \div 3 \div 4 \times 2.$
13. $20 \div 40 \div 30 \times 60.$
14. $45 \div 9 \times 3 \div 6.$
15. $120 \div 5 \div (3 \div 2).$
16. $\div 12 \times 4 \times 16 \times 3.$
17. $16 \div (4 \times 3) \times 3.$
18. $\div (12 \times 4) \times 16 \times 3.$
19. $16 \div 4 \times 3 \times 3.$
20. $(3 \div 4) \times (7 \div 5).$
21. $11 \div 3 \times 13 \div 6.$

(22) $(5 \div 2) \div (7 \div 9).$

Solution: $(5 \div 2) \div (7 \div 9) = 5 \div 2 \div 7 \times 9 = (5 \times 9) \div (2 \times 7) = 3\frac{3}{14}$, Ans.

23. $(7 \div 10) \div (3 \div 11).$
24. $(8 \div 5) \div (3 \times 2).$

25. $11 \div (10 \div 9) \times (8 \div 7 \times 6) \div (5 \div 4 \div 3 \times 2.)$

91. ***To combine numbers connected by the signs*** $+$, $-$, $\times$ ***and*** $\div$.

When several numbers are connected by the signs $+$, $-$, $\times$ and $\div$, as

$$8 + 6 \times 2 - 20 \div 4 - 12 \div 6 \times 3,$$

the operations indicated by $\times$ and $\div$ must, in general,* be performed before those denoted by $+$ and $-$, unless the numbers joined by the latter are united by a parenthesis.

Thus, to simplify the above expression, I write 12 in the place of 6×2, 5 in the place of $20 \div 4$, and 6 in the place of $12 \div 6 \times 3$, and obtain

$$8 + 12 - 5 - 6, \text{ which} = 9, \text{ Ans.}$$

Simplify:

26. $8 + 3 \times 5 - 10$.

27. $15 \div 3 \times 2 - 7$.

28. $9 \div 3 + 8 - 5$.

29. $9 + 7 - 6 \times 2$.

30. $10 - 6 \div 3 + 4$.

31. $40 - 10 - 7 \times 4$.

32. $5 \times 6 + 12 \div 3$.

33. $40 \div 10 + 7 \times 4$.

34. $10 - 12 \div 4 \times 2$.

35. $50 - 72 \div 6 \div 4$.

92. ***To find the average of two or more numbers.***

The **average** of two or more numbers is the quotient of their *sum* divided by the *number* of numbers.

The average of *two* numbers is *one-half* of their *sum*; of *three* numbers, *one-third* of their sum, etc.

What is the average:

36. Of 12 and 18?

37. Of 473 and 125?

38. Of 6, 10 and 14?

39. Of 58, 104 and 217?

40. The lengths of 3 poles are 7, 10 and 13 feet; what is their average length?

*See Note under Art. 129.

41. The weights of 3 bales of cotton are 425, 442 and 450 pounds; what is the average of their weights?

A merchant's receipts and expenses for a week were as follows:

Monday,	$375.	receipts,	and $52.75	expenses;
Tuesday,	$400.30	"	and $60.50	"
Wednesday,	$280.	"	and $55.	"
Thursday,	$500.25	"	and $40.35	"
Friday,	$392.15	"	and $65.20	"
Saturday,	$625.45	"	and $70.42	"

42. What was the average of the daily receipts?

43. What was the average of the daily expenses?

44. What was the average of the daily profits?

45. The ages of three boys are 8, 13 and 15 years; what is the average of their ages?

46. A merchant's profits the first year amounted to $1875; the second year to $2315.80; and the third year to $2837.65. What was the average of the yearly profits?

47. A boy sold 6 oranges at 5 cents apiece, and 3 of a better quality at 8 cents each; what average price per orange did he receive?

48. Find the average price of 28 acres of land at $36 an acre, and 35 acres at $27 an acre.

49. There were 50 apples on a tree, of which James took 9, Henry 11 and Charles 13; how many apples were left on the tree?

50. A teacher's salary is $1250, and his expenses are $753. If he buy a lot for $225, and pay $149 for improvements on it, how much of his salary will be left?

51. A newsboy bought papers at 3 cents each, and sold them at 5 cents apiece, thereby gaining 90 cents; how many *papers did he sell?*

52. A drover bought cattle at $49.63 per head, sold them at $63.50 per head, and gained $1511.83; how many head of cattle did he sell?

53. In how many days will 6 men do as much work as 8 men can do in 9 days?

54. Fifteen men have a job of work which they can do in 54 days. If they employ 12 men to assist them, in how many days will the work be completed?

55. William bought a hat for $2 50, a coat for $4.26, a knife for $1.00, and paid for them with oranges at 4 cents apiece; how many oranges did it take?

56. A farmer bought a wagon for $64.40, a carriage for $120.32, 2 plows at $7.08 each, and paid for them with cotton at 8 cents a pound; how many pounds of cotton did it take?

57. Eight boys and 12 girls went on an excursion, and their expenses, which were $2 each, were paid by the boys; how much did each boy pay?

58. Twenty-four gentlemen, 32 ladies and 48 children went on an excursion. The expenses, which were $3 each for the adults and $1 each for the children, were paid by the gentlemen; how much did each gentleman pay?

59. A boy bought a basket and 36 oranges for $1.65. He sold the oranges at 5 cents each and the basket for 15 cents; how much did he gain?

60. I paid $48 an acre for a wood lot of 60 acres. I sold the wood for $2378, and the land for $18 an acre; how much did I gain or lose?

61. A horse worth $65, and 3 cows at $15 each, were exchanged for 12 sheep and $50 in money; what was the price of the sheep?

62. A wagon worth \$82, a buggy worth \$95, and 6 plows at \$7.35 each, were exchanged for 3065 pounds of cotton, and 465 pounds of beef at 8 cents a pound; what was the price of the cotton?

63. What is the dividend when 7 is the divisor, 9 the quotient, and 3 the remainder?

64. When the divisor is 24, quotient 6306, and remainder 9, what is the dividend?

QUESTIONS.

How are numbers, connected by the signs + and —, combined? How may a parenthesis preceded by the sign —, be removed?

How are numbers, connected by the signs × and ÷, combined? How may a parenthesis enclosing such numbers and preceded by the sign ÷, be removed? State the relation of + to —. Of × to ÷.

How are numbers, connected by the signs +, —, × and ÷, combined?

What is the average of two or more numbers?

FACTORIAL PROPERTIES OF NUMBERS.

DEFINITIONS.

93. An **exact divisor** of a number is a number that is contained in it without a remainder.

(1) 4 is an exact divisor of 12; 5 of 20; and 3 of 21.

94. A **prime number** is one that has no exact divisors except itself and 1; and a **composite number** is one that has other exact divisors beside itself and 1.

(1) 2, 3, 5, 7, 11, 13, 17, etc., are prime numbers.
(2) 4, 6, 8, 9, 10, 12, etc., are composite numbers.

95. An **even number** is one that is exactly divisible by 2; and an **odd number** is one that is not exactly divisible by 2.

(1) 0, 2, 4, 6, 8, 10, etc., are even numbers.
(2) 1, 3, 5, 7, 9, 11, etc., are odd numbers.

96. A **factor** of a number is one of its exact divisors.

(1) 2, 3, 4, 6, 8 and 12 are factors of 24.

97. A **prime factor** of a number is a factor which is itself a prime number.

(1) 2, 3 and 5 are prime factors of 180.

98. All the prime factors of a number are the prime factors whose product is equal to the number.

(1) All the prime factors of 180 are 2, 2, 3, 3 and 5; since $2 \times 2 \times 3 \times 3 \times 5 = 180$.

99. A **multiple** of a number is the product obtained by taking it a certain number of times.

(1) 20 is a multiple of 5; 24 of 8; and 56 of 7.

Oral Exercises.

100. 1. Name the even numbers between 20 and 40.

2. Name the odd numbers between 30 and 50.

3. Name the prime numbers between 1 and 30. Between 30 and 60. Between 60 and 100.

4. Name the composite numbers between 1 and 31. Between 31 and 61. Between 61 and 1[illegible].

5. Name three exact divisors (omitting 1) of 12. Of 18. Of 30. Of 36. Of 42. Of 54. Of 56. Of 63. Of 72.

6. What is the smallest number, except 1, that will exactly divide 15? 21? 25? 49? 54? 121?

7. What is the largest factor, except itself, of 21? 33? 44? 35? 40? 52? 49? 121? 63? 72?

8. Name three multiples of 5. Of 6. Of 8. Of 9. Of 11. Of 15. Of 20. Of 50. Of 100.

9. What *are all the prime* factors of 6? Of 8? Of 12?

Of 15? Of 18? Of 25? Of 27? Of 36? Of 63? Of 64? Of 81?

10. Name an exact divisor of 6 and 8. Of 6 and 9. Of 15 and 25. Of 21 and 35. Of 42 and 54.

11. Name the greatest common factor of 8 and 12. Of 12 and 18. Of 12 and 24. Of 30 and 45. Of 36 and 48.

12. Name the least common multiple of 4 and 6. Of 8 and 12. Of 6 and 9. Of 4, 9 and 12. Of 3, 5 and 10. Of 4, 6 and 8. Of 4, 5, 6 and 12.

FACTORING.

101. **Factoring** is finding the factors of a number.

102. PRINCIPLES.—1°. *A factor of a number is a factor of any multiple of that number.*

(1) 2 and 5 are factors of 10; hence, they are factors, or exact divisors of any number of tens; as 7 tens, or 70; 13 tens, or 130.

(2) 4, 10 and 25 are factors of 100; hence, they are factors, or exact divisors of any number of hundreds; as, 17 hundreds, or 1700.

(3) 8 and 125 are factors of 1000; hence, they are factors of any number of thousands; as, 137 thousands, or 137000.

2°. *A factor of two numbers is a factor of their sum, and also of their difference.*

For, the two numbers may be expressed in terms of the common factor as a unit; hence, their sum and difference may also be expressed in terms of that factor as a unit. Thus:

Since 3 is a factor of 12 and 27, it is a factor of their sum and difference. For, 12 = 4 threes and 27 = 9 threes, hence, 27 + 12 = 13 threes, and 27 — 12 = 5 threes.

(1) *Any number is exactly divisible by 2 if its last figure is exactly divisible by 2.*

For, any number (as 476) is the sum of a multiple of 10 (47 tens) and its last figure (6). Now, 2 is an exact divisor of the first part (47 tens); hence, if it is an exact divisor of the second part (6), it is factor of the given number (476).

The same principle applies to 5 as a divisor.

(2) *Any number is exactly divisible by 4, if the number expressed by its last two figures is exactly divisible by 4.*

For, any number (as 1924) is the sum of a multiple of 100 (19 hundreds) and the number expressed by its last two figures (24). Now, 4 is an exact divisor of the first part (1900); hence if it is an exact divisor of the second part (24), it is a factor of the given number (1924.)

The same principle applies to 2, 5, 10, 20, 25 and 50, as divisors, since each of these is a factor of 100.

(3) *Any number is exactly divisible by 8 if the number expressed by its last three figures is exactly divisible by 8.*

For, any number is the sum of a multiple of 1000 and the number expressed by its last three figures. Now, the first part is exactly divisible by 8; hence, if the second part is so divisible, the given number is exactly divisible by 8.

The same principle applies to 2, 4, 5, 10, 20, 25, 40, 50, 100, 125, 200, 250 and 500 as divisors, since each of these is a factor of 1000.

3°. *If any multiple of 10, 100, 1000, etc., be divided by 9, the remainder will be equal to the number of tens, hundreds, etc.*

For, since $10 = 9 + 1$,
7 tens, or $70 = 7 \times 9 + 7$;
$700 = 70 \text{ tens} = 70 \times 9 + 70 = 77 \times 9 + 7$;
$7000 = 700 \text{ tens} = 700 \times 9 + 700 = 777 \times 9 + 7$.

That is, if 70, 700, 7000, etc., be divided by 9, the remainder in each case will be 7.

4°. *The sum of several numbers is exactly divisible by any divisor, if the sum of the remainders obtained by dividing each of the numbers by the divisor, is exactly divisible by that divisor.*

For, the sum of the numbers may be expressed in terms of the divisor as a unit.

If 10, 19 and 27 be divided by 4 the remainders will be respectively, 2, 3 and 3, and as the sum of these remainders is exactly divisible by 4, the sum of the *given numbers* (56) is also exactly divisible by 4.

$$\begin{aligned} 10 &= 2 \text{ fours} + 2 \\ 19 &= 4 \text{ fours} + 3 \\ 27 &= 6 \text{ fours} + 3 \\ \hline 56 &= 12 \text{ fours} + 8 \ (= 2 \text{ fours}) \\ 56 &= 14 \text{ fours} \end{aligned}$$

5°. *Any number is exactly divisible by* 9 *if the sum of its digits is exactly divisible by* 9.

For, take any number, as 75483, which $= 70000 + 5000 + 400 + 80 + 3$. If these parts be divided by 9 the remainders will be the digits 7, 5, 4, 8, 3, respectively. Now, since the sum of these digits (27) is exactly divisible by 9, the given number is also exactly divisible by 9.

This principle also applies to 3 as a divisor, since $10 = 3 \times 3 + 1$.

103. In addition to the preceding, it is well to remember that any number is exactly divisible:

(1) By 6, if it is even, and exactly divisible by 3.

(2) By 11, if the difference between the sums of the even and odd orders is divisible by 11.

(3) By 7, 11 or 13, if the difference between the sums of the even and odd periods is divisible by 7, 11, or 13 respectively.

104. ***To find the prime factors of a composite number.***

(1) Find the prime factors of 210.

Explanation.—The prime factors are the prime numbers which are exact divisors of 210. 2 is an exact divisor, because the last figure of the number is 0. 3 is an exact divisor of the quotient, 105, because the sum of its digits is divisible by 3. 5 will exactly divide the next quotient 35, because its last figure is five. The next quotient, 7, is a prime number. Hence, the prime factors are 2, 3, 5, 7.

Operation.

$$\begin{array}{r|l} 2 & 210 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

RULE. *Divide the given number by any prime number that will exactly divide it, and the quotient, if a composite number, in the same manner; and so continue dividing, until a prime number is obtained for a quotient. The several divisors and the last quotient will be the prime factors required.*

Note.—The composite factors of any number may be found by *multiplying together* two or more of its prime factors.

EXERCISE X.

Find the prime factors of:

2. 50.	**3.** 144.	**4.** 936.
5. 75.	**6.** 150.	**7.** 1155.
8. 64.	**9.** 189.	**10.** 70070.
11. 60.	**12.** 225.	**13.** 25725.
14. 96.	**15.** 420.	**16.** 39325.
17. 91.	**18.** 625.	**19.** 39424.

GREATEST COMMON DIVISOR.

105. A **common divisor** of two or more numbers is a number that will exactly divide each of them.

(1) 4 is a common divisor of 24 and 36.

106. The **greatest common divisor** (G. C. D.) of two or more numbers is the greatest number that will exactly divide each of them.

(1) 12 is the greatest common divisor of 24 and 36.

NOTE.—The greatest common divisor is often called the greatest common measure.

107. PRINCIPLES.—1°. *Any factor common to two or more numbers is a common divisor of those numbers.*

(1) The factors of 30 are 2, 3, 5; of 42, are 2, 3, 7. Hence, the common factors are 2, 3 and 2 × 3, or 6. Therefore the common divisors of 30 and 42 are 2, 3 and 6.

2°. *The product of all the prime factors common to two or more numbers is the greatest common divisor of those numbers.*

(1) The factors of 60 are 2, 2, 3, 5; of 84 are 2, 2, 3, 7; hence the common divisors of 60 and 84 are 2, 2, 3, 2 × 2, 2 × 3, 2 × 2 × 3, the greatest of which is 2 × 2 × 3, or 12, which is the G. C. D. of 60 and 84.

108. Numbers are **prime to each other** when they have no common divisor beside 1.

(1) 16 and 25 are prime to each other.

109. ***To find the greatest common divisor of two or more numbers.***

(1) Find the G. C. D. of 63, 105, 147.

Operation.

3	63,	105,	147
7	21,	35,	49
	3,	5,	7

Explanation.— The numbers are all exactly divisible by 3; hence 3 is a factor of the G. C. D. I divide the numbers by 3, and as the quotients are exactly divisible by 7, 7 is also a factor of the G. C. D. I divide the quotients by 7, and obtain a set of quotients which are prime to each other. Hence, the G. C. D. required is 3×7, or 21.

RULE. *Write the numbers in a horizontal line, divide by any prime number that will exactly divide all of them, divide the quotients in like manner, and so continue till all the quotients are prime to each other. The product of all the divisors is the G. C. D.*

EXERCISE XI.

Find the greatest common divisor of:

2. 20, 35, 40.
3. 252, 324.
4. 35, 56, 84.
5. 192, 224.
6. 21, 27. 81.
7. 378, 648, 918.
8. 33, 66, 77.
9. 175, 455, 315.
10. 12, 18, 42, 54.
11. 78, 234, 468, 702.
12. 50, 75, 100, 150.
13. 165, 385, 1331.
14. 24, 60, 96, 156.
15. 816, 1360, 2040.
16. 60, 90, 105, 135.
17. 1274, 3003, 13013.

110. When the numbers cannot be readily factored, the method of finding the G. C. D. is based on the following

PRINCIPLE.—*Any number which is a factor of a dividend and divisor is a factor of the divisor and remainder.*

(1) 49 contains 14, 3 times, with a remainder, 7; hence, $49 - 3 \times 14 = 7$. Now, any number which will exactly divide 49 and 14, will also exactly divide 49 and 42, and also their difference, 7 (Art. 102). That is, any number, as 7, which is a factor of 49 and 14, is also a factor of 14 and 7.

18. Find the G. C. D. of 301 and 430.

Operation.

```
301)430(1
    301
    129)301(2
        258
         43)129(3
            129
```

EXPLANATION. — I divide the greater number by the less, then divide the less number by the remainder, and continue to divide the last divisor by the last remainder until nothing remains; the last divisor, as 43, is the required G. C. D. For, any factor of 301 and 430 is a factor of 301 and 129; any factor of these is a factor of 129 and 43; and since 43 is the greatest number that will exactly divide these, it is also the G. C. D. of 301 and 430.

Find the G. C. D. of:

19. 477 and 636.
20. 779 and 1349.
21. 403 and 589.
22. 8771 and 6139.
23. 481 and 851.
24. 7241 and 10920.

NOTE.—To find the G. C. D. of more than two numbers, first find the G. C. D. of any two of them; then find the G. C. D. of the number thus found and a *third* number; and so on through all the numbers.

25. A boy has 24 red birds, 40 black birds and 64 blue birds; he wishes to put them into the fewest possible number of cages, putting an equal number of like birds in each; how many birds will he put in each cage, and how many cages will be required?

26. A farmer has 273 bushels of wheat, 1330 of corn, 707 of oats, 665 of rye; he wishes to put his grain into the largest bags of equal size that will exactly hold each kind; how many bags will he require?

27. A man rode 18 miles Monday, 30 miles Tuesday and 42 miles Wednesday, riding an exact number of hours each day. Required his greatest possible rate per hour?

28. A man built a fence with rails of equal length around a field whose sides are 165, 231, 385 and 341 feet. Required the greatest length of the rails.

29. A shepherd bought three lots of sheep, paying an equal integral number of dollars for each sheep. The first lot cost him $32, the second $48, and the third $72. Required the greatest possible cost of the sheep per head, and the total number bought?

30. A man filled four cribs holding 961, 1178, 775 and 1395 bushels, putting an exact number of loads in each. Required the greatest possible size of each load, and the number of loads.

LEAST COMMON MULTIPLE.

111. A **common multiple** of two or more numbers is a number exactly divisible by each of them.

(1) 48 is a common multiple of 4, 8 and 12.

112. The **least common multiple** (L. C. M.) of two or more numbers is the least number that is exactly divisible by each of them.

(1) 24 is the least common multiple of 4, 8 and 12.

113. PRINCIPLES.—*The least common multiple of two or more numbers is the least number that contains the prime factors of the numbers.*

(1) The prime factors of 12 are 2, 2, 3; of 15 are 3, 5; of 18 are 2, 3, 3. Now, $2 \times 2 \times 3 \times 3 \times 5$, or 180, is the least number that contains all the prime factors of each; hence, 180 is the L. C. M. of 12, 15 and 18.

114. ***To find the least common multiple of two or more numbers.***

(1) Find the L. C. M. of 6, 8, 18, 24, 45.

Explanation.—I resolve each of the numbers into its prime factors. To the factors of the greatest number (45) I annex those factors of the other numbers ($2 \times 2 \times 2$) which are not contained in 45, and the result $3 \times 3 \times 5 \times 2 \times 2 \times 2 = 360$ is the L. C. M. required.

Operation—1st method.

$$\begin{aligned} 6 &= 2 \times 3 \\ 8 &= 2 \times 2 \times 2 \\ 18 &= 2 \times 3 \times 3 \\ 24 &= 2 \times 2 \times 2 \times 3 \\ 45 &= 3 \times 3 \times 5 \\ \text{L. C. M.} &= 3 \times 3 \times 5 \times 2 \times 2 \times 2 \end{aligned}$$

Explanation.—As 6 is a factor of 18, it will exactly divide any number that 18 will; hence I cancel 6, also 8, as it is a factor of 24. I now divide 18 and 24 by the common prime factor 2, and bring down 45 with the quotients. 9 being a factor of 45, I cancel it, and divide 12 and 45 by the prime number 3. Now, the L. C. M. is the product of the divisors and final quotients, $2 \times 3 \times 4 \times 15 = 360$.

Operation—2d method.

$$\begin{array}{r|l} 2 & \cancel{6},\ \cancel{8},\ 18,\ 24,\ 45 \\ \hline 3 & \cancel{9},\ 12,\ 45 \\ \hline & 4,\ 15 \end{array}$$

RULE I. *Write the numbers in a horizontal line, cancel such of the smaller as are divisors of the larger, divide the remaining numbers by any common prime divisor of two or more of them, and write the quotients and undivided numbers underneath.*

II. *Proceed with these quotients and undivided numbers as with the given numbers, and so continue until the results are prime to each other. The continued product of all the divisors and undivided numbers is the L. C. M.*

EXERCISE XII.

Find the least common multiple of:

2. 2, 4, 6.
3. 4, 8, 14, 21, 28.
4. 3, 6, 12.
5. 7, 19, 57, 84, 112.
6. 2, 5, 6, 10.
7. 9, 25, 36, 42, 100.

8. 4, 9, 12, 18.	9. 13, 17, 33, 39, 119.
10. 4, 5, 8, 20.	11. 210, 315, 420, 525.
12. 4, 7, 8, 14.	13. 2183, 2479, 3953.
14. 6, 10, 12, 15.	15. 63, 105, 147, 231.
16. 8, 9, 12, 18.	17. 18, 55, 49, 33, 121.
18. 6, 18, 24, 36.	19. 720, 336, 576, 1820.
20. 5, 8, 16, 40.	21. 642, 876, 984, 2000.

22. Find the L. C. M. of the consecutive even numbers from 2 to 10 inclusive.

23. Find the least number that is exactly divisible by each of the nine digits.

24. Find the L. C. M. of the consecutive odd numbers from 1 to 9 inclusive.

25. Find the least number that is exactly divisible by each of the consecutive integers from 15 to 20, inclusive.

26. Find the least number of apples that may be equally divided among 12, 15 or 20 children.

27. Find the least sum of money that may be exactly expended for hogs at $7 a head, cattle at $56, or horses at $144.

28. I have four measures, which contain 6, 8, 12 and 18 quarts; required the size of the smallest vessel that may be filled by each one of these, taken a certain number of times full.

29. Required the length of the shortest piece of wire that can be cut into pieces 27, 35, 63, or 81 feet long.

30. James owes the least sum that can be exactly discharged in payments of $18, $15, or $30; how much does James owe?

31. The fore wheel of a buggy is .132 inches in circumference, and the hind wheel 154 inches; how far must the

buggy move in order that both wheels shall make an exact number of revolutions?

32. Three lads start at the same time and place to travel around a pond; one can travel the distance in 4 hours, the second in 5, and the third in 6. In what time will they first meet at the starting-place?

33. Three men travel around an island in 120, 144 and 180 hours, respectively; at what relative rates per hour did they travel, the rates being the least possible?

CANCELLATION.

115. Cancellation is the process of taking out equal factors from the dividend and divisor.

116. The **sign** of cancellation is an oblique line drawn across a figure; as $\cancel{2}$, $\cancel{6}$.

117. PRINCIPLES.—1°. *Cancelling a factor in any number, divides the number by that factor.*

2°. *Cancelling a factor in both dividend and divisor, does not change the quotient.*

118. ***To shorten operations involving division by cancellation.***

(1) Divide the product of 15, 8, 12 and 2 by the product of 3, 8, 5 and 7.

Operation.

$$\frac{\overset{\cancel{5}}{\cancel{15}} \times \cancel{8} \times 12 \times 2}{\cancel{3} \times \cancel{8} \times \cancel{5} \times 7} = \frac{24}{7} = 3\tfrac{3}{7}$$

Explanation.—I indicate the division by writing the dividend above a line, and the divisor below it. I first cancel the common factor 8,

I next reject the factor 3 from the divisor and from the factor 15 of the dividend, leaving the factor 5 in the place of the latter.

I next reject the common factor 5.

I now divide 24, the product of the remaining factors of the dividend, by 7, the remaining factor of the divisor, and obtain the *quotient*, $3\frac{3}{7}$.

(2) Divide 63 × 16 by 18 × 14.

Second method, using a vertical line.

Explanation.—I draw a vertical line; on the left place the factors of the dividend, and on the right those of the divisor. Since 2 will divide 16 and 14, I reject 2 as a factor of 16, retaining the factor 8; and also of 14, retaining the factor 7. Again, I reject the factor 7 from 7 and 63, retaining the factor 9. Again, I reject the factor 9 from 9 and 18, retaining the factor 2. Finally, rejecting the factor 2 from 2 and 8, there remains only the factor 4 in the dividend, which is the quotient required.

Operation.

$$\begin{array}{r|l} \cancel{9}\ \cancel{63} & \cancel{18}\ \cancel{2} \\ 4\ \cancel{8}\ \cancel{16} & \cancel{14}\ \cancel{7} \end{array}$$

RULE I. *Indicate the operations to be performed.*

II. *Cancel the factors common to the divisor and dividend, and divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

EXERCISE XIII.

3. Divide 48 by 6 × 4.
4. Divide 256 by 8 × 4.
5. Divide 9 × 5 × 4 by 2 × 5 × 3.
6. Divide 10 × 18 × 60 by 45 × 6 × 8.
7. Divide 12 × 10 × 7 by 14 × 15.
8. Divide 28 × 49 × 75 by 7 × 15 × 84.
9. Simplify 28 ÷ 15 × 90 ÷ 42 × 49 ÷ 2 ÷ 7.

NOTE.—In expressions like this, the factors of the dividend are the numbers preceded by the sign ×, and those whose sign is ÷ are the factors of the divisor. See Art. 90.

10. Simplify 9 ÷ 2 ÷ 6 × 8 × 6 ÷ 3.
11. Simplify 64 × 7 ÷ 25 ÷ 16 × 100 × 14 ÷ 49.
12. Simplify 8 × 6 ÷ 2 ÷ 3 ÷ 4 × 8 × 12 ÷ 6.
13. *Simplify* ÷ 7 ÷ 21 × 70 × 13 × 39 ÷ 26.

14. A lad bought 27 oranges at 5 cents each, and paid for them with apples at 3 cents each; how many apples did it require?

15. How many cattle at $56 apiece will pay for 91 horses at $120 each?

16. If a turkey is worth $2, how many coops of 6 turkeys each must be given for 9 pieces of muslin, each containing 50 yards, at 8 cents a yard?

17. How many hogsheads of sugar containing 1280 pounds, at 7 cents a pound, must be given for 35 bales of cotton, each weighing 448 pounds, at 8 cents a pound?

18. If 9 hats cost $24, what will 6 hats cost?

19. If 42 yards of cloth cost $147, what will 24 yards cost?

20. If 10 boys can build a fence in 9 days, how long will it take 15 boys to build it?

21. If 77 men can do a certain work in 39 days, how many men can do the same work in 91 days?

22. How many boxes, each containing 30 dozen of eggs, at $.25 per dozen, are worth as much as 100 pounds of butter at $.27 per pound?

23. Sold 12 tubs of butter, each containing 48 pounds, at $.35 a pound, and took in payment 8 barrels of sugar, each containing 336 pounds; what was the sugar worth a pound?

QUESTIONS.

What is an exact divisor? A prime number? A composite number? An even number? An odd number? A factor? What is factoring? What are the first two principles? When is a number divisible by 2? 3? 4? 5? 6? 8? 9? 7? 11? 13? Give the rule for factoring?

What is the G. C. D. of two or more numbers? What are its factors? Give the rule for finding it.

What is the L. C. M. of two or more numbers? Give the rule for finding it?

What is cancellation? What are the two principles? Give the rule.

Chapter II.

FRACTIONS.

Fractional Quotients, or Equal Parts of Integers.*

119. A **fractional quotient** is an indicated division.

(1) $\frac{12}{4}$ is a fractional quotient; it denotes $\frac{1}{4}$ of 12, and is so read.

120. The **value** of a fractional quotient is the result of performing the indicated division, or one of the equal parts into which the dividend is divided.

(1) The value of $\frac{12}{4}$ is 3; of $\frac{7}{2}$, $3\frac{1}{2}$; of $\frac{2}{5}$, 1 of the 5 equal parts of 2.

Note. For brevity, the term *quotient* will be used in this section for *fractional quotient*.

121. Any integer may be expressed as a quotient by writing 1 under it for a divisor; as $3 = \frac{3}{1}$.

122. The **terms** of a quotient are the dividend and divisor.

Oral Exercises.

123. 1. How would you express by figures 1 half of 10? 1 third of [illegible]? 1 fifth of 3? 1 tenth of 3? $3 \div 13$?

2. Read the following: [illegible].

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* The subject of common fractions is fully treated in the next section. The objects of this section are: First, to consider the properties of quotients, and, second, to prepare the student for the study of fractions. It may be omitted entirely, or taken up subsequently, at the discretion of the teacher.

3. What is the value of $\frac{6}{2}$? $\frac{15}{5}$? $\frac{20}{4}$? $\frac{23}{6}$? $\frac{3}{7}$? $\frac{37}{8}$? $\frac{47}{9}$? $\frac{9+6}{3}$? $\frac{10+11}{4}$? $\frac{20-2}{6}$? $\frac{30-1}{8}$? $\frac{20\times 2}{8}$? $\frac{15\times 2}{7}$? $\frac{24+3}{3}$? $\frac{93+3}{7}$? $\frac{64}{4\times 2}$? $\frac{65}{15+3}$?

4. Find the value of $\frac{6}{3}+\frac{12}{4}$; $\frac{10}{2}+\frac{27}{3}$; $\frac{26}{2}+\frac{63}{7}$; $\frac{18}{3}-\frac{8}{2}$; $\frac{63}{9}-\frac{56}{8}$; $\frac{70}{5}-\frac{54}{6}$; $\frac{12}{3}+\frac{35}{7}-\frac{21}{3}$; $\frac{42}{6}+\frac{54}{9}-\frac{36}{4}-\frac{20}{5}$.

5. Which is the more $\frac{42}{7}$ or $\frac{20}{4}$? $\frac{80}{5}$ or $\frac{90}{6}$?

6. Find the value of $\frac{6}{2}\times\frac{12}{6}$; $\frac{36}{4}\times\frac{35}{7}$; $\frac{48}{6}\times\frac{32}{8}$; $\frac{12}{2}\div\frac{15}{5}$; $\frac{40}{5}\div\frac{28}{7}$; $\frac{60}{3}\div\frac{60}{12}$; $\frac{18}{9}\times\frac{21}{7}\div\frac{25}{5}$; $\frac{70}{5}\div\frac{24}{12}\div\frac{49}{7}$.

7. What is the value of $\frac{5}{5}$? $\frac{6}{6}$? $\frac{13}{13}$?

8. When the dividend is equal to the divisor, what is the value of the quotient?

9. Is the value of $\frac{6}{5}$ more or less than 1? Of $\frac{11}{7}$?

10. When the dividend is greater than the divisor, the value of the quotient is greater than what?

11. Is 1 of the 4 equal parts of 3 less than 1? Is $\frac{3}{4}$ more or less than 1? Is $\frac{5}{6}$? $\frac{7}{10}$? $\frac{13}{14}$?

12. When the dividend is less than the divisor, the value of the quotient is what?

124.—PRINCIPLES OF QUOTIENTS.

Is the value of $\frac{6\times 5}{3\times 5}$ equal to the value of $\frac{6}{3}$?

Is the value of $\frac{20+5}{10+5}$ equal to the value of $\frac{20}{10}$?

Hence,

Multiplying or dividing both terms of a quotient by the same integer does not change the value of the quotient.

125. Reduction of quotients is changing their terms or forms without changing their values.

126. A quotient is in its **lowest terms** when its terms are prime to each other.

127. ***To reduce quotients to their lowest terms.***

(1) Reduce $\frac{12}{18}$ to its lowest terms.

Operation.

$$\tfrac{12}{18} = \tfrac{4}{6} = \tfrac{2}{3}$$

$$\tfrac{12}{18} = \tfrac{2}{3}.$$

Explanation.—Dividing both terms of a quotient by the same number does not change its value. Hence, I divide both terms by 3, and get $\frac{4}{6}$. Then I divide both terms of $\frac{4}{6}$ by 2, and get $\frac{2}{3}$. Now, since the terms are prime to each other, the quotient is in its lowest terms.

The same result might have been obtained at one division by dividing both terms by 6, the G. C. D. of the terms.

RULE.—*Divide both terms by any exact divisor of them, and continue the operation until the terms are prime to each other. Or,*

Divide both terms by their G. C. D.

EXERCISE XIV.

Reduce the following quotients to their lowest terms:

2. $\frac{4}{6}$.	3. $\frac{35}{40}$.	4. $\frac{77}{143}$.	5. $\frac{917}{1441}$.
6. $\frac{8}{10}$.	7. $\frac{42}{60}$.	8. $\frac{91}{117}$.	9. $\frac{3456}{4096}$.
10. $\frac{15}{20}$.	11. $\frac{16}{64}$.	12. $\frac{85}{136}$.	13. $\frac{6561}{8748}$.

128. ***To reduce quotients to equivalent quotients having a common divisor.***

(1) Reduce $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{4}{5}$ to a common divisor.

Operation.

$$\tfrac{2}{3} = \frac{2 \times 4 \times 5}{3 \times 4 \times 5} = \tfrac{40}{60}.$$

$$\tfrac{3}{4} = \frac{3 \times 3 \times 5}{4 \times 3 \times 5} = \tfrac{45}{60}.$$

$$\tfrac{4}{5} = \frac{4 \times 3 \times 4}{5 \times 3 \times 4} = \tfrac{48}{60}.$$

Explanation.—Since multiplying both terms of a quotient by the same integer does not change its value, I multiply both terms of $\frac{2}{3}$ by 4 and 5, the divisors of the other quotients, both terms of $\frac{3}{4}$ by 3 and 5, the other two divisors, and both terms of $\frac{4}{5}$ by 3 and 4, the other divisors, which evidently makes all the divisors 60, = the product of *the divisors 3, 4 and 5.*

RULE.—*Multiply both terms of each quotient by the divisors of the other quotients.*

EXERCISE XV.

Reduce each of the following to a common divisor:

2. $\frac{4}{2}$ and $\frac{6}{3}$.
3. $\frac{6}{3}$, $\frac{8}{4}$ and $\frac{4}{2}$.
4. $\frac{2}{3}$ and $\frac{1}{2}$.
5. $\frac{2}{3}$, $\frac{4}{5}$ and $\frac{1}{6}$.
6. $\frac{3}{7}$ and $\frac{5}{6}$.
7. $\frac{1}{4}$, $\frac{3}{5}$ and $\frac{5}{8}$.
8. $\frac{5}{8}$ and $\frac{3}{7}$.
9. $\frac{5}{6}$, $\frac{6}{7}$ and $\frac{7}{8}$.
10. 1 third of 2 and 1 fourth of 3.
11. 1 fifth of 3 and 1 sixth of 5.
12. 1 half of 1, 1 fifth of 3 and 1 tenth of 7.

129. ***To add quotients.***

(1) How many are 1 half of 1 and 1 third of 2?

Operation.

$$\tfrac{1}{2} = \frac{1 \cdot 3}{2 \cdot 3} = \tfrac{3}{6}.$$
$$\tfrac{2}{3} = \frac{2 \cdot 2}{3 \cdot 2} = \tfrac{4}{6}.$$
$$\tfrac{3}{6} + \tfrac{4}{6} = \frac{3 + 4}{6} = \tfrac{7}{6} = 1\tfrac{1}{6}.$$

Explanation.—1 half of 1 is $\frac{1}{2}$, and 1 third of 2 is $\frac{2}{3}$. I reduce these quotients to a common divisor, and obtain $\frac{3}{6}$ and $\frac{4}{6}$, which I wish to add together. $\frac{3}{6} + \frac{4}{6} =$ 1 sixth of $3 +$ 1 sixth of $4 =$ 1 sixth of $(3 + 4) =$ 1 sixth of $7 = 1\frac{1}{6}$.

RULE.—*Reduce the quotients to a common divisor, add the dividends, and divide the sum by the common divisor.*

NOTE.—In addition and subtraction of fractions, the operations indicated by $+$ and $-$ are in general performed before the operations indicated by $\div$.

EXERCISE XVI.

Find the sum of:

2. $1 \div 3$ and $1 \div 5$.
3. $1 \div 6$ and $3 \div 5$.
4. $3 \div 4$ plus $2 \div 3$.
5. $40 \div 8$ plus $15 \div 5$.
6. $(7 \div 2) + (5 \div 3)$.
7. $(9 \div 4) + (11 \div 6)$.
8. $\frac{3}{4} + \frac{2}{3} + \frac{1}{2}$.
9. $\frac{5}{8} + \frac{3}{4} + \frac{4}{5}$.

10. If I divide 17 by 3 and 28 by 5, what will the sum of the quotients be?

11. How much is the sixth of 37 plus the tenth of 11?

12. James has 1 fourth of \$71 and John 1 fifth of \$13; how much have both together?

13. William had \$$\frac{7}{8}$ and his father gave him \$$\frac{5}{6}$; how much did he then have?

14. James rode $\frac{1}{3}$ of 14 miles in one hour, and $\frac{1}{4}$ of 15 miles the next hour; how far did he ride in all?

130. ***To subtract one quotient from another.***

(1) From 1 fifth of 4 take 1 third of 2.

Operation.

$$\frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}.$$
$$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}.$$
$$\frac{12 - 10}{15} = \frac{2}{15}.$$

Explanation.—I reduce the quotients, $\frac{4}{5}$ and $\frac{2}{3}$, to a common divisor and obtain $\frac{12}{15}$ and $\frac{10}{15}$. Then, I say, $\frac{12}{15} - \frac{10}{15} = 1$ fifteenth of $12 - 1$ fifteenth of $10 = 1$ fifteenth of $(12 - 10) = \frac{2}{15}$.

Rule.—*Reduce quotients to a common divisor, take the difference of the dividends, and divide it by the common divisor.*

EXERCISE XVII.

2. From $\frac{8}{9}$ take $\frac{5}{9}$.

3. From $\frac{5}{6}$ take $\frac{2}{7}$.

4. From $\frac{2}{3}$ take $\frac{3}{8}$.

5. From 3 take $\frac{5}{11}$.

6. From $\frac{5}{6}$ take $\frac{3}{7}$.

7. From 1 take $\frac{3}{8}$.

8. From $\frac{1}{4}$ of 7 take $\frac{1}{6}$ of 2.

9. $\frac{3}{8} - \frac{1}{5} = ?$

10. From $\frac{1}{10}$ of 8 take $\frac{1}{15}$ of 4.

11. $\frac{5}{12} - \frac{3}{7} = ?$

12. James had \$$\frac{3}{4}$ and spent \$$\frac{1}{3}$: how much had he left?

13. How much more is \$$\frac{8}{9}$ than \$$\frac{3}{4}$?

14. How much does \$$\frac{3}{7}$ lack of being \$1?

15. John has \$$\frac{1}{3}$; how much more must he earn to have \$$\frac{5}{7}$?

16. A grocer bought eggs at the rate of 4 for 7 cts. and sold them at the rate of 5 for 11 cts.; how much did he make on each egg?

17. One man rides at the rate of 9 miles in 2 hours, and another 13 miles in 3 hours; how much farther does one ride per hour than the other?

18. A father divided \$359 equally among his 6 sons, and \$317 equally among his 7 daughters; how much more did each son receive than each daughter?

131. ***To multiply a quotient by an integer.***

(1) Multiply $\frac{5}{6}$ by 2.

Solution: $\frac{5}{6} \times 2 = 5 \div 6 \times 2$, which (Art. 90) is equal to $(5 \times 2) \div 6$, or $5 \div (6 \div 2)$, $= 1\frac{2}{3}$, Ans.

RULE.—*Multiply the dividend or divide the divisor by the integer.*

EXERCISE XVIII.

2. Multiply 1 sixth of 7 by 3.

3. Multiply 1 tenth of 3 by 5.

Find the following products:

4. $\frac{5}{8} \times 4$. **5.** $\frac{3}{7} \times 5$. **6.** $\frac{13}{45} \times 9$.

7. $\frac{3}{16} \times 8$. **8.** $\frac{6}{35} \times 7$. **9.** $\frac{11}{13} \times 7$.

How much money will it take to buy:

10. 5 chickens, at \$$\frac{3}{7}$ each?

11. 6 yards of cassimere, at \$$\frac{3}{2}$ per yard?

12. 9 cords of wood, at \$$\frac{7}{2}$ per cord?

132. ***To multiply an integer or quotient by a quotient.***

(13) Multiply 8 by 1 third of 2.

Solution: $8 \times \frac{2}{3} = 8 \times 2 \div 3$, which (Art. 90) is equal to $8 \times 2 \div 3 = 16 \div 3 = 5\frac{1}{3}$, Ans.

RULE.—*Multiply the integer by the dividend and divide the product by the divisor.*

(14) Multiply $\frac{3}{4}$ by $\frac{5}{7}$.

Solution: $\frac{3}{4} \times \frac{5}{7} = 3 \div 4 \times 5 \div 7$, which (Art. 90) is equal to $(3 \times 5) \div (4 \times 7) = \frac{15}{28}$, Ans.

RULE.—*Multiply the dividends together for the dividend of the product, and the divisors together for the divisor.*

NOTE.—Common factors in the dividends and divisors should always be cancelled.

EXERCISE XIX.

Find the value of:

15. $\frac{2}{8} \times \frac{5}{9}$.
16. $\frac{5}{8} \times \frac{6}{7} \times \frac{5}{9}$.
17. $\frac{3}{7} \times \frac{6}{5}$.
18. $\frac{4}{7} \times \frac{3}{5} \times \frac{35}{6}$.
19. $\frac{6}{11} \times \frac{5}{12}$.
20. $\frac{8}{15} \times \frac{10}{11} \times \frac{3}{16}$.

21. Multiply 1 third of 5 by 1 seventh of 2.

22. Multiply 1 tenth of 9 by 1 sixth of 5.

23. Multiply 1 eleventh of 4 by 1 fourth of 11.

What will be the cost:

24. Of 6 yards of cloth, at the rate of 2 yards for 11 cts?

25. Of 9 hats, at the rate of 4 hats for $13?

26. Of 7 caps, at the rate of 3 caps for $5?

27. Of 24 guns, at the rate of 7 guns for $80?

28. Of $\frac{2}{3}$ of a pound of butter, at 24 cents a pound?

29. Of $\frac{5}{8}$ of a day's work, at $\$\frac{5}{2}$ per day?

30. Of $\frac{9}{5}$ of a peck of corn, at $\$\frac{3}{8}$ per peck?

31. How much money will it take to buy $\frac{1}{4}$ of 13 yards of cassimere, at the rate of 3 yards for $10?

32. At the rate of $\frac{1}{3}$ of $8 per yard, what would be the cost of $\frac{1}{3}$ of 9 yards of silk?

33. How many cords of wood could a man cut in $\frac{1}{4}$ of 23 days, by cutting $\frac{1}{3}$ of 12 cords per day?

133. ***To divide a quotient by an integer.***

(1) Divide 1 seventh of 6 by 3.

Solution: 1 seventh of 6 is $\frac{6}{7}$. $\frac{6}{7} \div 3 = 6 \div 7 \div 3 = (6 \div 8) \div 7$, or $6 \div (7 \times 3)$, $= \frac{2}{7}$, Ans.

RULE.—*Divide the dividend or multiply the divisor by the integer.*

EXERCISE XX.

2. Divide 1 fourth of 15 by 5.

3. Divide 1 fifth of 4 by 7.

Find the value of:

4. $\frac{6}{7} \div 3$. **5.** $\frac{5}{11} \div 6$. **6.** $\frac{35}{41} \div 15$.

7. $\frac{7}{9} \div 4$. **8.** $\frac{21}{5} \div 7$. **9.** $\frac{60}{73} \div 32$.

What will be the cost:

10. Of 1 hat, at the rate of 5 hats for $\$\frac{13}{2}$?

11. Of 1 gun, at the rate of 3 guns for $\$\frac{78}{5}$?

12. Of 1 knife, at the rate of 8 knives for $\$\frac{34}{3}$?

134. ***To divide an integer or quotient by a quotient.***

135. To **invert** a quotient is to interchange the places of its terms.

(1) $\frac{5}{6}$ inverted is $\frac{6}{5}$; $\frac{3}{7}$ inverted is $\frac{7}{3}$; 8, is $\frac{1}{8}$.

(13) Divide 5 by 1 half of 3.

Explanation.—$5 \div \frac{3}{2} = 5 \div (3 \div 2) = 5 \div 3 \times 2 = (5 \times 2) \div 3 = 10 \div 3 = 3\frac{1}{3}$.

Operation.
$5 \times \frac{2}{3} = \frac{10}{3} = 3\frac{1}{3}$.

(14) Divide $\frac{3}{4}$ by $\frac{7}{5}$.

Explanation.—$\frac{3}{4} \div \frac{7}{5} = 3 \div 4 \div (7 \div 5) = 3 \div 4 \div 7 \times 5 = 3 \times 5 \div (4 \times 7) = \frac{15}{28}$.

Operation.
$\frac{3}{4} \times \frac{5}{7} = \frac{15}{28}$.

RULE.—*Invert the divisor and proceed as in multiplication.*

15. Divide 6 by $\frac{2}{3}$; 8 by $\frac{4}{5}$; 12 by $\frac{5}{6}$.

16. Divide 12 by $\frac{3}{4}$; 15 by $\frac{7}{8}$; 25 by $\frac{9}{11}$.

17. Divide $\frac{4}{5}$ by $\frac{3}{8}$; $\frac{3}{7}$ by $\frac{7}{5}$; $\frac{6}{11}$ by $\frac{12}{7}$.

18. Divide $\frac{3}{4}$ by $\frac{7}{8}$; $\frac{6}{5}$ by $\frac{9}{4}$; $\frac{18}{25}$ by $\frac{6}{15}$.

How many times:

19. Does $\frac{2}{3}$ contain $\frac{1}{2}$? Ans. $\frac{4}{3}$, or $\frac{1}{3}$ of 4 times.

20. Does $\frac{2}{3}$ contain 5? Ans. $\frac{2}{15}$, or $\frac{1}{15}$ of 2 times.

21. Does $\frac{5}{4}$ contain $\frac{3}{8}$? $\frac{7}{5}$ contain $\frac{9}{2}$? $\frac{3}{7}$ contain 2?

22. Does 1 contain $\frac{7}{9}$? $\frac{7}{9}$ contain 5? 5 contain $\frac{2}{3}$?

23. Does 1 third of 5 contain 1 fifth of 9?

24. If 4 knives are worth $\$\frac{8}{7}$, what is one knife worth?

25. If I pay $\$\frac{6}{7}$ for 4 books, what do I pay for 1 book?

26. If 5 tops are worth $\$\frac{3}{8}$, what is one top worth?

27. If 7 melons cost $\$\frac{9}{11}$, what will 1 melon cost?

How many:

28. Tops can I buy for $\$\frac{5}{4}$, at $\$\frac{1}{8}$ per top?

29. Books can I buy for \$4, at $\$\frac{2}{5}$ per book?

30. Slates can I buy for $\$\frac{18}{7}$, at $\$\frac{3}{7}$ per slate?

31. Pencils can I buy for $\$\frac{27}{4}$, at $\$\frac{3}{16}$ per pencil?

32. How many hours will it take a boy to travel $\frac{1}{2}$ of 15 miles, at the rate of 5 miles in 3 hours?

33. A gentleman paid $\frac{1}{2}$ of \$6825 for a farm, at the rate of \$25 for 2 acres; how many acres were in the farm?

QUESTIONS.

What is a fractional quotient? The value of a fractional quotient? The terms of a quotient? The principle of quotients? Reduction?

How do you reduce a quotient to its lowest terms? Quotients to a common divisor? Add quotients? Subtract? Multiply? Divide?

COMMON FRACTIONS, OR EQUAL PARTS OF UNITS.

136. 1. Can any integral unit, as 1 apple, 1 bar of soap, or 1 dollar, be divided into equal parts? What is the name of one of the parts when one is divided into two equal parts? Into three equal parts? Into four equal parts?

2. How many halves in one? How many thirds? Fourths? Fifths? Sevenths? Tenths? Twentieths?

3. How is one half expressed by figures? One third? One fifth? One twelfth? One fifteenth?

4. Does $\frac{1}{2}$ express 1 of 2 equal parts of one?

5. Does $\frac{3}{5}$ express 3 of 5 equal parts of one?

6. What does $\frac{1}{3}$ express? $\frac{1}{4}$? $\frac{1}{6}$? $\frac{3}{7}$? $\frac{5}{11}$? $\frac{8}{17}$?

137. A **fractional unit** is one of the equal parts of an integral unit.

(1) $\frac{1}{3}$ of a yard, $\frac{1}{5}$ of a dollar, $\frac{1}{9}$, are fractional units.

138. A **fractional number**, or **fraction**, is a fractional unit, or a collection of fractional units.

(1) $\frac{1}{6}$ of a mile, $\frac{3}{4}$ of a pound, $\frac{7}{8}$, are fractions.

139. A **common fraction** is expressed by two numbers, one above and the other below a horizontal line.

140. The **terms** of a fraction are the two numbers used to express it.

141. The **denominator** is the term below the line. It shows the number of parts into which the integral unit is divided, and thus names the parts expressed by the fraction.

142. The **numerator** is the term above the line. It shows how many parts are taken, and thus numbers the parts expressed by the fraction.

(1) The terms of the fraction $\frac{7}{10}$ are 7 and 10; the denominator is 10, which shows the size of the parts *tenths*, or the fractional unit 1 *tenth*; the numerator is 7, which shows the number of parts taken.

143. There are two ways of considering any fraction, as $\frac{7}{10}$.

1°. It may imply 1 tenth of seven. In this sense it is a quotient, 7 being the dividend and 10 the divisor. Fractions were so regarded and treated under the heading of Fractional Quotients.

2°. It may imply 7 tenths of one. It is in this sense that fractions are now to be regarded and treated.

Read each of the following fractions, and name the terms, the numerator, the denominator, the fractional unit, the number of fractional units, and the signification:

$\frac{2}{3}$, $\frac{1}{7}$, $\frac{5}{8}$, $\frac{13}{12}$, $\frac{20}{6}$, $\frac{3}{17}$, $\frac{8}{8}$, $\frac{10}{16}$.

144. The **value** of a fraction is its relation to one.

145. When classified with respect to **value**, fractions are *proper*, *improper*, and *mixed*.

146. A **proper fraction** is one whose numerator is less than its denominator; an **improper fraction** is one whose numerator is equal to, or greater than its denominator: and a **mixed number** is a number composed of an integer and a fraction.

(1) $\frac{1}{3}$, $\frac{3}{5}$, $\frac{7}{8}$, $\frac{5}{11}$, are proper fractions.

(2) $\frac{4}{3}$, $\frac{8}{8}$, $\frac{12}{8}$, $\frac{11}{11}$, are improper fractions.

(3) $1\frac{1}{3}$, $4\frac{3}{5}$, $9\frac{1}{4}$, $13\frac{3}{7}$, are mixed numbers.

147. When classified with respect to **form**, fractions are *simple*, *complex*, and *compound*.

148. A **simple fraction** is one whose terms are integers; a **complex fraction** is one which has one or both of its terms fractional; and a **compound fraction** is a fraction of a fraction.

(1) $\frac{2}{3}$, $\frac{4}{5}$, $\frac{1}{6}$, $\frac{12}{4}$, are simple fractions.

(2) $\frac{2\frac{1}{2}}{3}$, $\frac{3}{5\frac{1}{2}}$, $\frac{1\frac{1}{4}}{2\frac{1}{3}}$, are complex fractions.

(3) $\frac{1}{2}$ of $\frac{2}{3}$, $\frac{3}{4}$ of $\frac{4}{5}$, are compound fractions.

149. An integer may be expressed as a fraction by writing 1 under it as a denominator.

(1) $8 = \frac{8}{1}$, which is read eight ones.

150. The **reciprocal** of a fraction is the result of inverting it. See Art. 135.

PRINCIPLES OF FRACTIONS.

151. 1°. *Multiplying the numerator of a fraction by any integer multiplies the value of the fraction by that integer.*

Explanation.—Multiplying the numerator by 2, doubles the number of parts taken.

Illustration.

$\frac{3}{5}$ is 3 fifths of one.

$\frac{2 \times 3}{5}$ is 6 fifths of one.

2°. *Dividing the denominator of a fraction by any integer multiplies the value of the fraction by that integer.*

Explanation. — Dividing the denominator by 2, doubles the size of the parts taken.

Illustration.

$\frac{5}{6}$ is 5 sixths of one.

$\frac{5}{6 \div 2}$ is 5 thirds of one.

3°. *Dividing the numerator of a fraction by any integer divides the value of the fraction by that integer.*

Explanation. — Dividing the numerator by 2, halves the number of parts taken.

Illustration.

$\frac{6}{5}$ is 6 fifths of one.

$\frac{6 \div 2}{5}$ is 3 fifths of one.

4°. *Multiplying the denominator of a fraction by any integer divides the value of the fraction by that integer.*

Explanation.—Multiplying the denominator by 2, halves the size of the parts taken.

Illustration.

$\frac{5}{3}$ is 5 thirds of one.

$\frac{5}{2 \times 3}$ is 5 sixths of one.

5°. *Multiplying or dividing both terms of a fraction by the same integer does not change its value.*

Explanation.—Doubling the number of parts and at the same time halving the size of the parts does not change the value of the fraction.

Again, halving the number of parts and at the same time doubling the size of the parts, does not change the value of the fraction.

Illustration.

$\frac{2}{3}$ is 2 thirds of one.

$\frac{2 \times 2}{2 \times 3}$ is 4 sixths of one.

$\frac{6}{10}$ is 6 tenths of one.

$\frac{6 \div 2}{10 \div 2}$ is 3 fifths of one.

Oral Exercises.

152. 1. What is the effect of multiplying the numerator of $\frac{5}{6}$ by 3? Of dividing the denominator by 3?

2. In what two ways may $\frac{7}{8}$ be multiplied by 4? Are the products equal? Why?

3. Which is the better method of multiplying $\frac{5}{12}$ by 3? By 4? By 7?

4. What is the effect of dividing the numerator of $\frac{10}{11}$ by 5? Of multiplying the denominator by 5?

5. In what two ways may $\frac{15}{7}$ be divided by 3? Are the quotients equal? Why?

6. Which is the better method of dividing $\frac{16}{21}$ by 8? By 4? By 3? By 5?

7. How many times greater is $\frac{5 \times 2}{3}$ than $\frac{2}{3}$? Is $\frac{2}{63 \div 7}$ than $\frac{2}{63}$? Is $\frac{7 \times 9}{11}$ than $\frac{7}{11}$? Is $\frac{6}{77 \div 11}$ than $\frac{6}{77}$?

8. How many times less is $\frac{56 \div 7}{5}$ than $\frac{56}{5}$? Is $\frac{3}{4 \times 9}$ than $\frac{3}{9}$? Is $\frac{18 \div 3}{12}$ than $\frac{18}{12}$? Is $\frac{48}{2 \times 4}$ than $\frac{48}{2}$?

9. What is the effect of multiplying both terms of $\frac{16}{28}$ by 5? Of dividing both terms by 5?

10. How can $\frac{8}{12}$ be changed to sixths? To thirds?

11. How can $\frac{3}{5}$ be changed to tenths? To fifteenths?

13. $\frac{12}{16}$ of a lemon are how many eighths of a lemon? How many fourths? How many halves?

REDUCTION.

153. Reduction of fractions consists in changing their forms without changing their values.

154. A simple fraction is in its **simplest form** when its terms are prime to each other.

155. ***To reduce fractions to their simplest form.***

The process is the same as that of reducing a quotient to its lowest terms, Art. 127.

EXERCISE XXI.

Reduce the following fractions to their simplest form:

2. $\frac{6}{9}$.	3. $\frac{14}{108}$.	4. $\frac{432}{576}$.	5. $\frac{923}{1491}$.
6. $\frac{12}{28}$.	7. $\frac{25}{175}$.	8. $\frac{494}{806}$.	9. $\frac{2261}{4123}$.
10. $\frac{25}{40}$.	11. $\frac{162}{844}$.	12. $\frac{315}{693}$.	13. $\frac{1767}{4557}$.
14. $\frac{54}{63}$.	15. $\frac{156}{195}$.	16. $\frac{336}{504}$.	17. $\frac{6160}{40480}$.
18. $\frac{96}{132}$.	19. $\frac{288}{480}$.	20. $\frac{420}{980}$.	21. $\frac{9702}{18522}$.

156. ***To reduce an integer or a mixed number to an improper fraction.***

(1) Reduce 18 to fifths.

Analysis: $1 = 5$ fifths; hence, $18 = 18 \times 5$ fifths, or 90 fifths.

Operation.

$18 \times 5 = 90$

$18 = \frac{90}{5}$

RULE.—*Multiply the integer by the denominator, and write the product over the denominator.*

(2) Reduce $18\frac{3}{5}$ to fifths.

Analysis: $18\frac{3}{5} = 18$ and 3 fifths.

$= 90$ fifths and 3 fifths $= 93$ fifths.

RULE.—*Multiply the integer by the denominator of the fraction, to the product add the numerator, and under the sum write the denominator.*

Operation.

$$\begin{array}{r} 18\frac{3}{5} \\ 5 \\ \hline 90 \\ 3 \\ \hline 93 \end{array} \quad \frac{93}{5}$$

EXERCISE XXII.

Reduce the following mixed numbers to improper fractions:

3. $3\frac{1}{2}$.	**4.** $17\frac{3}{25}$.	**5.** $116\frac{1}{4}$.	**6.** $831\frac{19}{20}$.
7. $7\frac{2}{3}$.	**8.** $24\frac{5}{14}$.	**9.** $116\frac{31}{61}$.	**10.** $31\frac{11}{125}$.
11. $9\frac{3}{7}$.	**12.** $136\frac{5}{8}$.	**13.** $100\frac{100}{199}$.	**14.** $4695\frac{2}{3}$.
15. $11\frac{2}{5}$.	**16.** $208\frac{15}{22}$.	**17.** $98\frac{48}{97}$.	**18.** $73\frac{893}{1728}$.
19. $18\frac{4}{7}$.	**20.** $521\frac{3}{10}$.	**21.** $718\frac{45}{97}$.	**22.** $99\frac{53}{1000}$.

157. ***To reduce an improper fraction to an integer or mixed number.***

(1) Change 27 thirds to an integer.

Analysis: 3 thirds $= 1$. $27 \div 3 = 9$; hence, 27 thirds $= 9$.

(2) Change 28 thirds to a mixed number.

Analysis: 28 thirds $=$ 27 thirds $+$ 1 third.
$= 9 +$ 1 third, or $9\frac{1}{3}$.

Operation.
$28 \div 3 = 9\frac{1}{3}$

RULE.—*Divide the numerator by the denominator.*

NOTE.—The fractional remainder, if any, should be reduced to its lowest terms, or simplest form.

EXERCISE XXIII.

Reduce the following to whole or mixed numbers:

3. $\frac{42}{6}$.	**4.** $\frac{98}{12}$.	**5.** $\frac{2025}{25}$.	**6.** $\frac{399}{28}$.
7. $\frac{49}{8}$.	**8.** $\frac{57}{9}$.	**9.** $\frac{1561}{13}$.	**10.** $\frac{720}{25}$.
11. $\frac{75}{4}$.	**12.** $\frac{300}{15}$.	**13.** $\frac{8349}{24}$.	**14.** $\frac{2028}{45}$.
15. $\frac{98}{6}$.	**16.** $\frac{465}{25}$.	**17.** $\frac{9328}{11}$.	**18.** $\frac{3275}{20}$.

158. ***To reduce a fraction to an equivalent fraction having a given denominator.***

(1) Reduce $\frac{3}{4}$ to a fraction whose denominator is 24.

Explanation.—Multiplying both terms of a fraction by the same number does not change its value. Hence, I multiply both terms of $\frac{3}{4}$ by such a number, $24 \div 4$, as will *make the denominator* 24.

Operation,
$24 \div 4 = 6$

$$\frac{3}{4} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24}.$$

RULE.—*Divide the required denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.*

EXERCISE XXIV.

Reduce:

2. $\frac{3}{4}$ to 12ths.	3. $\frac{5}{9}$ to 45ths.	4. $\frac{12}{13}$ to 156ths.
5. $\frac{4}{5}$ to 20ths.	6. $\frac{7}{15}$ to 60ths.	7. $\frac{5}{17}$ to 187ths.
8. $\frac{7}{8}$ to 24ths.	9. $\frac{5}{14}$ to 70ths.	10. $\frac{23}{28}$ to 252ds.

11. Reduce $\frac{3}{4}$, $\frac{2}{3}$ and $\frac{1}{2}$ to 12ths.
12. Reduce $\frac{5}{6}$, $\frac{1}{3}$ and $\frac{7}{9}$ to 18ths.
13. Reduce $\frac{5}{6}$, $\frac{3}{4}$, $\frac{7}{12}$ and $\frac{11}{15}$ to 60ths.
14. Reduce $\frac{3}{4}$, $1\frac{1}{2}$ and $2\frac{1}{3}$ to 12ths.
15. Reduce 2, $3\frac{1}{3}$, $\frac{5}{9}$ and $1\frac{1}{6}$ to 18ths.

159. ***To reduce fractions to equivalent fractions having the least common denominator.***

(1) Reduce $\frac{5}{6}$, $\frac{7}{8}$ and $\frac{2}{3}$ to equivalent fractions having the least possible common denominator.

Explanation.—The required least common denominator is evidently the L. C. M. of the denominators. Hence, I first find the L. C. M. of the denominators, which is 24, then reduce each of the fractions to 24ths, as in the last article.

Operation.

$$\begin{array}{r|l} 2 & 6,\ 8,\ 3 \\ \hline & \cancel{3},\ 4,\ 3 \end{array}$$

$2 \times 4 \times 3 = 24.$

$\frac{5}{6} = \frac{20}{24}$

$\frac{7}{8} = \frac{21}{24}$

$\frac{2}{3} = \frac{16}{24}.$

RULE.—*Find the least common multiple of all the denominators for the least common denominator. Divide this least common denominator by the denominator of each fraction, and multiply the quotient by its numerator; the several products will be the numerators required.*

NOTE.—In applying this Rule, each fraction should be reduced to its *lowest* terms and mixed numbers to improper fractions, before commencing the operation.

EXERCISE XXV.

Reduce the following to equivalent fractions having the least common denominator.

2. $\frac{3}{4}$, $\frac{5}{6}$. 3. $\frac{3}{10}$, $\frac{7}{15}$, $\frac{1}{6}$. 4. $\frac{2}{3}$, $\frac{1}{2}$, $\frac{5}{6}$, $\frac{3}{4}$.
5. $\frac{1}{2}$, $\frac{3}{8}$. 6. $\frac{5}{8}$, $\frac{7}{12}$, $\frac{2}{3}$. 7. $\frac{4}{5}$, $\frac{3}{10}$, $\frac{5}{12}$, $\frac{5}{20}$.
8. $\frac{3}{7}$, $\frac{6}{9}$. 9. $\frac{3}{4}$, $\frac{5}{9}$, $\frac{7}{18}$. 10. $\frac{4}{7}$, $\frac{5}{14}$, $\frac{8}{21}$, $\frac{5}{8}$.

ADDITION.

160. Like fractions are those which have the same fractional unit, or the same denominator.

(1) 3 fourths and 5 fourths, or $\frac{3}{4}$ and $\frac{5}{4}$, are like fractions.

161. Unlike fractions are those which have different fractional units, or different denominators.

(1) 3 fourths and 5 tenths, or $\frac{3}{4}$ and $\frac{5}{10}$, are unlike fractions.

162. ***To add two or more fractions.***

PRINCIPLE.—*Only like fractions can be added.*

(1) Find the sum of $\frac{7}{10}$, $\frac{5}{10}$ and $\frac{3}{10}$.

Explanation.—As the fractions have the same denominator, they have the same fractional unit 1 tenth. $\frac{7}{10}$ = 7 tenths, $\frac{5}{10}$ = 5 tenths, $\frac{3}{10}$ = 3 tenths. 7 tenths + 5 tenths + 3 tenths = 15 tenths = $\frac{15}{10}$ = $1\frac{1}{2}$, Ans.

Operation.

10, the com. denominator.

7, 1st. numerator.
5, 2d. "
3, 3d. "

15, sum of numerators.

$\frac{15}{10} = 1\frac{1}{2}$, sum of fractions.

(2) Find the sum of $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$.

Explanation.—As these are unlike fractions, they cannot be added in their present forms. Hence, I reduce them to like fractions, or to a common denominator. The L. C. M. of 3, 4 and 6 is 12. Reducing each fraction to 12ths, I have $\frac{2}{3} = \frac{8}{12}$, $\frac{3}{4} = \frac{9}{12}$, $\frac{5}{6} = \frac{10}{12}$. $\frac{8}{12} + \frac{9}{12} + \frac{10}{12} = \frac{27}{12} = 2\frac{1}{4}$.

Operation.

The L. C. M. of 3, 4, 6, is
12, the com. denominator.

8, 1st. numerator.
9, 2d. "
10, 3d. "

27, sum of numerators.

$\frac{27}{12}$, or $2\frac{1}{4}$, the sum of the fractions.

RULE.—*Reduce the fractions to equivalent fractions having a common denominator, add the numerators, and divide the sum by the denominator.*

NOTE.—It is not necessary to write the work out in full. It may be abbreviated as shown in the solution of the preceding example.

EXERCISE XXVI.

Find the sum of:

3. $\frac{2}{3}$, $\frac{3}{4}$.
4. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$.
5. $\frac{5}{7}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{5}{21}$.
6. $\frac{1}{2}$, $\frac{5}{6}$.
7. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$.
8. $\frac{4}{5}$, $\frac{3}{14}$, $\frac{3}{10}$, $\frac{5}{7}$.
9. $\frac{5}{6}$, $\frac{7}{8}$.
10. $\frac{4}{5}$, $\frac{1}{6}$, $\frac{7}{10}$.
11. $\frac{7}{12}$, $\frac{5}{18}$, $\frac{5}{9}$, $\frac{7}{24}$.
12. $\frac{7}{12}$, $\frac{11}{15}$.
13. $\frac{4}{9}$, $\frac{7}{12}$, $\frac{13}{20}$.
14. $\frac{3}{8}$, $\frac{1}{22}$, $\frac{7}{24}$, $\frac{29}{88}$.

(15) What is the sum of $4\frac{2}{3}$, $5\frac{5}{6}$, $7\frac{1}{4}$?

Explanation.—When there are mixed numbers, add the fractions first, and then add this sum to the sum of the integers. Thus, $\frac{3}{12} + \frac{10}{12} + \frac{8}{12} = \frac{21}{12} = 1\frac{3}{4}$. Write the $\frac{3}{4}$ below, and add 1 to the sum of 7, 5, 4, which makes 17. Hence, the entire sum is $17\frac{3}{4}$.

Operation.

$$4\frac{2}{3} = 4\frac{8}{12}$$
$$5\frac{5}{6} = 5\frac{10}{12}$$
$$7\frac{1}{4} = 7\frac{3}{12}$$
$$17\frac{3}{4}$$

Find the value of:

16. $9\frac{1}{5} + 5\frac{2}{3}$.
17. $\frac{3}{4} + \frac{5}{6} + 4\frac{3}{4}$.
18. $15\frac{1}{2} + 27\frac{1}{3}$.
19. $\frac{7}{9} + 3\frac{1}{4} + 5\frac{3}{7}$.
20. $25\frac{3}{4} + 32\frac{2}{3}$.
21. $4\frac{3}{8} + 7\frac{8}{9} + 6\frac{3}{5}$.
22. $30\frac{2}{5} + 13\frac{1}{4}$.
23. $8\frac{5}{9} + 6\frac{3}{4} + 11\frac{7}{12}$.

24. If you pay $\$\frac{1}{2}$ for figs and $\$\frac{5}{8}$ for raisins, what do you pay for both?

25. Henry paid $\$\frac{1}{2}$ for a whip, $\$\frac{1}{3}$ for a ball, $\$\frac{1}{4}$ for a bat, and $\$\frac{1}{6}$ for a top; how much money did he pay for all?

26. There are 5 boys in a class and each boy has $\$\frac{1}{3}$; how much have all the boys?

27. If each of four sides of a field is $\frac{13}{18}$ of a mile in length, how far is *it around the* field?

28. A grocer sold $5\frac{2}{3}$ pounds of coffee and $7\frac{5}{6}$ pounds of tea; how many pounds of both did he sell?

29. A farmer sold a quantity of wheat for $\$36\frac{2}{3}$, corn for $\$42\frac{1}{2}$, oats for $\$45\frac{7}{8}$, and rye for $\$35\frac{3}{4}$; what was the amount of his sales?

30. If you go $5\frac{3}{4}$ miles east and I go $7\frac{5}{6}$ miles west from this schoolhouse, how far apart will we be?

31. Two men have equal sums of money; if one of them earn $\$73\frac{13}{16}$ and the other spends $\$45\frac{11}{16}$ how much more will the one have than the other?

32. A lad sold three fishes. The first weighed $4\frac{3}{4}$ pounds, the second $3\frac{5}{8}$ pounds, and the third $5\frac{1}{2}$ pounds; how many pounds of fish were sold?

33. A man traveled $68\frac{3}{8}$ miles Monday, $22\frac{1}{4}$ miles Tuesday, $37\frac{4}{5}$ miles Wednesday; how far did he travel in all?

34. A lady paid $\$2\frac{7}{8}$ for gloves, $\$1\frac{3}{4}$ for a lace collar and $\$4\frac{1}{2}$ for a parasol; what was the amount of her bill?

35. A man sold $48\frac{3}{5}$ acres of land to one man, $62\frac{1}{4}$ acres to a second, $56\frac{2}{3}$ acres to a third, and had $73\frac{1}{2}$ acres left; how many acres had he at first?

36. A German came to America when he was $9\frac{3}{4}$ years old, $12\frac{5}{8}$ years afterward he married, $7\frac{1}{4}$ years later his wife died, and she has been dead $7\frac{1}{2}$ years. What is his age?

37. From A to B it is $168\frac{4}{5}$ miles, from B to C $199\frac{19}{20}$ miles, from C to D $783\frac{9}{10}$ miles; how far is it from A to D, through B and C?

38. A train ran from New Orleans to Baton Rouge in $2\frac{3}{4}$ hours, from Baton Rouge to Vicksburg in $5\frac{1}{6}$ hours, and from Vicksburg to Memphis in $6\frac{2}{3}$ hours. What was the time from New Orleans to Memphis?

39. Monday I deposited $\$217\frac{7}{10}$ in bank, Tuesday $\$328\frac{5}{6}$,

Wednesday $\$425\frac{1}{3}$, and Thursday $\$487\frac{1}{5}$; what was the amount of my deposits for the four days?

40. A boy sold $\frac{1}{5}$ of his oranges to one man, $\frac{1}{4}$ of them to another, and $\frac{1}{6}$ of them to a third; what part of his oranges did he sell?

41. A man gave his wife $\frac{1}{2}$ of his estate, his son $\frac{5}{17}$ of it, his daughter $\frac{16}{136}$ of it and his servant $\frac{3}{34}$ of it; what part of his estate did he give away?

42. In a park are an elm $20\frac{1}{2}$ feet high, an oak $30\frac{2}{5}$ feet higher than the elm, and a pine $40\frac{3}{4}$ feet higher than the oak; what is the height of each tree?

43. A merchant bought 3 crates of crockery; for the first he paid $\$368\frac{3}{4}$, for the second $\$81\frac{1}{2}$ more than the first, and for the third $\$49\frac{7}{8}$ more than the second. What did he pay for all?

SUBTRACTION.

163. *To subtract one fraction from another.*

PRINCIPLE.—*Only like fractions can be subtracted, the one from the other.*

(1) From $\frac{3}{4}$ take $\frac{2}{3}$.

Explanation.—As $\frac{3}{4}$ and $\frac{2}{3}$ are unlike fractions, they must be reduced to a common denominator. The L. C. M. of 4 and 3 is 12, hence, I reduce each fraction to 12ths. $\frac{3}{4} = 9$ twelfths, $\frac{2}{3} = 8$ twelfths. 9 twelfths — 8 twelfths = 1 twelfth $= \frac{1}{12}$.

Operation.

The L. C. M. of 4 and 3 is
12, the com. denominator.

9, 1st. numerator.
8, 2d. "

1, diff. of numerators.
$\frac{1}{12}$, diff. of fractions.

RULE.—*Reduce the fractions to a common denominator, subtract the numerator of the subtrahend from the numerator of the minuend, and place the difference over the denominator.*

EXERCISE XXVII.

2. From $\frac{1}{2}$ take $\frac{1}{3}$.
3. From $\frac{5}{7}$ take $\frac{3}{8}$.
4. From $\frac{3}{4}$ take $\frac{2}{5}$.
5. From $\frac{13}{15}$ take $\frac{11}{24}$.
6. From $\frac{5}{6}$ take $\frac{2}{9}$.
7. From $\frac{19}{2}$ take $\frac{17}{3}$.
8. Take $\frac{3}{8}$ from $\frac{5}{4}$.
9. Take $\frac{17}{56}$ from $\frac{21}{32}$.
10. Take $\frac{2}{15}$ from $\frac{1}{6}$.
11. Take $\frac{23}{60}$ from $\frac{37}{45}$.
12. Take $\frac{3}{8}$ from $\frac{7}{12}$.
13. Take $\frac{13}{54}$ from $\frac{43}{72}$.

(14) From $16\frac{2}{3}$ take $7\frac{1}{5}$.

Explanation.—When there are mixed numbers subtract the fractions first, and annex their difference ($\frac{7}{15}$) to the difference of the integers (9).

Operation.

$$\begin{aligned} 16\tfrac{2}{3} &= 16\tfrac{10}{15} \\ 7\tfrac{1}{5} &= \underline{\;7\tfrac{3}{15}} \\ & \quad 9\tfrac{7}{15} \end{aligned}$$

(15) From $11\frac{1}{4}$ take $9\frac{2}{3}$.

Explanation.—As $\frac{8}{12}$ cannot be taken from $\frac{3}{12}$, take 1, or $\frac{12}{12}$, from 11 leaving 10, and add it to $\frac{3}{12}$ making $\frac{15}{12}$. Then $\frac{8}{12}$ from $\frac{15}{12}$ leaves $\frac{7}{12}$, and 9 from 10 leaves 1.

Operation.

$$\begin{aligned} 11\tfrac{1}{4} &= 11\tfrac{3}{12} \\ 9\tfrac{2}{3} &= \underline{\;9\tfrac{8}{12}} \\ & \quad 1\tfrac{7}{12} \end{aligned}$$

Find the value of:

16. $8\frac{3}{5} - 7\frac{1}{3}$.
17. $5 - 2\frac{1}{4}$.
18. $6\frac{2}{3} - \frac{3}{4}$.
19. $9\frac{7}{8} - 5\frac{1}{2}$.
20. $7 - 5\frac{3}{8}$.
21. $2\frac{1}{3} - 1\frac{5}{6}$.

What is the value of:

22. $16\frac{3}{7} - 5\frac{13}{14}$?
23. $41\frac{5}{7} - 2\frac{2}{9}$?
24. $93\frac{1}{2} - 6\frac{21}{41}$?
25. $25\frac{1}{6} - 7\frac{5}{8}$?
26. $84\frac{5}{16} - 73\frac{4}{21}$?
27. $8\frac{13}{40} - 6\frac{1}{3}$?

28. A lad went into a store with $\$2\frac{1}{2}$, of which he spent $\$1\frac{1}{3}$ for a knife; how much had he left?

29. A man's salary per day is $\$9\frac{3}{4}$ and his expenses $\$4\frac{3}{5}$: how much does he save daily?

30. From a rod $12\frac{1}{2}$ feet long, $9\frac{5}{6}$ feet were cut off; how much remained?

31. From a jar of honey weighing $25\frac{1}{5}$ pounds a grocer sold $16\frac{2}{3}$ pounds; how much remained?

32. A father divided a number of apples among his 3 sons; to the first he gave $\frac{2}{5}$ of them, to the second $\frac{1}{4}$, and the remainder to the third; what part did the third receive?

33. Four men engage to do a certain work; the first is to do $\frac{2}{7}$ of it, the second $\frac{1}{6}$, the third $\frac{2}{9}$, and the fourth the balance; what part of the work is the fourth to do?

34. Five boys together have \$7. Each of the first two has \$$1\frac{1}{3}$, and each of the second two \$$1\frac{3}{5}$; how much has the fifth boy?

35. From a bin containing $675\frac{1}{2}$ bushels of wheat there were sold $84\frac{2}{5}$ bushels at each of two sales, and $78\frac{3}{4}$ bushels at each of three sales; how many bushels remained?

36. James and John together have \$$11\frac{1}{3}$, and James lacks \$$2\frac{2}{5}$ of having \$8; now much has each?

37. If A had \$$635\frac{4}{5}$ more he would have a \$1000, and he and B together have \$$987\frac{3}{4}$; how much has A and B each?

38. A boy agreed to pay \$$11\frac{1}{4}$ for a gun; at one time he paid \$$3\frac{2}{3}$, and at another \$$5\frac{1}{5}$; how much does he still owe?

39. From a cistern containing $697\frac{17}{32}$ gallons there were drawn at one time $256\frac{5}{8}$ gallons, and at another time $297\frac{3}{4}$ gallons; how much remained?

40. William had \$$5\frac{1}{3}$ of which he spent \$$2\frac{1}{4}$ and then earned \$$3\frac{2}{5}$; how much had he then?

41. A farmer having $246\frac{7}{10}$ acres of land, bought $57\frac{1}{4}$ acres more, and then sold $120\frac{2}{3}$ acres. How much had he left?

42. How much money shall I have left of \$$8\frac{1}{4}$ after paying my grocer \$$6\frac{2}{5}$?

43. What is the error in stating that the circumference of the earth at the equator is 25,000 miles, when it is exactly $24{,}899\frac{11}{500}$ miles?

44. If 1 be added to each term of $\frac{3}{5}$, is the value of the fraction *increased or diminished*? How much?

45. If 2 be subtracted from each term of $\frac{5}{8}$, is the value of the fraction increased or diminished? How much?

46. From the sum of $1\frac{2}{3}$ and $2\frac{1}{2}$ subtract the difference between $3\frac{1}{2}$ and $1\frac{1}{4}$.

47. From the sum of $\frac{5}{7}$ and $3\frac{1}{2}$ subtract the difference between $4\frac{1}{3}$ and $5\frac{1}{4}$.

QUESTIONS.

What is a tractional unit? A fraction? The terms? Numerator? Denominator? The two significations of a fraction? Value of a fraction?

How many kinds of fractions with respect to value? With respect to form? Give an example of each.

What is the rule for reducing fractions to lowest terms? Improper fractions to mixed numbers? Mixed numbers to improper fractions? Fractions to their least common denominator?

What are like fractions? Unlike fractions? Give rule for addition of fractions. For subtraction.

MULTIPLICATION.

164. ***To multiply a fraction by an integer or an integer by a fraction.***

(1) Multiply $\frac{7}{12}$ by 4.

1° Process.

$$\frac{7 \times 4}{12} = \frac{28}{12} = 2\tfrac{4}{12} = 2\tfrac{1}{3}$$

2° Process.

$$\frac{7}{12 \div 4} = \frac{7}{3} = 2\tfrac{1}{3}.$$

Explanation.—A fraction is multiplied by an integer by multiplying its numerator or by dividing its denominator by the integer.

Hence, in the first process I multiply the numerator by 4, and in the second I divide the denominator by 4; and reduce.

When the denominator of the fraction is exactly divisible by the multiplier, the second process is preferable.

(2) Multiply 18 by $\frac{5}{6}$.

18 multiplied by $\frac{5}{6}$ is $\frac{5}{6}$ of 18. For, to multiply is to take one number as many times as there are units in another. In $\frac{5}{6}$ there is $\frac{5}{6}$ of a *unit; hence,* to multiply 18 by $\frac{5}{6}$ is to take 18, $\frac{5}{6}$ of 1 time, or to find *of 1 time 18,* which is $\frac{5}{6}$ of 18.

Explanation.— $\frac{1}{6}$ of 18 is 3; $\frac{5}{6}$ of 18 is 5 times 3, or 15.

Again, $\frac{5}{6}$ of 18 is $\frac{1}{6}$ of 5 times 18, or $\frac{1}{6}$ of 90, which is 15.

1° Process.

$18 \div 6 = 3, \times 5 = 15.$

2° Process.

$18 \times 5 = 90, \div 6 = 15.$

RULE I. *Divide the integer by the denominator of the fraction and multiply the quotient by the numerator.* Or,

II. *Multiply the integer by the numerator of the fraction and divide the product by the denominator.*

NOTES I.—When practicable, the first of the preceding rules is preferable.

II. When the word "of" follows a fraction, it is equivalent to the sign $\times$.

EXERCISE XXVIII.

Perform the following indicated operations:

3. $12 \times \frac{3}{4}$.	4. $54 \times \frac{8}{9}$.	5. $\frac{9}{13} \times 7$.
6. $13 \times \frac{4}{5}$.	7. $75 \times \frac{13}{18}$.	8. $15 \times \frac{9}{14}$.
9. $\frac{5}{6} \times 30$.	10. $\frac{11}{17} \times 85$.	11. $\frac{25}{36}$ of 9.
12. $\frac{7}{9} \times 54$.	13. $\frac{15}{16} \times 25$.	14. $\frac{15}{16} \times 10$.
15. $\frac{3}{11}$ of 33.	16. $\frac{23}{27}$ of 63.	17. $6 \times \frac{11}{18}$.
18. $\frac{3}{7}$ of 5.	19. $\frac{19}{24}$ of 84.	20. $\frac{13}{25}$ of 85.

165. *To multiply one fraction by another.*

(21) Multiply $\frac{3}{7}$ by $\frac{4}{5}$.

Explanation.— $\frac{3}{7}$ multiplied by $\frac{4}{5}$ is $\frac{4}{5}$ of $\frac{3}{7}$, see Art. 164. $\frac{1}{5}$ of $\frac{3}{7}$ is $\frac{3}{5 \times 7}$, and $\frac{4}{5}$ of $\frac{3}{7}$ is 4 times $\frac{3}{5 \times 7}$, which is $\frac{4 \times 3}{5 \times 7}$, or $\frac{12}{35}$.

Operation.

$\frac{4 \times 3}{5 \times 7} = \frac{12}{35}.$

RULE I. *Reduce all integers and mixed numbers to improper fractions.*

II. *Multiply the numerators for the numerator, and the denominators for the denominator, of the product.* Or.

III. *Draw a vertical line; on the left place the numer-*

ators, on the right the denominators, and proceed according to the rules of cancellation.

NOTE.—This rule embraces also the two former cases. Whatever method is used, it is best to cancel common factors in the numerators and denominators before multiplying.

What is the value of:

22. $\frac{1}{2}$ of $\frac{4}{5}$? 23. $\frac{3}{4}$ of $\frac{8}{9}$? 24. $\frac{16}{7} \times \frac{21}{4}$?
25. $\frac{2}{3}$ of $\frac{6}{7}$? 26. $\frac{5}{6}$ of $\frac{12}{25}$? 27. $3\frac{1}{8} \times 3\frac{1}{5}$?

Find the product of the following:

28. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.
29. $\frac{3}{7}$, $8\frac{3}{4}$, $\frac{2}{5}$.
30. $\frac{4}{5}$, $\frac{5}{6}$, $\frac{9}{10}$.
31. $\frac{8}{9}$, $71\frac{4}{5}$, $6\frac{3}{4}$.
32. $\frac{5}{7}$, $\frac{3}{5}$ of $\frac{2}{3}$.
33. $\frac{9}{10}$, $17\frac{3}{11}$, $5\frac{1}{2}$.
34. $\frac{3}{4}$ of $\frac{5}{7}$, $\frac{28}{15}$.
35. $3\frac{1}{2}$, $3\frac{1}{2}$, $\frac{5}{14}$.
36. $\frac{2}{3}$, $\frac{5}{11}$ of $2\frac{3}{4}$.
37. $3\frac{4}{5}$, $2\frac{1}{3}$, $\frac{15}{133}$.
38. $\frac{1}{5}$, 16, $26\frac{2}{3}$.
39. $3\frac{5}{8}$, $2\frac{2}{3}$, $\frac{1}{4}$ of $\frac{3}{29}$.

Find the value of:

40. $\frac{7}{8} \times 2\frac{1}{2}$.
41. $\frac{8}{9}$ of $7\frac{1}{5}$ of $3\frac{3}{4}$.
42. $\frac{7}{12} \times 3\frac{3}{7}$.
43. $\frac{2}{3}$ of $7\frac{1}{4} \times \frac{7}{9}$ of $11\frac{3}{4}$.
44. $4\frac{1}{2} \times 5\frac{1}{3}$.
45. $\frac{4}{7} \times 8\frac{3}{10}$ of $\frac{4}{7}$ of $9\frac{1}{4}$.
46. $4\frac{9}{10} \times 1\frac{3}{7}$.
47. $9 \times \frac{3}{5} \times \frac{2}{7}$ of 17.

Multiply:

48. $\frac{1}{2} + \frac{1}{3}$ by $\frac{1}{2} - \frac{1}{3}$.
49. $2\frac{1}{2} + 3\frac{1}{3}$ by $5\frac{1}{2} - 4\frac{1}{3}$.
50. $\frac{3}{4} + \frac{1}{2}$ by $\frac{2}{5} + \frac{7}{10}$.
51. $5\frac{3}{4} + 7\frac{1}{2}$ by $3\frac{2}{5} + 1\frac{1}{4}$.
52. $\frac{5}{6} \times \frac{3}{4}$ by $\frac{3}{5} \times \frac{4}{7}$.
53. $3\frac{1}{3} \times 2\frac{2}{5}$ by $7\frac{1}{3} \times 1\frac{1}{11}$.

How much money will it take to buy:

54. $\frac{3}{4}$ of a yard of cloth, at $\$\frac{4}{5}$ a yard?
55. $4\frac{1}{2}$ barrels of cider, at $\$2\frac{2}{3}$ a barrel?
56. $1\frac{1}{5}$ pounds of coffee, at $17\frac{1}{2}$ cts. a pound?
57. $2\frac{2}{5}$ barrels of flour, at $\$5\frac{3}{4}$ a barrel?
58. $8\frac{3}{4}$ pounds of tea, at $\$\frac{4}{7}$ a pound?
59. $18\frac{3}{4}$ acres of land, at $\$18\frac{3}{4}$ per acre?
60. $11\frac{1}{9}$ cords of wood, at $\$4\frac{1}{2}$ per cord?
61. $16\frac{2}{3}$ pounds of lard, at $16\frac{2}{3}$ cts. per pound?

How far will a man walk:

62. In $3\frac{1}{3}$ hours, at the rate of $1\frac{1}{2}$ miles per hour?

63. In $3\frac{1}{4}$ hours, at the rate of $3\frac{1}{3}$ miles per hour?

What part of a ship did a man sell:

64. Who owned $\frac{4}{9}$ of it, and sold $\frac{3}{8}$ of his share?

65. Who owned $\frac{7}{30}$ of it, and sold $\frac{4}{9}$ of his share?

What part of a factory belongs to a man:

66. Who owned $\frac{3}{8}$ of it, and sold $\frac{2}{3}$ of his share?

67. Who owned $\frac{5}{9}$ of it, and sold $\frac{4}{7}$ of his share?

68. John had $15, of which he spent $\frac{2}{3}$ of $\frac{6}{5}$; how much had he left?

69. A lady had $\frac{7}{9}$ of 16\frac{1}{5}$ in her purse and spent $\frac{2}{3}$ of it; how much remained?

70. Henry and John start from the same place and travel in the same direction; if Henry goes $5\frac{5}{6}$ yards faster per minute than John, how far apart will they be in $4\frac{2}{7}$ minutes?

71. A clock gains $11\frac{1}{4}$ minutes a day; if it is right at 8 o'clock A. M. on Monday, what time will the clock indicate on the following Saturday at noon?

72. How much money will it take to pay for $6\frac{2}{3}$ loads of wood, at 4\frac{2}{5}$ a load?

73. Find the cost of $19\frac{3}{5}$ acres of land, at 24\frac{5}{12}$ per acre.

74. A railroad train ran for 29 hours at the average rate of $19\frac{4}{5}$ miles per hour; find the distance passed over.

75. A farmer sold 6 bales of cotton each weighing $456\frac{1}{2}$ pounds, at $8\frac{2}{3}$ cts. a pound; and 12 hogsheads of sugar each weighing $1125\frac{2}{3}$ pounds, at $6\frac{3}{4}$ cts. a pound; what did he receive for all?

DIVISION.

166. ***To divide a fraction by an integer.***

(1) Divide $\frac{6}{7}$ by 3.

Explanation.—A fraction is divided by an integer by dividing its numerator or by multiplying its denominator by the integer. Hence, in the first process I divide the numerator by 3, and in the second I multiply the denominator by 3, and reduce.

In the third process I invert the divisor, and proceed as in multiplication.

1° Process.

$$\frac{6 \div 3}{7} = \frac{2}{7}.$$

2° Process.

$$\frac{6}{7 \times 3} = \frac{6}{21} = \frac{2}{7}.$$

3° Process.

$$\frac{\overset{2}{\cancel{6}}}{7} \times \frac{1}{\cancel{3}} = \frac{2}{7}.$$

RULE. —*Divide the numerator or multiply the denominator of the fraction by the integer.*

167. ***To divide an integer by a fraction.***

(2) Divide 3 by $\frac{4}{5}$.

Explanation.— $\frac{4}{5}$ is 4 times $\frac{1}{5}$; $\frac{1}{5}$ is contained in 3 3×5, or 15 times; hence, $\frac{4}{5}$ is contained in 3, $\frac{1}{4}$ of 15 times.

In the second process I invert the divisor and proceed as in multiplication.

1° Process.

$$5 \times 3 = 15, \div 4 = 3\tfrac{3}{4}.$$

2° Process.

$$\frac{3}{1} \times \frac{5}{4} = \frac{15}{4} = 3\tfrac{3}{4}.$$

RULE.—*Multiply the integer by the denominator of the fraction and divide the product by the numerator.*

EXERCISE XXIX.

3. Divide 8 by $\frac{4}{5}$.	4. $(\frac{1}{2} + \frac{1}{3}) \div 4 = ?$
5. Divide 12 by $\frac{6}{7}$.	6. $(\frac{2}{3} - \frac{1}{6}) \div 3 = ?$
7. Divide 20 by $\frac{4}{5}$.	8. $(\frac{3}{4}$ of $\frac{1}{2}) \div 7 = ?$
9. Divide $\frac{6}{7}$ by 2.	10. $8 \div (\frac{1}{3} + \frac{1}{4}) = ?$
11. Divide $\frac{45}{51}$ by 9.	12. $6 \div (\frac{2}{5} - \frac{1}{8}) = ?$
13. *Divide* $\frac{11}{18}$ by 5.	14. $5 \div (\frac{2}{3}$ of $\frac{6}{7}) = ?$

(15) I paid $\$2\frac{1}{4}$ for 5 bushels of oats; how much was that per bushel?

1 bushel costs 1 fifth as much as 5 bushels, or 1 fifth of $\$2\frac{1}{4}$; 1 fifth of $\$2\frac{1}{4}$ is 1 fifth of $\$\frac{9}{4}$, which is $\$\frac{9}{20}$.

16. $\$2\frac{1}{4}$ for 3 turkeys is how much a piece?

17. $\$3\frac{1}{3}$ for 13 dozen eggs is how much per dozen?

18. $\$1\frac{2}{3}$ for 4 yards of cloth is how much per yard?

19. $37\frac{3}{4}$ yards for 5 suits is how much per suit?

20. John received $\$3\frac{1}{3}$ for 4 days work; how much was that per day?

21. A railroad train ran $168\frac{1}{4}$ miles in 7 hours; what was its average rate per hour?

168. ***To divide a fraction by a fraction.***

(22) Divide $\frac{3}{4}$ by $\frac{5}{6}$.

Explanation.—I divide $\frac{3}{4}$ first by 5 and obtain $\frac{3}{20}$; but 5 is 6 times as great as the true divisor $\frac{5}{6}$, hence, the quotient $\frac{3}{20}$ is 6 times too small. Therefore the true quotient is 6 times $\frac{3}{20}$, which is $\frac{18}{20}$, or $\frac{9}{10}$.

Operation.

$$\frac{3}{4} \times \frac{6}{5} = \frac{18}{20} = \frac{9}{10}.$$

GENERAL RULE. I. *Reduce integers and mixed numbers to improper fractions,*

II. *Invert the divisor and proceed as in multiplication.*

Perform the following indicated divisions:

23. $\frac{2}{3} \div \frac{3}{8}$.

24. $(\frac{1}{2} + \frac{1}{3}) \div (\frac{1}{2}$ of $\frac{3}{4})$.

25. $\frac{3}{4} \div \frac{5}{6}$.

26. $(\frac{2}{3} - \frac{1}{5}) \div (\frac{3}{8} - \frac{2}{7})$.

27. $\frac{7}{11} \div \frac{3}{9}$.

28. $(\frac{5}{8}$ of $16) \div (\frac{2}{3} \times \frac{6}{8})$.

29. $\frac{5}{6} \div \frac{2}{3}$.

30. $(\frac{3}{4} + \frac{5}{7}) \div (16 - 2\frac{3}{4})$.

31. $\frac{7}{8} \div \frac{5}{16}$.

32. $(2\frac{1}{2} + 3\frac{2}{5}) \div (5\frac{1}{2} - 3\frac{2}{3})$.

33. $1\frac{1}{2} \div \frac{7}{8}$.

34. $(\frac{5}{8} \div \frac{3}{4}) \div (\frac{2}{3} \times \frac{6}{8})$.

35. $3\frac{1}{3} \div 5\frac{1}{2}$.

36. $(\frac{4}{5}$ of $3\frac{1}{2}) \div (\frac{3}{8} \div 1\frac{3}{7})$.

37. $6 \div 3\frac{1}{4}$.

38. $(10\frac{4}{5} \div 2\frac{4}{7}) \div (7 + \frac{7}{8})$.

39. $3\frac{1}{4} \div 2$.

40. $(\frac{2}{3} + \frac{3}{4}) \div (3\frac{1}{3} \times \frac{4}{9})$.

41. $5\frac{1}{6} \div \frac{7}{8}$.

42. $[4\frac{2}{3} - 6\frac{2}{3} \times \frac{3}{5}] \div [2\frac{3}{4} + 8\frac{3}{4} \div 13\frac{1}{8}]$.

169. Complex fractions.

(43) Find the value of $\dfrac{5\frac{1}{2} - 2\frac{1}{3}}{\frac{3}{4} \text{ of } 2\frac{1}{2}}$.

Explanation.—This example simply means that the fractional numerator is to be divided by the fractional denominator. Hence, to find its value is a simple problem in division of fractions. Thus:

$$\frac{5\frac{1}{2} - 2\frac{1}{3}}{\frac{3}{4} \text{ of } 2\frac{1}{2}} = (5\tfrac{1}{2} - 2\tfrac{1}{3}) \div (\tfrac{3}{4} \text{ of } 2\tfrac{1}{2}) = 3\tfrac{1}{6} \div \tfrac{15}{8} = \tfrac{19}{6} \times \tfrac{8}{15} = 1\tfrac{31}{45}.$$

Find the value of:

44. $\dfrac{18\frac{3}{4}}{\frac{15}{19}}$.

45. $\dfrac{\frac{8}{9} - \frac{5}{6}}{\frac{3}{4} - \frac{2}{3}}$.

46. $\dfrac{2\frac{2}{5} \text{ of } 2\frac{5}{8}}{2\frac{3}{5} - 2\frac{1}{4}}$.

47. $\dfrac{\frac{15}{17}}{3\frac{5}{6}}$.

48. $\dfrac{\frac{4}{5} \div \frac{5}{6}}{\frac{4}{5} \times \frac{5}{6}}$.

49. $\dfrac{\frac{4}{5} \text{ of } (2\frac{1}{3} + 1\frac{1}{6})}{\frac{4}{13} \text{ of } (5\frac{1}{6} - 3\frac{1}{15})}$.

50. $\dfrac{\frac{12}{13}}{9\frac{1}{3}}$.

51. $\dfrac{8\frac{5}{8} \div 18\frac{2}{5}}{10 - \frac{5}{11}}$.

52. $\dfrac{\frac{1}{5} \text{ of } (4 - \frac{6}{5} + \frac{24}{25})}{\frac{1}{3} \text{ of } (2 - \frac{2}{3} + \frac{52}{27})}$.

170. Multiplication and division of fractions may often be shortened by cancellation.

(53) Divide $7\frac{1}{5} \times 5\frac{5}{6} \times 3\frac{3}{7}$ by $1\frac{1}{8} \times 2\frac{2}{3}$.

Explanation.—I draw a vertical line, on the left of it write the factors of the dividend and on the right the factors of the divisor, reducing mixed numbers to improper fractions as I bring them down. I next transfer each denominator to the opposite side (which is equivalent to multiplying both dividend and divisor by the same integer), and then proceed according to the rule of cancellation.

Operation.

$$\begin{array}{r|l r|l}
 & & 4\ \cancel{36} & \cancel{9} \\
\frac{36}{5} & \frac{9}{8} & \cancel{7}\ \cancel{35} & \cancel{8} \\
\frac{35}{6} & \frac{8}{3} & 4\ \cancel{24} & \cancel{5} \\
\frac{24}{7} & & \cancel{8} & \cancel{6} \\
 & & 3 & \cancel{7} \\
\hline
 & & & 48, \text{ Ans.}
\end{array}$$

If preferable a horizontal line may be used instead of a vertical one.

Divide:

54. $\frac{2}{3}$ of $\frac{7}{11}$ of $\frac{3}{8}$ by $1\frac{3}{13}$.

55. $3\frac{3}{4}$ by $\frac{4}{5} \times 8\frac{1}{4} \times \frac{6}{11} \times 3\frac{3}{10}$.

56. $3\frac{1}{2}$ of $\frac{5}{8}$ by $2\frac{1}{10}$ of $\frac{1}{13}$.

57. $2\frac{5}{8} \times \frac{4}{5} \times 19\frac{1}{3} \div 4\frac{5}{6} \div \frac{3}{10} \div 8 = ?$

58. At $\$\frac{2}{5}$ apiece, how many hats will $\$3\frac{1}{5}$ buy?

59. At $\$9\frac{4}{5}$ each, how many acres will $\$73\frac{1}{2}$ buy?

60. At $\$\frac{3}{4}$ apiece, how many chickens will $\$7\frac{1}{2}$ buy?

61. A laborer's wages per month are $\$25\frac{5}{6}$; in how many months will he earn $\$54\frac{43}{48}$?

62. In how many days will a horse eat $4\frac{1}{2}$ pecks of oats, if he eat $\frac{3}{8}$ of a peck daily?

63. In how many hours will a man walk $19\frac{1}{2}$ miles at the rate of $1\frac{7}{8}$ miles per hour?

64. What is the price of flour per barrel when I can buy $\frac{2}{5}$ of a barrel for $\$2\frac{1}{2}$?

65. Mary is $16\frac{1}{2}$ years old, which is $\frac{11}{13}$ of Susan's age; how old is Susan?

66. $8\frac{1}{3}$ yards are $\frac{5}{11}$ of the distance across a bridge; what is the length of the bridge?

67. A owns $\frac{7}{12}$ of a bale of cotton, and his part is worth $\$26\frac{1}{4}$; what is the whole bale worth?

68. A farmer exchanged $2\frac{1}{2}$ dozen eggs, worth 24 cts. per dozen, for 8 pounds of sugar; what was the price of sugar per pound?

69. Butter is worth 35 cts. a pound; what is the price of lard if $\frac{6}{7}$ of a pound of lard cost the same as $\frac{8}{15}$ of a pound of butter?

70. A newsboy bought papers at $2\frac{1}{2}$ cts., and sold them at $4\frac{1}{3}$ cts., and thereby made 22 cts. on all he sold; how many did he sell?

71. A and B start from the same place and travel in the same direction at the rates of $3\frac{2}{3}$ and $4\frac{1}{7}$ miles per hour; in how many *hours will they be* $15\frac{2}{3}$ miles apart?

72. Two bales of cotton weigh $437\frac{3}{8}$ and $443\frac{1}{5}$ pounds; what is the worth of both bales if the second bale is worth $30\frac{1}{2}$ cts. more than the first?

73. By what number must $3\frac{1}{3}$ be multiplied in order that the product shall be equal to $4\frac{1}{6}$ times $7\frac{1}{5}$?

74. How many pounds of coffee at $18\frac{3}{4}$ cts. will pay for $15\frac{1}{2}$ pounds of bacon at $10\frac{1}{2}$ cts?

75. A man owes $\frac{5}{8}$ of (\$$175\frac{3}{4}$, \$$262\frac{2}{5}$, and \$$581\frac{7}{12}$); in how many months can he discharge his indebtedness by making monthly payments of \$$79\frac{2}{3}$ each?

QUESTIONS.

How is a fraction multiplied by an integer? An integer by a fraction? A fraction by a fraction?

How is a fraction divided by an integer? An integer by a fraction? A fraction by a fraction? How is a complex fraction simplified?

GREATEST COMMON DIVISOR.

171. The greatest common divisor of two or more fractions is the greatest fraction that will exactly divide each of them.

172. ***To find the greatest common divisor of two or more fractions.***

173. One fraction is an exact divisor of another when its numerator is a divisor, and its denominator is a multiple, of the corresponding terms of the latter.

(1) $\frac{3}{16}$ is an exact divisor of $\frac{9}{8}$. $\frac{9}{8} \div \frac{3}{16} = \frac{9}{8} \times \frac{16}{3} = 6$.

Hence, a fraction is a common divisor of two or more fractions when its numerator is a divisor, and its denominator is a multiple, of the corresponding terms of each of the *given fractions.*

Therefore, *the G. C. D. of two or more fractions is the G. C. D. of their numerators divided by the L. C. M. of their denominators.*

(1) Find the G. C. D. of $\frac{2}{3}$, $\frac{4}{5}$ and $\frac{8}{15}$.

Solution. The G. C. D. of 2, 4 and 8 is 2.
The L. C. M. of 3, 5 and 15 is 15.

Hence, The G. C. D. of $\frac{2}{3}$, $\frac{4}{5}$ and $\frac{8}{15}$ is $\frac{2}{15}$.

Mixed numbers should be reduced to improper fractions.

EXERCISE XXX.

Find the G. C. D.

2. Of $\frac{3}{2}$, $\frac{6}{5}$ and $\frac{9}{10}$.
3. Of $14\frac{7}{12}$ and $95\frac{3}{8}$.
4. Of $\frac{2}{3}$, $\frac{4}{5}$ and $\frac{6}{7}$.
5. Of $23\frac{7}{16}$ and $213\frac{13}{24}$.
6. Of $3\frac{1}{3}$ and $4\frac{3}{4}$.
7. Of $261\frac{13}{14}$ and $652\frac{11}{21}$.
8. Of $2\frac{4}{7}$, $8\frac{2}{5}$ and $1\frac{1}{35}$.
9. Of $33\frac{3}{4}$, $67\frac{1}{2}$ and $70\frac{7}{8}$.

10. A man has three vessels holding respectively $4\frac{1}{2}$, 6 and $7\frac{1}{2}$ gallons; what is the capacity of the largest vessel which will exactly fill them if emptied into each an exact number of times?

11. A farmer sells $33\frac{3}{4}$ bushels of yellow corn, $67\frac{1}{2}$ bushels of white corn, and $70\frac{7}{8}$ bushels of mixed corn; required the size and number of the largest sacks that can be used in shipping, so as to keep the corn from being mixed.

LEAST COMMON MULTIPLE.

174. The least common multiple of two or more fractions is the least number that each of them will exactly divide.

175. ***To find the least common multiple of two or more fractions.***

176. One fraction is a multiple of another when its numerator is a multiple, and its denominator a divisor, of the corresponding terms of the latter.

(1) $\frac{20}{3}$ is a multiple of $\frac{5}{9}$. $\frac{20}{3} \div \frac{5}{9} = \frac{20}{3} \times \frac{9}{5} = 12$.

Hence, a fraction is a common multiple of two or more fractions when its numerator is a multiple, and its denominator a divisor, of the corresponding terms of each of the given fractions.

Therefore, *the L. C. M. of two or more fractions, in their simplest terms, is the L. C. M. of the numerators divided by the G. C. D. of their denominators.*

(1) Find the L. C. M. of $\frac{3}{4}$, $\frac{7}{8}$ and $\frac{9}{16}$.

Solution. The L. C. M. of 3, 7 and 9 is 63.
The G. C. D. of 4, 8 and 16 is 4.

Hence, The L. C. M. of $\frac{3}{4}$, $\frac{7}{8}$ and $\frac{9}{16}$ is $\frac{63}{4} = 15\frac{3}{4}$.

EXERCISE XXXI.

Find the L. C. M.

2. Of $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{12}$.
3. Of $3\frac{15}{16}$, $7\frac{7}{8}$, $5\frac{1}{4}$.
4. Of $\frac{3}{5}$, $\frac{5}{7}$, $\frac{2}{9}$.
5. Of $2\frac{3}{4}$, $3\frac{5}{8}$, $5\frac{7}{12}$.
6. Of $2\frac{1}{3}$, $1\frac{1}{6}$, $3\frac{8}{9}$.
7. Of $4\frac{4}{7}$, $5\frac{11}{14}$, $6\frac{17}{21}$.
8. Of $1\frac{3}{4}$, $1\frac{4}{5}$, $2\frac{5}{8}$.
9. Of $2\frac{5}{8}$, $8\frac{3}{4}$, $16\frac{2}{3}$, $31\frac{1}{2}$.

10. Three wheels are respectively $\frac{5}{9}$, $\frac{7}{12}$ and $\frac{14}{15}$ inches in circumference; what is the least distance in which each will revolve an exact number of times?

11. A lady, having been asked her age replied: "The number of years in my age is the least number that $1\frac{1}{4}$, $2\frac{1}{8}$ and $5\frac{5}{16}$ will exactly divide." What was her age?

12. A lad went from A to B at the rate of 15 miles in 4 hours; returned at the rate of 25 miles in 6 hours, and made the trip both ways in an exact number of hours; required the least distance from A to B.

13. A man can discharge a debt in an exact number of days by paying at the rate of \$$21\frac{1}{4}$ in 3 days, or \$$63\frac{3}{4}$ in 7 days, or \$$42\frac{1}{2}$ in 17 days; what is the least amount of the debt?

FRACTIONAL PARTS OF NUMBERS.

177. ***To find what part one number is of another.***

(1) What part of 8 is 2?

Solution: 1 is $\frac{1}{8}$ of 8; hence, 2 is 2 times $\frac{1}{8}$, or $\frac{1}{4}$ of 8. Therefore, the answer is $\frac{1}{4}$.

RULE.—*Divide the part by the number of which it is a part.*

EXERCISE XXXII.

2. What part of 4 is 2? Of 6 is 3? Of 12 is 5?

What part

3. Of 24 is 8?

4. Is \$5 of \$60?

5. Of $3\frac{1}{8}$ is $2\frac{1}{2}$?

6. Is \$15 of \$24.60?

7. Of $\frac{3}{4}$ is $\frac{1}{2}$?

8. Is ($\frac{1}{2}$ of $\frac{1}{3}$) of $\frac{1}{2} + \frac{1}{3}$?

9. Of $3\frac{1}{3}$ is $\frac{7}{12}$?

10. Is $\frac{5}{8} - \frac{1}{3}$ of $\frac{5}{8} + \frac{1}{3}$?

11. Of $5\frac{1}{4}$ is $2\frac{5}{8}$?

12. Is $\frac{5}{4} \div \frac{3}{2}$ of $\frac{5}{4} \times \frac{3}{2}$?

13. Of $8\frac{3}{4}$ is $1\frac{2}{3}$?

14. Is \$5.25 of \$180?

15. A horse, which cost \$90, was sold at a gain of \$6; what part of the cost was the gain?

16. Bought a buggy for \$$87\frac{1}{2}$ and sold it at a loss of \$$4\frac{3}{4}$; what part of the cost was the loss?

17. A father divided \$45 among his three sons, giving the first \$20, the second \$15 and the third the balance; what part of the whole did each receive?

18. A, B, C and D built a bridge at a cost of \$162, of which A paid \$75.60, B \$54.45, C \$27, and D the remainder; what part of the whole did each pay?

178. ***To find a number when a part of it is given.***

(19) $\frac{5}{8}$ of a number is 30; what is the number?

Explanation.—Since $\frac{5}{8}$ of the number is 30, $\frac{1}{8}$ of it is $\frac{1}{5}$ of 30, which is 6, and $\frac{8}{8}$ of it, or the whole of it, is 8 times 6, equal to 48.

Operation.

$30 \times \frac{8}{5} = 48$, Ans.

Formula. If $\frac{5}{8}$ of a number is 30, then

the number is 8 times $\frac{1}{5}$ of 30, = 48.

RULE.—*Divide the number which is the part by the given fractional part.*

What is the number:

20. Of which $\frac{7}{9}$ is 28?

21. Of which $\frac{11}{15}$ is 125?

22. Of which $\frac{5}{12}$ is 45?

23. Of which $\frac{3}{4}$ of $\frac{5}{8}$ is $17\frac{1}{2}$?

24. Of which $\frac{6}{13}$ is 54?

25. Of which $\frac{7}{9}$ less $\frac{1}{2}$ is $\frac{2}{3}$ of $2\frac{1}{4}$?

26. How old is John if $\frac{5}{7}$ of his age is 10 years?

27. $\frac{13}{25}$ of a bale of cotton was sold for \$20.75; what would the entire bale have brought at that rate?

28. If a boy can do $\frac{4}{7}$ of a certain work in 12 hours, how long will it take him to do the entire work?

29. $\frac{3}{5}$ of the time past noon is 4 hours and 30 minutes; what o'clock is it?

30. A jeweler paid \$45 for a watch, which was $\frac{9}{13}$ of what he received for it; what was his profit?

31. The cost of a buggy, viz: \$180, was $\frac{15}{19}$ of the sale; what part of the cost was the gain?

32. $\frac{3}{7}$ of John's age is 6 years, and $\frac{4}{5}$ of Henry's age is 12 years; how much older is Henry than John?

33. If the cost of an article is $\frac{7}{8}$ of the sale, what did a merchant receive for a hat which cost $\$3\frac{1}{3}$ and a saddle which cost $\$6\frac{2}{3}$?

34. A grocer sold a quantity of bacon at a profit of \$17.40, receiving for $\frac{11}{15}$ of it as much as all of it cost; find the cost.

QUESTIONS.

What is the G. C. D. of two or more fractions? How is it found? What is the L. C. M. of two or more fractions? How is it found? How do you find what part one number is of another? How do you find a number when a part of it is given?

ANALYSIS.

179. Analysis is a method of solving problems in which we reason,

1°. *From a part to the whole, or from the whole to a part;* or,

2°. *From a given cause to its effect, or from a given effect to its cause,*

according to the relations betweeen the known and required quantities.

EXERCISE XXXIII.

(1) If 4 hats cost $14, how much will 10 hats cost?

Analysis: If 4 hats cost $14, 1 hat will cost $\frac{1}{4}$ of $14, which is $3½. As 1 hat cost $3½, 10 hats will cost 10 times $3½, or $35, Ans.

Or, 10 hats are $\frac{10}{4}$ or $\frac{5}{2}$ of 4 hats; therefore 10 hats will cost $\frac{5}{2}$ of $14; $\frac{1}{2}$ of $14 is $7, and $\frac{5}{2}$ of $14 is 5 times $7, or $35.

2. If 8 sheep cost $40, what will 12 sheep cost?

3. If 30 hogs cost $76.80, what will 185 hogs cost?

4. What will 15 pounds of coffee cost if 6 pounds cost $1?

5. What cost 87½ pounds of tea, if 2 pounds cost $1.75?

6. If a man can cut 7½ cords of wood in 3 days, how many cords can he cut in 24 days?

7. If 70 acres of land produce 1875 bushels of corn, how much will 168 acres produce?

8. If a vertical staff 5 feet high cast a shadow 6 feet long, how long a shadow will a pole 110 feet high cast at the same time?

9. If a field 35 rods wide contains 22 acres, how much does another field *of equal depth* and 59½ rods wide contain?

(10) If $\frac{3}{4}$ of a bale of cotton cost \$36, what will 5 bales cost?

Analysis: Since $\frac{3}{4}$ of a bale cost \$86, $\frac{1}{4}$ will cost $\frac{1}{3}$ of \$86, or \$12, and $\frac{4}{4}$, or 1 bale, 4 times \$12, or \$48. Now, at \$48 a bale, 5 bales will cost 5 times \$48, or \$240, Ans.

11. If $\frac{3}{8}$ pound tea cost 21 cents, what cost 10 pounds?

12. If $\frac{11}{15}$ acres land cost \$35.20, what cost $8\frac{2}{5}$ acres?

13. If $\frac{5}{7}$ pound candy cost \$.15, what cost $2\frac{1}{3}$ pounds?

14. If $3\frac{1}{2}$ pounds coffee cost \$.$52\frac{1}{2}$, what cost $12\frac{4}{5}$ pounds?

15. If 5 pounds ginger cost \$$\frac{10}{7}$, what cost 14 pounds?

16. If 18 pounds ginger cost \$$4\frac{1}{2}$, what cost $15\frac{1}{5}$ pounds?

17. If $\frac{5}{6}$ of a number is 15, how much is $\frac{2}{3}$ of it?

18. If $\frac{8}{11}$ of a stone weighs $196\frac{2}{5}$ pounds, how much does $\frac{9}{20}$ of it weigh?

(19) If 3 hats cost \$10.50, how many hats can be bought for \$24.50?

Analysis: Since 3 hats cost \$10.50, 1 hat will cost $\frac{1}{3}$ of \$10.50, or \$3.50. Now, if 1 hat cost \$3.50, \$24.50 will buy as many hats as \$3.50 are contained times in \$24.50, or 7 hats, Ans.

20. If 9 sheep cost \$27, how many sheep can be bought for \$45?

21. If 17 acres land cost \$69.70, how many acres can be bought for \$287?

22. If $\frac{5}{6}$ bushel oats cost 50 cents, how many bushels can be bought for \$2.40?

23. If $3\frac{3}{14}$ acres land cost \$$37\frac{1}{2}$, how many acres can be bought for \$70.

24. If a pole 10 feet high cast a shadow 12 feet long, what *is the height* of a tree that casts a shadow 96 feet long?

25. If $7\frac{1}{3}$ pounds sugar are equal to $4\frac{8}{9}$ pounds butter, how much sugar is equal to $17\frac{1}{4}$ pounds butter?

(26) How much coffee, at $16\frac{2}{3}$ cents a pound, will pay for 8 pounds sugar at $12\frac{1}{2}$ cents a pound?

Analysis: 8 pounds sugar at $12\frac{1}{2}$ cents a pound are worth 8 times $12\frac{1}{2}$ cents, or 100 cents. Now since $16\frac{2}{3}$ cents will pay for 1 pound coffee, 100 cents will buy as many pounds as $16\frac{2}{3}$ cents are contained times in 100 cents, or 6 pounds, Ans.

27. How many oranges, at 6 cents each, will pay for $11\frac{1}{4}$ yards muslin, worth 8 cents a yard?

28. A farmer exchanged 126 pounds of cotton, valued at $8\frac{1}{3}$ cents a pound, for sugar at $7\frac{1}{2}$ cents a pound; how many pounds of sugar did he receive?

29. How many slate pencils worth $\frac{1}{2}$ cent each, can be bought for 75 steel pens worth 4 cents per dozen?

30. A farmer bought $237\frac{1}{2}$ pounds pork at $7\frac{1}{5}$ cents a pound, and $33\frac{1}{4}$ yards prints at $5\frac{1}{7}$ cents a yard, and paid the bill with cotton at $8\frac{1}{3}$ cents a pound; how many pounds of cotton were required?

(31) If 4 men in 3 days earn \$60, how much will 7 men earn in 5 days?

Analysis: If 4 men in 3 days earn \$60, 1 man in 3 days will earn $\frac{1}{4}$ of \$60, or \$15; if 1 man in 3 days earn \$15, 1 man in 1 day will earn $\frac{1}{3}$ of \$15, or \$5; if 1 man in 1 day earn \$5, 1 man in 5 days will earn 5 times \$5, or \$25; and if 1 man in 5 days earn \$25, 7 men in 5 days will earn 7 times \$25, or \$175, Ans.

32. If 3 horses in 5 days eat 45 pecks of oats, how much will 4 horses eat in 7 days?

33. If 15 persons consume 6 barrels of flour in 9 months, how much will 20 persons consume in 12 months?

34. If 2 boys in 3 hours can whitewash 120 pickets, in how many *hours will 5 boys* whitewash 400 pickets?

35. If a pasture of 72 acres supplies 120 sheep 30 days, how long will one of 48 acres supply 150 sheep?

36. If $100 gain $6 in 12 months, how many dollars are required to gain $2 in 8 months?

37. If 6 men in 24 days, working 10 hours a day, can build a wall 48 feet by 10 feet by 3 feet, how many men can build a wall 72 feet by 9 feet by 2 feet in 18 days, working 8 hours a day?

(38) If 4 oxen or 5 horses consume 40 bushels of corn in a certain time, how much will 16 oxen and 7 horses consume in the same time?

Analysis: Since 4 oxen consume as much as 5 horses, 1 ox will consume $\frac{5}{4}$ as much as 1 horse, and 16 oxen 16 times $\frac{5}{4}$, or 20 times as much as 1 horse, or as much as 20 horses. Hence, the problem may be stated thus: If 5 horses consume 40 bushels in a certain time, how much will 20 + 7, or 27 horses consume? Since 5 horses consume 40 bushels, 1 horse will consume 8 bushels, and 27 horses will consume 27 times 8 bushels, or 216 bushels, Ans.

39. If 2 pounds coffee or 3 pounds sugar cost 30 cents, how much are 6 pounds coffee and 11 pounds sugar worth?

40. If 8 men or 15 boys earn $55 in 7 days, how much will 32 men and 10 boys earn in 6 days?

41. If 3 hats, or 5 caps, or 10 ties cost $10, what will be the cost of 9 hats, 10 caps and 7 ties?

42. If 15 men, or 24 women, or 33 boys can do a certain work in 11 days of 8 hours each, in how many days of 9 hours each can 35 men, 16 women and 22 boys perform another work 3 times as great?

(43) A can cut a cord of wood in 6 hours and B in 8 hours, how long will it take both together to cut a cord?

Analysis: Since A can cut a cord in 6 hours he can cut $\frac{1}{6}$ of a cord in 1 hour; and as B can cut a cord in 8 hours, he can cut $\frac{1}{8}$ of a cord

in 1 hour; hence, both can cut $\frac{1}{8} + \frac{1}{6}$ or $\frac{7}{24}$ of a cord in one hour. That is, if the work were divided into 24 equal parts, they could perform 7 of the parts in 1 hour; hence, it will take them as many hours as 7 is contained times in 24, or $3\frac{3}{7}$ hours, Ans.

How long will it take A and B together to reap a field:

44. If A can reap it in 4 days and B in 12 days?

45. If A can reap it in 9 days and B in 8 days?

46. If A can reap it in $2\frac{1}{2}$ days and B in $3\frac{1}{3}$ days?

47. If A can reap it in $6\frac{1}{4}$ days and B in $4\frac{2}{7}$ days?

How long will it take A, B and C together to do a piece of work, if they can do it alone:

48. In 3, 6 and 9 days, respectively?

49. In $6\frac{2}{3}$, $7\frac{1}{2}$ and $11\frac{1}{4}$ days, respectively?

50. A can do a work in 3 days, but with the help of B he can do it in 2 days, how long would it take B to do it alone?

51. A box will hold 20 oranges; after putting 7 oranges and 2 peaches in the box, I observe that it is $\frac{5}{12}$ full; how many peaches will the box hold?

52. How long will it take a boy to do a certain work if, after performing $\frac{1}{5}$ of it, it takes him 8 hours to finish it?

53. Two persons, A and B, could finish a work in 12 days; they worked together 5 days, when A was called off and B finished it in $8\frac{3}{4}$ days. In what time can each do it?

NOTE.—In Fahrenheit's thermometer, the freezing point of water is marked 32°, and the boiling point 212°; and in the Centigrade the freezing point is 0°, and the boiling point 100°.

(54) How many degrees C. are equivalent to 59° F.?

Analysis: The number of degrees F. between the freezing and boiling points is 180; therefore 1° F. = $\frac{100}{180}$° C. = $\frac{5}{9}$ C.	Operation.
	$212° - 32° = 180°$
	$59° - 32° = 27°$
Again, 59° F. is 27° F. above freezing point, and $\frac{5}{9}$ of 27° = 15°. *Therefore* 59° F. = 15° C.	$\frac{100}{180} = \frac{5}{9}$; $\frac{5}{9}$ of $27° = 15°$.

(55) How many degrees F. are equivalent to 50° C.?

Analysis: Since 100° C. = 180° F. 1° C. = $\frac{180}{100}$° F. = $\frac{9}{5}$° F., and 50° C. = 50 × $\frac{9}{5}$° F. = 90° F. = the height above the freezing point. But 90° above freezing point is 90° + 32° = 122° F. Therefore 50° C. = 122° F.

Operation.
$\frac{9}{5}$ of 50° = 90°,
90° + 32° = 122°, Ans.

NOTE.—In the following exercises, the sign — denotes below the zero point. Thus, — 5° means 5° below 0.

Find the missing number in the following:

56. 50° Fahrenheit = ( ) Centigrade.

57. 65° Centigrade = ( ) Fahrenheit.

58. 86° F. = ( ) C. **59.** 75° C. = ( ) F.

60. 131° F. = ( ) C. **61.** 110° C. = ( ) F.

62. 14° F. = ( ) C. **63.** — 10° C. = ( ) F.

64. — 12° F. = ( ) C. **65.** — 3° C. = ( ) F.

66. A boy owned $\frac{5}{12}$ of a number of apples, of which he gave his mother $\frac{2}{5}$, and his sister $\frac{1}{3}$ of the remainder; what part of the apples did he still own?

67. A man owned $\frac{5}{7}$ of an estate. He sold $\frac{3}{4}$ of his share to one man and gave $\frac{3}{8}$ of the remainder to his son; what part of the estate did he then own?

68. A tailor cut 7 suits of $7\frac{1}{5}$ yards each from a bolt of cassimere and $\frac{2}{3}$ of a suit remained; how much did the bolt contain?

69. A farmer gathered his corn in loads of $26\frac{1}{4}$ bushels each, and after $8\frac{3}{5}$ loads $\frac{4}{7}$ of a load remained; how much corn did he make?

70. A boy bought 24 apples at the rate of 8 for 9 cents and sold them at the rate of 3 for 4 cents; how much did *make?*

71. A grocer bought 25 dozen eggs at the rate of 19 cents per dozen and sold them at the rate of 39 cents per score; how much did he make?

72. If 3 pounds of sugar are worth $19\frac{1}{2}$ cts., and $4\frac{1}{2}$ pounds of coffee 63 cts., how much money will it take to buy a pound of each?

73. In $8\frac{1}{4}$ hours A can cut $2\frac{3}{4}$ cords of wood, and B can cut $2\frac{2}{3}$ cords in $6\frac{2}{5}$ hours; how much can both together cut in an hour?

74. A lad saved $\$4\frac{4}{5}$ from a week's earnings; if his daily expenses were $\$\frac{2}{5}$, what were his wages per day?

75. How much does a saleswoman earn per day whose weekly expenses are $\$3\frac{3}{4}$, if she saves $78 in 52 weeks?

76. I paid 10 cts. a pound for a fish that weighed $7\frac{1}{2}$ pounds, and the waste in dressing was $\frac{1}{5}$ of its weight; how much a pound did the dressed fish cost me?

77. I paid 12 cts. a pound for a live pig that weighed $17\frac{7}{8}$ pounds, and the waste in dressing was $\frac{3}{13}$ of its weight; how much a pound did the dressed meat cost me?

78. James spent $\frac{1}{4}$ of his money and lost $\frac{1}{3}$ of the remainder, after which he had $6; what had he at first?

79. After paying $\frac{3}{5}$ of a debt, and $\frac{2}{3}$ of the remainder there was a balance of $150.40; what was the original debt?

80. $\frac{3}{4}$ of a number is 20 more than $\frac{2}{3}$ of it; what is the number?

81. A tank is $\frac{5}{8}$ full of water, and if 153 gallons more were poured in it would then be $\frac{7}{10}$ full; how many gallons does it hold?

82. A lad spent $\frac{5}{9}$ of his money, which was $4 more than $\frac{3}{4}$ of what he had left; *how much had he?*

83. A and B together own a lot, of which A owns $\frac{5}{12}$, and $\frac{3}{8}$ of B's part is worth \$100 more than $\frac{2}{7}$ of A's part; what is the value of the lot?

84. If it is worth \$$2\frac{1}{3}$ to cut wood, which is 8 feet long, into pieces 4 ft. long; how much is it worth to cut it into pieces 2 ft. long?

85. If a piece of land containing $3\frac{1}{2}$ square rods is worth \$$2\frac{3}{4}$, what is the worth of a piece which is $3\frac{1}{2}$ rods square?

86. John sold 12 oranges, which was $\frac{2}{7}$ of all he had; he then divided the remainder equally among his 5 sisters; how many did each receive?

87. A man spent \$45, which was $\frac{9}{17}$ of all he had; he then spent the rest for 12 yards of cloth; what was the cloth worth per yard?

88. $\frac{3}{8}$ of a rope, at 6 cts. a yard, is worth 90 cts.; what is the length of the rope?

89. A man sold $\frac{6}{13}$ of his sheep at \$$4\frac{1}{2}$ each, and received \$135; how many sheep had he?

90. James killed 4 birds and Henry 3, which they sold for 35 cts.; how much should each receive?

91. A and B buy a carriage together for \$500, of which A paid \$370 and B the remainder; they sold it at a gain of \$75; what was each one's share of the gain?

92. A pole stands $\frac{1}{4}$ in the mud, $\frac{1}{5}$ in the water, and 22 feet above the water; how long is the pole?

93. In an orchard $\frac{1}{3}$ are apple trees, $\frac{1}{4}$ peach trees, $\frac{1}{6}$ plum trees, and the remaining 15 are cherry trees; how many trees are in the orchard?

94. Jack sold 12 papers which was $\frac{2}{5}$ of all he had; how many had he left after selling $\frac{4}{9}$ of the remainder?

95. A man gave his check for $\$1675\frac{3}{4}$, which was $\frac{5}{8}$ of what he had on deposit; the next day he bought a carriage and then had $\frac{5}{24}$ of what he had at first; what did the carriage cost?

96. A father divided $\$4\frac{1}{2}$ among his sons, giving each $\$\frac{3}{4}$; how much would it have taken to give each $\$\frac{1}{3}$ more?

97. A stationer paid $\$11\frac{2}{3}$ for books, at $\$1\frac{1}{6}$ a piece; what will he receive for them if he gains $\$\frac{3}{10}$ on each?

98. James has $\frac{3}{5}$ as many marbles as Gordon, and both have 40; how many has each?

99. A speculator sold land for $2\frac{3}{8}$ times the cost, and gained $\$1795\frac{4}{9}$; required the cost and sale?

100. J can whitewash a fence in 4 hours, and H in 6 hours; how long would it take both together?

101. A tank has 3 pipes, the first will fill it in 8 hours, the second in 9 hours, and the third will empty it in 10 hours; in how many hours will the tank be filled if all run together?

102. F and G are a certain distance apart, and go towards each other; in how many minutes will they meet if F can go the whole distance in 10 minutes and G in 12 minutes?

103. Ten boxes, or 24 barrels, or 40 kegs will fill a certain room; what equal number of boxes, barrels and kegs will fill the same room?

104. A and B together can buy $\frac{1}{2}$ of a farm; A and C, $\frac{1}{3}$ of it; and B and C, $\frac{1}{4}$ of it; what part of it can each alone buy?

105. A and B can do a certain work in $4\frac{4}{5}$ days, A and C in 6 days, and B and C in 8 days; in how many days can each do the work?

106. A boy has enough money to buy 140 oranges and 140 apples, or 180 oranges and 180 peaches, or 210 apples and 210 peaches; how many of each could he buy?

107. If a man travels $6\frac{1}{4}$ hours per day, in how many days will he travel 100 miles, at the rate of $3\frac{1}{5}$ miles per hour?

108. In how many days will a man earn $\$15\frac{3}{4}$ by cutting wood at $\$\frac{7}{8}$ per cord, if he can cut $3\frac{3}{5}$ cords per day?

109. A boy sold 12 pigs for $\$26\frac{2}{15}$, and lost $\$5\frac{1}{15}$; what did the pigs cost apiece?

110. On $6\frac{3}{5}$ tons of coal a grocer made a profit of $\$5\frac{3}{8}$ by selling it all for $\$58\frac{7}{40}$; what was the cost per ton?

111. A boy sold $\frac{1}{2}$ of his melons at $\$\frac{1}{3}$ each, and the remainder at $\$\frac{1}{4}$ each, receiving $14 for all; how many melons had he?

112. A farmer sold $\frac{1}{3}$ of his wheat at $\$1\frac{9}{20}$ per bushel, $\frac{1}{2}$ of the remainder at $\$1\frac{2}{25}$ per bushel, and what was left at $\$\frac{83}{100}$ per bushel, receiving $\$6073\frac{1}{5}$ for all; how many bushels were there?

113. A owned $\frac{3}{10}$ of a vessel and sold $\frac{3}{20}$ of his share, B owned $\frac{9}{20}$ of the vessel and sold $\frac{3}{25}$ of his share; they together received $1287; what was the value of the vessel at that rate?

114. In an orchard $\frac{1}{10}$ of the trees are apple, $\frac{1}{6}$ of the remainder, peach; $\frac{3}{5}$ of the number left, pear; and the balance 432, plum; how many trees in all?

115. $\frac{3}{8}$ of a farm was sold at $\$25\frac{1}{2}$ per acre, $\frac{2}{5}$ of the remainder at $\$30\frac{3}{4}$ per acre, and the remainder, $49\frac{1}{6}$ acres, at $\$32\frac{5}{8}$ per acre; the gain was $\$96\frac{7}{20}$; what was the cost per *acre?*

116. If A give B $\frac{1}{2}$ as much money as B has, then B give A $\frac{1}{3}$ as much as A would then have, each would have \$12; how much has each?

117. A merchant has 3 casks, A, B and C, each containing wine. He pours from A into B and C each $\frac{1}{3}$ as much as they contain, then pours from B into A and C each $\frac{1}{4}$ as much as they contain, then pours from C into A and B each $\frac{1}{2}$ as much as they contain, and each then contains 60 quarts; how many quarts did each contain at first?

DECIMALS.

180. A **decimal fraction**, or **decimal** is a fraction whose denominator is 10, 100, 1000, etc.

(1) $\frac{3}{10}$, $\frac{17}{100}$, $\frac{103}{10000}$, are decimal fractions.

181. For convenience, the denominator of a decimal is not written, but is indicated by prefixing a period, called the **decimal point**, and a sufficient number of ciphers to the numerator, so that

The number of figures in the decimal is equal to the number of ciphers in the denominator.

.03	indicates 2	ciphers	in the denominator,		or	$\frac{3}{100}$.
.019	" 3	"	"	"	"	$\frac{19}{1000}$.
.0203	" 4	"	"	"	"	$\frac{203}{10000}$.

State what is indicated by the following, and write in full the value of each:

.5	.023	.0506	.000154
.07	.0270	.00035	.0205604

Express the following decimally:

$\frac{4}{10}$.	$\frac{85}{100}$.	$\frac{17}{1000}$.	$\frac{803}{10000}$.
$\frac{6}{100}$.	$\frac{245}{1000}$.	$\frac{3}{10000}$.	$\frac{5380}{10000000}$.

182. The unit of the *first* figure on the right of the decimal point is *tenths;* of the *second*, *hundredths;* of the *third*, *thousandths;* of the *fourth*, *ten-thousandths;* of the *fifth*, *hundred-thousandths*, etc.

Illustration: $.327 = \frac{327}{1000} = \frac{300+20+7}{1000} = \frac{300}{1000} + \frac{20}{1000} + \frac{7}{1000} = \frac{3}{10} + \frac{2}{100} + \frac{7}{1000}$ = 3 tenths 2 hundredths 7 thousandths.

183. Principle.—*The orders or units of decimal figures decrease from left to right by the scale of ten, just as integers do.*

In .37594, what is the value of 3? of 5? of 7? of 9? of 75? (Ans., 75 thousandths; the unit is that of the last figure). Of 59? of 94? of 375? of 759? of 37594?

The denominator of a decimal is the name of its right-hand order.

184. Since decimals are expressed according to the fundamental laws of integers (see Art. 24),

An integer and a decimal may be written together in a single number.

$300 + 30 + 3 + \frac{3}{10} + \frac{3}{100}$ may be written 333.33.

5 and $\frac{3}{10}$ is written 5.3. 9 and $\frac{3}{100}$ is written 9.03.

7 and $\frac{13}{100}$ is written 7.13. 13 and $\frac{5}{1000}$ is written 13.005.

185. Mixed numbers, also called decimals, are numbers composed of integers and decimals.

(1) 17.4, 203.015, are mixed numbers.

186. The relation of integral and decimal orders of the scale is clearly shown in the following

TABLE.

NAMES.	***Millions.***	Hund.-thousands.	Ten-thousands.	***Thousands.***	Hundreds.	Tens.	UNITS.	Tenths.	Hundredths.	***Thousandths.***	Ten-thousandths.	Hund.-thousandths.	***Millionths.***
ORDERS.	7th.	6th.	5th.	4th.	3d.	2d.	1st.	2d.	3d.	4th.	5th.	6th.	7th.
UNITS.	3	3	3	3	3	3	3	.3	3	3	3	3	3
	INTEGERS.							DECIMALS.					

The number is read, 3 million 333 thousand 333, *and* 333 thousand 333 *millionths.*

All integral orders on the left of the unit's place are *multiples* of this unit, all decimal orders on the right are *decimal parts* of this unit, and the values of the units, equally distant from this *fundamental* unit, are *reciprocals* of each other.

Thus, the 2d order on the left is 1 *ten*, and the 2d order on the right is 1 *tenth;* the 3 order on the left is 1 *hundred*, and the 3d order on the right is 1 *hundredth.*

187. ***To write decimals.***

(1) Write 83 hundred-thousandths.

Explanation.—Hundred-thousandths requires five places of decimals, since there are five 0's in 100,000, the denominator; and as there are only two figures or places in 83, the numerator, it is necessary to prefix three 0's. Hence, I write the decimal point, three 0's and 83.

Operation.
.00083.

RULE.—*Make the decimal point, and after it write the requisite number of ciphers and the numerator.*

In writing mixed numbers, write the integer first, and then the decimal. The decimal point is called *and*, and emphasized to indicate that what follows *is the decimal* part.

EXERCISE XXXIV.

Write the following decimally:

2. 31 hundredths. 3. 35 thousandths.

4. 124 thousandths. 5. 305 millionths.

6. 25 *and* 6 tenths. 7. 31 *and* 8 hundredths.

8. 425 *and* 517 hundred-thousandths.

9. Forty-three thousand twenty-five *and* sixty-four ten-thousandths.

10. Three hundred fifty-six *and* forty-nine thousand six hundred ninety-seven billionths.

11. Five million *and* twelve ten-millionths.

12. Three thousand seven *and* six million seven hundred forty-two thousand six billionths.

13. Sixty-five *and* eleven hundred-thousandths.

14. Forty-nine *and* six thousand five ten-thousandths.

15. $\frac{16}{1000}$. 16. $\frac{207}{1000}$. 17. $\frac{34025}{1000000}$.

18. $5\frac{3}{100}$. 19. $24\frac{39}{1000}$. 20. $408\frac{307}{10000}$.

21. $17\frac{1}{1000}$. 22. $4091\frac{20}{10000}$. 23. $3\frac{3}{100000000}$.

188. ***To read decimals.***

(1) Read .073.

Considered as an integer, the number is 73, and the name of the lowest order is thousandths. Hence, the decimal is read, seventy-three thousandths.

RULE.—*Read the decimal as an integer, and give to it the name of the lowest order.*

EXERCISE XXXV.

Read the following:

2. .06.	3. 10.010.	4. 251.3.
5. .005.	6. 12.012.	7. 64.037.
8. .042.	9. 365.007.	10. 444.444.
11. .507.	12. 40.0040.	13. .161616.
14. .4716.	15. 3.33331.	16. 1.010001.
17. .00837.	18. 85.62471.	19. 385.0095.

DECIMAL CURRENCY.

189. The **currency** of a nation is its money.

190. A **decimal currency** is a currency whose denominations increase and decrease by the decimal scale.

191. The **United States Money** is a decimal currency, as shown by the following

TABLE.

10 Mills (m)	= 1 Cent.	c. or ct.	
10 Cents	= 1 Dime.	d.	
10 Dimes	= 1 Dollar.	$	
10 Dollars	= 1 Eagle.	E.	

$1 = 10 d. = 100 ct. = 1000 m.

Thousands.	Hundreds.	Tens.	Units.		Tenths.	Hundredths.	Thousandths.
1	1	1	1	.	1	1	1
Dolls.					Cts.		M.

In business operations *eagles* are regarded as *tens of dollars*, and *dimes* as *tens of cents*. Thus, 4 *eagles* is written $40; and 3 *dimes*, 30 cents.

192. The dollar is the **unit**, and the only denominations used in practice are *dollars* and *cents*. Cents are hundredths, and mills thousandths, of a dollar.

Hence, dollars are written with the sign ($) prefixed to them, and the point (.) placed after them, and cents and mills are written in the hundreths and thousandths places respectively on the right of the point.

Thus, we write 12 dollars 23 cents and 6 mills, $12.236;
12 dollars 4 cents and 3 mills, $12.043.

Generally, in business calculations, if in the final result the mills are less than 5 they are not regarded; if 5 or more than 5, they are considered a cent.

Thus, we write $12, 23 cts. 6 m., $12.24.

193. PRINCIPLE.—*The system of notation, and all the operations in decimal currency, are the same as corresponding operations in integers and decimals.*

REDUCTION OF DECIMALS.

194. Principles.—1°. *Annexing a cipher to a decimal, or removing one from its right, does not change its value.*

(1) Since $\frac{3}{10} = \frac{30}{100} = \frac{300}{1000} = \frac{3000}{10000}$, etc., we have

$.3 = .30 = .300 = .3000$, etc.

2°. *Two or more decimals may be reduced to a common denominator by annexing ciphers at the right until the decimal places are all equal.*

Thus, .34 and .017 = .340 and .017 = $\frac{340}{1000}$ and $\frac{17}{1000}$.

3°. *Removing the decimal point one, two, three, etc., places to the right, is equivalent to multiplying the decimal by* 10, 100, 1000, *etc.*

Thus, $10 \times 3.74 = 37.4$.

For, $10 \times (3 + \frac{7}{10} + \frac{4}{100}) = 30 + 7 + \frac{4}{10} = 37.4$.

Again, reversing the preceding principle, we have

4°. *Removing the decimal point one, two, three, etc., places to the left is equivalent to dividing the decimal by* 10, 100, 1000, *etc.*

195. ***To reduce a decimal to a common fraction.***

(1) Reduce .675 to a common fraction.

Solution: $.675 = \frac{675}{1000} = \frac{135}{200} = \frac{27}{40}$, Ans.

Rule.—*Express the decimal as a common fraction, and reduce it to its simplest form.*

EXERCISE XXXVI.

Reduce to common fractions:

2. .45
3. .125
4. .03125
5. .75
6. .875
7. .015625
8. .32
9. .512
10. .001975

11. $.14\frac{2}{7}$. Solution. $.14\frac{2}{7} = \frac{14\frac{2}{7}}{100} = \frac{100}{700} = \frac{1}{7}$.

12. $.6\frac{1}{4}$. 13. $.038\frac{1}{8}$. 14. $.923\frac{1}{13}$.

15. $.9\frac{3}{8}$. 16. $.47\frac{1}{7}$. 17. $.01339\frac{2}{7}$.

Express by an integer and common fraction:

18. \$6.4. 19. \$5.75. 20. \$7.375.

21. \$3.6. 22. \$6.33$\frac{1}{3}$. 23. \$2.0284$\frac{1}{11}$.

196. ***To reduce a common fraction to a decimal.***

(1) Change $\frac{3}{8}$ to an equivalent decimal.

Operation.

$$\frac{3}{8} = \frac{3000}{8000} = \frac{375}{1000} = .375. \quad \text{Or, } \frac{3}{8} = \frac{3.000}{8} = .375.$$

RULE.—*Annex ciphers to the numerator, divide by the denominator, and point off as many decimal places in the quotient as there are ciphers annexed.*

EXERCISE XXXVII.

Reduce to decimals:

2. $\frac{3}{4}$. 3. $\frac{14}{25}$. 4. $\frac{33}{750}$.

5. $\frac{5}{8}$. 6. $\frac{3}{16}$. 7. $\frac{7}{125}$.

8. $\frac{1}{2}$ of $\frac{2}{5}$. 9. $\frac{3}{4}$ of $\frac{5}{8}$. 10. $\frac{3}{5}$ of $\frac{17}{32}$.

11. $\dfrac{\frac{1}{2}}{1 - \frac{1}{3}}$. 12. $\dfrac{1\frac{1}{3} - \frac{1}{2}}{1\frac{2}{3} + \frac{5}{6}}$. 13. $\dfrac{\frac{5}{8} \div \frac{3}{4}}{\frac{2}{3} \text{ of } \frac{5}{8} \text{ of } \frac{7}{160}}$.

197. When the numerator, with ciphers annexed, is exactly divisible by the denominator, the result is called a **perfect decimal**. Thus, .25, .375 are *perfect decimals*.

198. When the denominator of a fraction in its lowest terms contains any other prime factor than 2 or 5, the division of the numerator by the denominator cannot be exact, and the result is called a **circulating decimal**; and the figure or set of figures repeated is called the **repetend**.

Thus, .333 + and .4545 + are *circulating decimals*, and the repeating figures, 3 and 45, are the *repetends*.

For circulating decimals, see Appendix, Art. 460.

ADDITION AND SUBTRACTION.

199. ***To find the sum of two or more decimals.***

(1) Find the sum of 6.24, 17.3, and 9.786.

Analysis.	Operation.
6240 thousandths =	6.24
17300 thousandths =	17.3
9786 thousandths =	9.786
33326 thousandths =	33.326

RULE I.—*Write the numbers so that units of the same order stand in the same column and points in the same line.*

II.—*Add as in integers, and place the decimal point before the order of tenths in the sum.*

200. ***To subtract one decimal from another.***

(2) From 16.24 subtract 9.7538.

Analysis.	Operation.
162400 ten-thousandths =	16.24
97538 ten-thousandths =	9.7538
64862 ten-thousandths =	6.4862

RULE I. *Write the subtrahend under the minuend, so that units of the same order stand in the same column.*

II. *Subtract as in integers, and place the decimal point before the order of tenths in the remainder.*

EXERCISE XXXVIII.

Find the sum:

3. Of 35., 3.5, .35, and .035.
4. Of .083, 21.01, 2.5 and 94.5.
5. Of 18.37, 23.504, 14.375 and 16.7.
6. Of 28.256, 45.309, 427.48 and 32.5346.
7. Of 6 *and* 428 thousandths, 23 *and* 73 hundredths, 18 *and* 456 thousandths, and 15 *and* 907 thousandths.

Find the value:

8. Of 3.6167 + 21.611 + 6888.32.

9. Of 165.13 + 46.67 + 61.87 + 676.167895 + 165.13 + 1121.6116.

Find the difference:

10. Of 91.61 and 2.6671. 11. Of 425.824 and 226.75.

12. Of 97.7 and 27.028. 13. Of 125.26 and 47.508.

14. From 75 and 4 tenths take 53 and 647 thousandths.

15. Find the value of 6.1 — 1.99999.

16. Take one hundred fifteen and seven hundredths from three hundred fifteen and twenty-seven ten-thousandths.

17. From one million take one millionth.

18. One melon weighs 13.75 pounds, a second 17.7 pounds, and a third 16.125 pounds; what is the united weight?

19. A farmer owed $217.925; at one time he paid $62.75, at another $53.8 and at a third $48.675; how much did he still owe?

20. A lad went into a store with $18.25, of which he spent $3.875 for a vest, $1.80 for a knife, $.45 for a bat, and $8.5 for a saddle; how much had he left?

21. A man traveled 25269.111505 miles in 4 years; the first year he traveled 59.059 miles, the second 25000.0025 miles, and the third 205.05 miles; how far did he travel the fourth year?

22. A farmer planted 174.13 acres in corn, 48.63 acres less in cotton than in corn, 85.017 acres in oats and enough in wheat to have 500 acres planted in all; how many acres were planted in wheat?

23. The mercury in a thermometer stood at 8.0°; the first day it fell 2.6535°, the second it fell .37615° less than on the first, and on the third it fell .25128° less than on the second; how much above 0 did it then stand?

24. If a barrel of flour cost $\$6.3\frac{1}{8}$, a gun $\$19\frac{4}{5}$, a saddle $\$10.8\frac{3}{4}$, and a suit of clothing $21.375; what is the cost of all?

25. Find the value of $750 - (476.0\frac{5}{8} - 87\frac{7}{25})$.

26. The mercury in a barometer stood at 16.5 inches Monday and at 23.465 inches Friday; on Tuesday it rose 5.237 inches and on Thursday it rose 2.8395 inches; did it rise or fall on Wednesday, and how much?

MULTIPLICATION.

201. ***To find the product of two decimals.***

(1) Multiply .346 by .08.

Analysis.	Operation.
$.346 = \frac{346}{1000}$; $.08 = \frac{8}{100}$.	.346
	.08
$\frac{346}{1000} \times \frac{8}{100} = \frac{2768}{100000} =$	.02768

RULE.—*Multiply as in whole numbers, and in the product point off as many decimal figures from the right as there are decimal places in both factors, prefixing ciphers when necessary to supply the deficiency.*

EXERCISE XXXIX.

2. Multiply 8.47 by 4.6.
3. Multiply .357 by .27.

Find the value:

4. Of 16.5×16.5.
5. Of $.0046 \times .1234$.
6. Of $324.7 \times .75$.
7. Of $61.76 \times .0071$.

Find the cost:

8. Of 4.75 acres of land, at $5.35 per acre.
9. Of 4.5 pounds of lard, at $.1725 per pound.
10. Of 15.225 yards of wire, at $.067 per yard.

11. A railroad train ran 7.161 hours, at the average rate of 36.5 miles per hour; required the distance passed over.

12. A man owned .6711 of a factory and sold .6543 of his share; what part of the factory did he sell?

13. If the pulling force of a loaded wagon is equal to .2658 of the load, what force is exerted by a team when drawing a wagon loaded with 4 bales, each weighing 562.5 pounds?

14. If 7.985 acres of land produce an average of 35.75 bushels of wheat per acre; what is the wheat worth at $1.375 per bushel?

15. Find the value of $(184.6 - 120.3\frac{3}{4} + .1\frac{7}{8}) \times .02\frac{3}{25}$.

16. A sample of lignite or brown coal from Clark County, Ala., was analyzed at one of the stations of the Louisiana State University, and found to contain .2875 of moisture, .2945 of volatile matter, .2885 of fixed carbon, and the remainder ash; how many pounds of each are there in a sample of 145.25 pounds?

202. Oughtred's Method. In the multiplication of decimals it is frequently not necessary that all the decimal places of the product should be found; in which case the work may be abbreviated as follows:

(17) Find the cost of 27.653 pounds of sugar at 9.157 cts. a pound, retaining two places of decimals.

Ordinary Method.

$27.65	3
9.15	7
19	3571
138	265
276	53
24887	7
$253.21	8521

Abbreviated Method.

$27.653	
751.9	
24888	= 2765 × 9 + 3.
276	= 276 × 1 + 0.
138	= 27 × 5 + 3.
19	= 2 × 7 + 5.
$253.21	= required product.

Explanation.—The ordinary method gives 6 places of decimals, or 4 more than the required number, thus rendering unnecessary the several figures *on the right of* the vertical line.

In the abbreviated method, I write the units figure of the multiplier under that figure of the multiplicand whose place it is proposed to retain in the product, and write the other figures of the multiplier in reverse order, so that the product of each figure by the one of the multiplicand above it, will be of the order required in the decimal.

Then, in multiplying, I begin, for each partial product, with that figure of the multiplicand which stands above the multiplying figure, observing to add to the product the number nearest to that which would have been carried if the places at the right had not been rejected. Write down the several partial products, so that the right-hand figure of each shall be in the same column, and their sum will be the product required.

NOTES.—1. In obtaining the number to be carried to each contracted partial product, it is generally necessary to multiply (mentally) only one figure at the right of the figure above the multiplying figure; but when the figures are large, the multiplication should commence at least *two* places to the right.

2. Observe, that when the number of units in the highest order of the rejected part of the product is between 5 and 15, carry 1; if between 15 and 25 carry 2; if between 25 and 35 carry 3; and so on.

3. There is always a liability to an error of one or two units in the last place, which may be obviated by performing the operation as if one more decimal place were required.

(18)* Multiply .24367 by 36.75, retaining 2 decimal places; and 4256.785 by .00564, retaining 3 decimal places.

Operation.	Operation.
0.24367	4256.785
5763	46500.0
731	21283
146	2554
17	170
1	24.007
8.95	

19. Multiply 3.141592 by 52.7438, retaining 4 places of decimals in the product.

20. Multiply .0716 by 1.326, retaining 3 decimal places in *the product.*

21. Multiply 26.58 by $5.6\frac{1}{4}$, extending the product to 2 decimal places.

22. Find the correct value of $.12\frac{17}{50} \times 12.3\frac{2}{5}$ to 3 decimal places.

23. What will 28.75 tons of hay cost, at $12.578 per ton, omitting mills in the answer.

24. Multiply 64.01082 by .3537, preserving 5 decimal places in the product.

25. Of 175.24 pounds of cane juice, .1683 is total solids, of which .8037 is sucrose, and there is .0735 as much glucose as sucrose; find the amount of total solids, sucrose and glucose, to 4, 3 and 2 decimal places, respectively.

DIVISION.

203. ***To divide one decimal by another.***

(1) Divide .315 by .7.

Analysis.	Operation.
$.315 = \frac{315}{1000}$; $.7 = \frac{7}{10}$	$.7 \mid .315$
$\frac{315}{1000} \div \frac{7}{10} = \frac{315}{100} \times \frac{1}{7} =$	$.45$

RULE.—*Divide as in whole numbers, and in the quotient point off as many decimal figures from the right as the decimal places of the divideud exceed those of the divisor, prefixing ciphers, if necessary, to supply the deficiency.*

NOTES.—1. When there are more decimal places in the divisor than in the dividend, make them equal by annexing ciphers to the dividend before dividing.

2. If there is a remainder, ciphers may be annexed to it as decimals, and the division continued at pleasure.

3. When there is a remainder at the close of the operation, the sign + should be annexed to the quotient to show that it is not complete.

EXERCISE XL.

2. Divide 25.65 by 5.7. 3. Divide 182.75 by 4.25.

4. Divide .00432 by .009. 5. Divide 270 by .72.

Find the value:

6. Of 7.25406 ÷ 9.57. 7. Of .0949416 ÷ 1.326.

8. Of 30614.4 ÷ .9567. 9. Of 15227.56 ÷ .1234.

10. Divide 41448651.06 by 78205002.

11. Divide .000019737 by 153 thousandths.

12. Divide 7.2091365 by .5201.

13. Divide 1 by 3.141592.

14. If 14.75 bushels of wheat cost $18.4375, what is the cost of one bushel?

15 At $125.78 per hogshead, how many hogsheads of sugar can I buy for $3616.175?

16. How many acres of land can be bought for $245.792, at $76.81 per acre?

17. If .0716 of a farm is worth $94.9416, what is the entire farm worth?

18. If the freight on 5865.875 tons of coal is $6599.109375, what is freight per ton?

19. An American deposited $76988.03 in the bank of England; how many sovereigns had he in bank, the value of a sovereign being $4.8665?

20. A quantity of oyster shell marl from New Orleans was found to contain .681 of lime and .0012 of magnesia, and there were 84.975 pounds more of lime than magnesia; what quantity of marl was analyzed?

204. Oughtred's Method. If the quotient is not required to contain figures below a certain denomination, the work *may be abridged* as follows:

(21) Divide 75.46254 by 3.20354, extending the quotient to 3 decimal places.

Operation.

```
3.2035|4)75.462|54(23.556.
           64070
           11391
            9610
            1781
            1601
             180
             160
              20
              19
               1
```

Explanation.—I first consider how many figures, in all, the quotient must contain. There will evidently be 2 places of whole numbers, and since 3 decimals are required, there will be 5 places in all.

Then, for the first contracted divisor, I take as many significant figures from the left of the given divisor as there are places required in the quotient; and at each subsequent division I reject one place from the right of the last preceding divisor.

In multiplying by the several quotient figures, I carry from the rejected figures of the divisor as in contracted multiplication.

NOTE.—There is often some uncertainty about the last figure of the decimal, which may be obviated as in contracted multiplication. Thus, in the preceding example, had the division been carried to 4 decimal places, and the last figure of the quotient rejected, we should have obtained the true answer, 23.555.

22. Divide 12.9 by 8.256, retaining 3 decimal places.

23. Divide 7.2091365 by .5201, carrying the quotient to 3 decimals.

24. Divide 252070.520751 by 591.57, extending the quotient to 4 decimal places.

25. If 589 pounds of sugar cost $36.6947, what will one pound cost, disregarding tenths of mills in the answer?

26. Find the value of $478\frac{13}{40} \div 1.43\frac{2}{3}$ to 2 places of decimals.

27. How many times is 1.4142136 contained in 200000, omitting decimals in the answer.

28. The circumference of the earth at the equator is $24899\frac{11}{500}$ miles; find to 3 decimal places the length of one degree of longitude at the equator, each degree being $\frac{1}{360}$ of the circumference.

SHORT BUSINESS METHODS.

205. **Quantity,** in business transactions, is the amount of any commodity bought or sold.

206. **Price** is the value in money of a given unit of the quantity.

207. **Cost** is the value in money of the entire quantity.

208. An **aliquot part** of a number is an exact divisor of that number.

Aliquot Parts of One Dollar.

50 cts. $= \frac{1}{2}$ of \$1.	$33\frac{1}{3}$ cts. $= \frac{1}{3}$ of \$1.
25 cts. $= \frac{1}{4}$ of \$1.	$16\frac{2}{3}$ cts. $= \frac{1}{6}$ of \$1.
20 cts. $= \frac{1}{5}$ of \$1.	$12\frac{1}{2}$ cts. $= \frac{1}{8}$ of \$1.
10 cts. $= \frac{1}{10}$ of \$1.	$8\frac{1}{3}$ cts. $= \frac{1}{12}$ of \$1.
5 cts. $= \frac{1}{20}$ of \$1.	$6\frac{1}{4}$ cts. $= \frac{1}{16}$ of \$1.

209. ***To find the cost when the quantity is given, and the price is an aliquot part of \$1.***

NOTE.—The symbol @ stands for *at*, and is usually written before the price.

(1) What cost 18 yards of cloth, @ $12\frac{1}{2}$ cts?

Explanation.—At \$1 per yard the cost would be \$18; hence, as the price is $\frac{1}{8}$ of \$1, the cost is $\frac{1}{8}$ of \$18, or $\$2\frac{1}{4}$.

Operation.

$\frac{1}{8}$ of $\$18 = \$2\frac{1}{4}$, Ans.

RULE.—*Take such a part of the given quantity, regarded as dollars, as the price is of* \$1.

EXERCISE XLI.

Find the cost:

2. Of 6 slates, @ 20 cts.
3. Of 37 bushels of apples, @ $33\frac{1}{3}$ cts.
4. Of 14 melons, @ $16\frac{2}{3}$ cts.
5. Of 73 pounds of lard, @ $16\frac{2}{3}$ cts.

6. Of 26 readers, @ 25 cts.
7. Of 57 quarts of milk, @ $6\frac{1}{4}$ cts.
8. Of 13 ties, @ 50 cts.
9. Of $3\frac{5}{8}$ yards of delaine, @ 50 cts.
10. Of 18 pencils, @ $8\frac{1}{3}$ cts.
11. Of 30 spellers, @ $12\frac{1}{2}$ cts.

(12) What cost 23 knives, @ $1.25?

Explanation.—At $1 a knife, the cost would be $23; hence, as the price is $1 + $$\frac{1}{4}$ the cost is $23 + $\frac{1}{4}$ of $23, which is $28.75.

Operation.

4)$23,
5.75
$28.75

Find the cost:

13. Of 25 hams, @ $1.50; of 18 yards of silk, @ $$1.33\frac{1}{3}$.
14. Of 97 books, @ $$1.16\frac{2}{3}$; of 450 feet of hose, @ $$1.12\frac{1}{2}$.

210. ***To find the quantity when the cost is given, and the price is an aliquot part of $1.***

(15) How many books, at 25 cents, can be bought for $7?

Explanation.—If the price were $1 a book, $7 would buy 7 books; hence, since the price is $\frac{1}{4}$ of $1, $7 will buy 4 times 7 books.

Operation.

4×7 books = 28 books.

RULE.—*Multiply the number of dollars in the cost by the number of times the price is contained in one dollar.*

16. How many yards of cloth can be bought for $8, @ $12\frac{1}{2}$ cts.? @ $16\frac{2}{3}$ cts.? @ 20 cts.?

17. How many pounds of butter can be bought for $17.75, @ 25 cts.? @ $33\frac{1}{3}$ cts.? @ 50 cts.?

18. A grocer received $5 for a cheese, @ $16\frac{2}{3}$ cts.; what did the cheese weigh?

19. How many dozen peaches can be bought for $6.50, @ $6\frac{1}{4}$ cts. per dozen? @ $8\frac{1}{3}$ cts. per dozen?

20. How many hats, @ $$1.33\frac{1}{3}$, will $20 buy?

21. How many hoes, @ $1.20, will $$7\frac{1}{5}$ buy?

22. How many pounds of cotton, @ $8\frac{1}{3}$ cents, will pay for 120 yards of ribbon, @ 25 cts.?

23. Bought 642 bushels of apples @ $33\frac{1}{3}$ cts., and paid for them with oats, @ $62\frac{1}{2}$ cts. a bushel; how many bushels were required?

211. ***To find the cost, when the quantity is sold by the 100 or 1000.***

(24) What cost 575 rails at 75 cts. per hundred?

Explanation.—575 = 5.75 times 100; hence, 575 rails cost 5.75 times as much as 100 rails, or $4.3125.

Operation.

575 ÷ 100 = 5.75
5.75 × 75 cts. = $4.3125.

RULE.—*Multiply the price per 100 or 1000 by the hundredth or thousandth part of the given quantity.*

The letter *C* is sometimes used for *hundred*, and *M* for *thousand*.

25. What is the cost of 2540 boards, at $1.80 per C.?

26. What will be the freight on 465 pounds of merchandise at $.62 per C.? At $1.85 per M.?

27. What will 3840 pounds of sugar cost, @ $8.50 per C.?

28. At $35 per M., what will 40450 cigars cost?

29. At $8 per M., what will 25675 bricks cost?

30. What cost 2540 cedar posts, at $9.25 per C.?

31. What cost 4550 envelopes, at $3.25 per M.?

212. ***To find the cost when the thing or quantity is sold by the ton of 2000 pounds.***

(32) At $5.60 a ton, what will 1750 pounds of coal cost?

Explanation.—Since $5.60 is the price of 2000 pounds, $\frac{1}{2}$ of $5.60, or $2.80, is the price of 1000 lbs. The cost of 1750 lbs. is now determined as in Art. 211.

Operation.

$5.60 ÷ 2 = $2.80
1.750 × $2.80 = $4.90.

RULE.—*Multiply ½ of the cost per ton by the one thousandth part of the given quantity.*

33. At $12 per ton, what is the value of 5 loads of hay, each weighing 1640 pounds?

34. What cost 15450 pounds of guano, @ $75 per ton?

35. What is the freight on a quantity of goods weighing 10350 pounds, @ $5.40 per ton?

36. Find the cost of 5450 pounds of cotton seed, at $8.10 per ton.

Accounts and Bills.

213. An **account** is a record of business transactions between two parties.

214. A **debt** is money, goods, or services due from one party to another. A *debtor* is a person who owes a debt, a *creditor* one to whom a debt is due.

215. A **bill of goods** is a written statement given by the seller to the buyer, containing the date of the purchase, the names of the buyer and seller, a list of the goods bought, with their prices and the total amount.

216. An **item** is any article in the bill; *extending* an item is finding its cost, and the *footing* is the entire cost of all the items in a bill.

217. A bill is **receipted** when the words "received payment" are written at the bottom, and it is signed by the creditor, or by some one duly authorized.

The following abbreviations are often used: Account (Acct. or %), Amount (Amt.), At (@), Balance (Bal.), By (per), Creditor (Cr.), Debtor (Dr.), Merchandise (Mdse.), Payment (Payt.), Paid (Pd). Pound (lb.), Received (Recd.), The same (Do.), Next month (Prox.), *This month (Inst.),* Last month (Ult.).

EXERCISE XLII.

218. Copy and find the footings of the following bills:

(1) BATON ROUGE, Jan. 26, 1889.

Mr. J. M. Hart,

Bought of Louis Kretz.

2	doz.	Cups and Saucers	@ $1.25		
$1\frac{1}{2}$	"	Plates.................	@ 1.10		
$1\frac{3}{4}$	"	Ind. Salts...............	@ .35		
$\frac{1}{3}$	gro.	Clothes Pins.............	@ .25		
$1\frac{1}{2}$	doz.	No. 1, Chimneys........	@ .50		
$1\frac{1}{3}$	"	Ind. Butters............	@ .35		
1	ea.,	Pitchers, @ $.35, $.60, $.85....			

Received Payment,

Louis Kretz.

(2) BATON ROUGE, Feb. 2, 1889.

Mr. S. B. Jones,

Bought of Jolly & Reynaud.

$12\frac{1}{2}$	gr.	Hyd. Cocoaine...........	@ $.03		
7	"	Sulph. Atropia..........	@ $.02\frac{1}{2}$		
16	"	Codeia..................	@ $.03\frac{3}{4}$		
$10\frac{1}{4}$	"	Acetate Morphine........	@ .02		
8	"	Pure Grain Musk.........	@ .15		
$3\frac{3}{5}$	ozs.	Sulph. Quinine..........	@ .65		

Received Payment,

Jolly & Reynaud.

Make receipted bills in correct form:

3. J. H. Jordan bought of Adkins & Marsalis, Arcadia, La., June 1, 1889, 4 gals. syrup @ $.63; 14 lbs. starch @ $9\frac{1}{2}$ cts.; 48 yds. sheeting @ 16 cts.; 3 lbs. Y. H. tea @ $.90; 8 doz. eggs @ 21 cts.; 42 lbs. sugar @ 13 cts.

4. Judge C. C. Cordill bought of J. Moore & Bro., St. Joseph, La., May 10, 1889; 42 yds. muslin, @ 56 cts., 35 sacks of oats, @ $\$2\frac{1}{10}$; 16 pairs cotton hose, @ $87\frac{1}{2}$ cts.; 25 pairs shoes, @ $\$3\frac{1}{4}$; 28 yds. silk @ $\$1\frac{3}{4}$.

5. Prof. T. S. Sligh bought of Kidd & Lewis, Ruston, La., July 18, 1888, 85 lbs. coffee, @ $\$\frac{1}{4}$; 56 lbs. bar-soap, @ $\$.06\frac{1}{4}$; 36 lbs. tea, @ $.94; 75 yds. cassimere, @ $\$\frac{7}{8}$; 63 gals. molasses, @ $\$.37\frac{1}{2}$; 125 lbs. rice, @ $\$.08\frac{1}{3}$.

6. Guilbeau & Martin, Breaux Bridge, La., sold Arcade Patin, Dec. 3, 1888, $94\frac{3}{4}$ lbs. bacon, @ 9 cts.; 75 lbs. boneless bacon, @ 14 cts.; 256 lbs. ham, @ $12\frac{1}{2}$ cts.; 325 lbs. shoulder, @ 10 cts.; 68 lbs. dried beef, @ 16 cts.; $32\frac{3}{4}$ lbs. Bologna sausage, @ 18 cts.

7. Dr. W. H. N. Magruder bought of David, Garig & Co., Baton Rouge, La., Dec. 2, 1888, 37 lbs. sugar, @ $\$.06\frac{1}{4}$; 138 lbs. coffee, @ $\$.17\frac{3}{4}$; $25\frac{1}{4}$ lbs. rice, @ $.06; $6\frac{1}{2}$ lbs. starch, @ $\$.03\frac{1}{2}$; $3\frac{1}{2}$ gals. syrup, @ $.45; 60 lbs. flour, @ $\$.03\frac{1}{2}$; 42 lbs. grits, @ $\$.01\frac{3}{4}$; 2 lbs. tea, @ $.57; 15 lbs. dried apples, @ $\$.07\frac{1}{2}$. The amt. due was paid to Burnett, Agent.

8. A. E. Read, Esq., bought of Doherty & Co., Baton Rouge La., May 5, 1889, $\frac{1}{3}$ gross knives and forks, @ $7.75; $\frac{3}{4}$ gross tablespoons, @ $12.25; 250 lbs. rod iron (1 in.), @ $\$.02\frac{5}{8}$; $\frac{2}{3}$ doz. axes, @ $6; 2 kegs nails (10d.), @ $2.50; $\frac{1}{2}$ doz. L. H. shovels, @ $6.35; 2 B. F. Avery & Son stubble-diggers, @ $75; 55 lbs. sad irons, @ $\$.03\frac{1}{4}$; 2 J. H. Hall cotton plows, @ $4.90. The amt. due was paid to Hereford *for the Firm.*

MISCELLANEOUS PROBLEMS.

EXERCISE XLIII.

219. 1. What common fraction is equivalent to .08125?

2. Reduce $.37\frac{3}{16}$ to a simple decimal.

3. Reduce $\frac{5}{6}$ of $1\frac{5}{16}$ to a decimal.

4. Reduce 3.2, $.237\frac{1}{2}$ and $13.2\frac{1}{160}$ to a common denominator.

5. Simplify $(64.5 + .015) - (64.5 \times .015)$.

6. Simplify $.0512 \div .032 \div .005$.

7. From $25.6 \div .064$ take $32.4 \times .015$.

8. How much is $.082\frac{1}{2} \div .006\frac{1}{4} \times 1.25 \div .016$?

9. Find the cost of 327.25 pounds of sugar, @ 8.75 cts.

10. Find the cost of $23.6\frac{1}{4}$ tons of coal, @ $\$5.87\frac{1}{2}$.

11. Find the cost of $\frac{3}{7}$ of 327.6 acres of land, at $\frac{2}{5}$ of $124 an acre.

12. Find the cost of 2432.5 lbs. of cotton, at $\$.70\frac{3}{4}$ per hundred.

13. Find the cost of 3175.75 lbs. of hay, at $\$12\frac{2}{5}$ per ton.

14. What will be the cost of $5.1\frac{1}{4}$ lbs. of rice, at the rate of ($7.6875 ÷ 187.5) per pound?

15. If 19.25 acres are divided into lots of $.8\frac{3}{4}$ of an acre each, what are all the lots worth at $5.35 a piece?

16. At $12.75 per ton, what is the value of $9\frac{1}{2}$ loads of hay, each weighing 1640.5 pounds?

17. How much will a merchant receive for broadcloth which cost $6651, at $7.39 per yard, if he makes $\$\frac{1}{4}$ on each yard?

18. A grocer sold 252 barrels of flour for $2047.50, and gained $\$.37\frac{1}{2}$ on each barrel; what did he pay for the flour per barrel?

19. A farmer paid $791.2375 for 65 sheep @ $4.50, 18 cows @ $23.35, and 6275 rails; what did the rails cost per *hundred?*

20. A man's salary is $1800 a year, of which he spends .125 for board, .097$\frac{2}{9}$ for clothes, and $220 for incidentals; in how many years will he have $3687.50?

21. A farmer bought 115 sheep, @ $3, of which he sold .82$\frac{14}{23}$ at a profit of $1.25 each, and the remainder at a loss of $.50 each; how much did he gain by the operation?

22. A boy gave .40625 of his money for a gun and a saddle; the gun cost $7.75 and the saddle $.875 less than the gun; how much money had he?

23. Find the two numbers whose sum is 93.26 and whose difference is 1.357.

24. A man lost .376 of his money, and after gaining .3875 as much as he lost, had $3848.50; how much had he at first?

25. Find the G. C. D. of 12.375, 9.75 and 7.5.

26. Find the L. C. M. of 2.25, 4.5 and 9.375.

27. A can do a certain work in 8.4 days and B can do the same in 9.24 days; how long will it take both together to do the work?

28. A and B together can cut 6.09375 cords of wood in 6.25 hours, and A alone can cut 5.9 cords in 9.8$\frac{1}{3}$ hours; in how many hours can B alone cut 2.75 cords?

QUESTIONS.

What is a decimal? How is the denominator indicated? Give the rule for writing decimals. For reading decimals.

What is currency? Decimal currency? Repeat the table of United States money. What denominations are used in practice?

How are decimals reduced to a common denominator? Decimals to common fractions? Common fractions to decimals? How are decimals added? Subtracted? Multiplied? Divided?

In business transactions, what is quantity? Price? Cost? An aliquot part? Name some of the aliquot parts of a dollar.

What is an account? A debt? Bill of goods? An item? How is a bill receipted?

CHAPTER III.

DENOMINATE NUMBERS.

MEASURES.*

220. **Quantity** is anything which can be measured.

(1) Time, distance and money are quantities.

221. Quantities may be classified into six kinds, viz.: **values**, **extension**, **capacity**, **weight**, **time** and **angles**.

222. To **measure** a quantity is to find how many times it contains another known quantity of the same kind, called the **measure.**

223. **Standard measures** are measures established by law or custom.

(1) 1 mile, 1 peck and 1 ounce are standard measures.

224. A **principal unit** of measure is the basis of a table of standard measures.

(1) 1 dollar, 1 Troy pound, 1 solar day, 1 meter, etc., are principal units of measures.

NOTE.—In the following tables, the principal unit of measure is printed in capital letters, as 1 POUND.

225. A **denominate number** is a number whose unit is a standard measure.

MEASURES OF VALUE.

226. **Value** is the worth of one thing as compared with another.

The commercial value of any thing is its purchasing power, or its worth in market.

Money is a standard of value, and the medium of exchange.

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or other measures see Appendix, Art. 474.

227. United States money consists of stamped metals, called coins or specie, and printed bills or notes, called paper money **(191)**.

For a table of the name, value, composition and weight of each coin, see Appendix, Art. 472.

228. Canada money is expressed in dollars, cents and mills, which have the same nominal value as the corresponding denominations of United States money.

229. French money is the legal currency of France.

TABLE.

10 millimes‡ (m)	make	1 centime	(c.)
10 centimes†	"	1 decime	(d.)
10 decimes*	"	1 FRANC	(fr.)

The franc is equal in value to $.193, or about $\frac{1}{5}$ dollar; and is also used in Switzerland, Belgium, and, under other names, in Italy, Spain and Greece.

230. English or Sterling money is the currency of Great Britain.

TABLE.

4 farthings (qr. or far.)	make	1 penny	(d.)
12 pence	"	1 shilling	(s.)
20 shillings	"	1 POUND or SOVEREIGN	(£.)

The pound sterling, which is represented by a gold sovereign, is equal in value to $4.8665.

For a table of coins, etc., see Appendix, Art. 473.

231. German money is the currency of the German Empire.

TABLE.

100 pfennige (Pf.) make 1 MARK (RM.)

The mark, which is a silver coin, is equal in value to $.238, or about $\frac{1}{4}$ dollar.

For table of foreign coins see Art. 486.

Pronounced: *dey-seem. †son-teem. ‡mil-leem.

Oral Exercises.

In the following, name the missing numbers:

1. 2 dollars = ( ) dimes; \$5 = ( ) d.; \$7 = ( ) d.
2. 6 dimes = ( ) cts.; 12 d. = ( ) cts.; $4\frac{1}{2}$ d. = ( ) cts.
3. 8 cents = ( ) m.; $6\frac{1}{2}$ cts. = ( ) m.; 13 d. = ( ) cts.
4. £5 = ( ) shillings; £7 = ( ) s.; £2 = ( ) s.
5. 4 shillings = ( ) pence; 9 s. = ( ) d., = ( ) far.
6. 40 dimes = \$( ); 65 d. = \$( ); 75 cts. = ( ) d.
7. 850 cents = ( ) dimes, = ( ) dollars, = ( ) mills.
8. 12 farthings=( ) d.; 36 far.=( ) d.; 41 far. =( ) d.
9. 72 pence = ( ) shillings; 50 d. = ( ) s., = ( ) far.
10. 120 pence = ( ) s., = ( ) £, = ( ) far.

MEASURES OF EXTENSION.

232. Extension is that which has one or more of the dimensions, *length*, *breadth*, and *thickness*.

Measures of extension are of three kinds: measures of **lines**, measures of **surfaces**, and measures of **solids**, in each of which the imperial yard of Great Britain is the *standard unit.*

I. Linear Measures.

233. A **line** has only one dimension, length.

234. Long measure is used in measuring lines or distances.

TABLE.

12 inches (in.)	make	1 foot	(ft.)
3 feet	"	1 YARD	(yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ ft.	make	1 rod	(rd.)
320 rods	make	1 statute mile	(mi.)

In measuring cloth, ribbons, etc., the yard is divided into halves, fourths, eighths, etc. In estimating duties in the Custom House, the yard is divided into tenths and hundredths.

235. Surveyors' linear measures are used by land surveyors in measuring *roads* and *boundaries* of land.

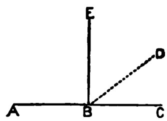
TABLE.

7.92	inches (in.)	make	1 link	(l.)
100	links	"	1 CHAIN	(ch.)
80	chains	"	1 mile	(mi.)

The chain, called Gunter's chain, is 4 rods or 66 feet long. Hence, 25 links = 1 rod, and 4 rods = 1 chain.

II. Surface Measures.

236. An **angle** is the opening between two right lines that proceed from a common point, called the **vertex.**



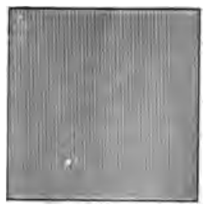
Thus, ABD and DBC are *angles*, and B is their *vertex.*

When one line meets another line so as to make the *adjacent* angles equal, each angle is a *Right Angle*, and the lines are said to be *perpendicular* to each other.

Thus, ABE and EBC are right angles.

237. A **surface** has *two* dimensions, *length* and *breadth;* as the face of a blackboard.

238. A **square** is a plane surface, bounded by four equal sides, and having four right angles.


A SQUARE INCH.

A *Square Inch* is a square, each side of which is 1 *in.* in length.

A square measure is called the square of the linear measure.

239. Square measures are used in measuring surfaces.

TABLE.

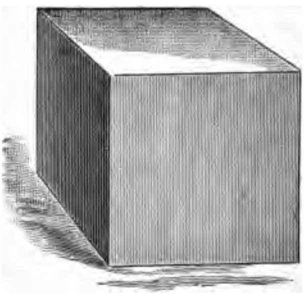
144	square inches (sq. in.)	make	1 square foot	(sq. ft.)
9	square feet	"	1 SQUARE YARD	(sq. yd.)
$30\frac{1}{4}$	square yards	"	1 square rod	(sq. rd.)
160	square rods	"	1 acre (A.)	

240. Surveyor's square measures are used chiefly in government surveys.

TABLE.

625 square links (sq. l.)	make	1 square rod (sq. rd.)
16 square rods	"	1 SQUARE CHAIN (sq. ch.)
10 square chains	"	1 acre (A.)
640 acres	"	1 square mile (sq. mi.)
36 square miles	"	1 township (Tp.)

III. Cubic Measure.



A CUBIC INCH.

241. A solid is a body, volume, or space, that has three dimensions, *length*, *breadth*, *and thickness;* as a box, a room, or a book.

242. A cube is a solid bounded by six equal squares, called *Faces*. The sides of these squares are called the *Edges* of the cube.

A cubic inch is a cube, each edge of which is 1 *in*. in length.
A cubic foot is a cube, each edge of which is 1 *ft*. in length.

243. Cubic measures are used in measuring solids.

TABLE.

1728 cubic inches (cu. in.)	make	1 cubic foot (cu. ft.)
27 cubic feet	"	1 CUBIC YARD (cu. yd.)
128 cubic feet	"	1 cord (C.)

A cord of wood is a pile 8 ft. long, 4 ft. wide, and 4 ft. high; and *a cord foot* is one foot in length of such a pile; hence, 1 cord foot = *16 cu. ft.* A cord contains 128 cu. ft.

Oral Exercises.

Read the following, calling the missing numbers:

1. 2 feet = ( ) in.; 6 ft. = ( ) in.; $3\frac{1}{2}$ ft. = ( ) in.
2. 48 inches = ( ) ft.; 84 in. = ( ) ft.; 30 in. = ( ) ft.
3. 5 yd. = ( ) ft., = ( ) in.; 20 ft. = ( ) yd., = ( ) in.
4. 4 rods = ( ) yd.; 7 rd. = ( ) yd.; 33 yd. = ( ) rd.
5. 5 chains = ( ) rd., = ( ) yds., = ( ) ft., = ( ) in.
6. 400 chains = ( ) miles, = ( ) rd.; 5 mi. = ( ) rd.
7. $\frac{1}{4}$ mile = ( ) rd.; $8\frac{1}{3}$ yd. = ( ) ft.; $7\frac{3}{8}$ ft. = ( ) in.
8. 3 chains = ( ) links, = ( ) rd., = ( ) yd., = ( ) ft.
9. 2 sq. feet = ( ) sq. in.; 5 sq. ft. = ( ) sq. in.
10. 1440 sq. in. = ( ) sq. ft.; 432 sq. in. = ( ) sq. ft.
11. $\frac{1}{2}$ acre = ( ) sq. rd., = ( ) sq. ch.; $\frac{3}{4}$ A. = ( ) sq. rd.
12. $\frac{1}{2}$ sq. mi. = ( ) acres; $\frac{1}{5}$ sq. mi. = ( ) acres.

MEASURES OF CAPACITY.

244. **Capacity** signifies extent of *room* or *space*.

245. Measures of capacity are of two kinds: measures of **liquids**, and measures of **dry substances**.

246. **Liquid measures** are used in measuring *liquids*.

TABLE.

4 gills (gi.)	make	1 pint	(pt.)
2 pints	"	1 quart	(qt.)
4 quarts	"	1 GALLON	(gall.)

The gallon contains 231 cubic inches.

The barrel and hogshead have no fixed capacity, but vary when used for commercial purposes. Usually, especially in measuring cisterns, 1 barrel (bar.) = $31\frac{1}{2}$ galls., and 1 hogshead (hhd) = 63 galls.

247. **Apothecaries' fluid measure** is used for measuring all liquids that enter into the composition of medical prescriptions.

TABLE.

60 minims (m.)	make	1 fluid drachm	(fʒ.)
8 fʒ	"	1 fluid ounce	(f℥.)
16 f℥	"	1 pint	(O.)
8 O	"	1 GALLON	(cong.)

NOTES.—1. Cong. is an abbreviation for *congiarium*, the Latin for gallon; O. is the initial of *octans*, the Latin for *one-eighth*, the pint being one-eighth of a gallon.

2. For ordinary purposes, 1 tea-cup = 2 wine-glasses = 8 table-spoons = 32 tea-spoons = 4 f℥.

248. Dry measures are used for measuring quantities that are not liquid; such as grain, potatoes, etc.

TABLE.

2 pints (pt.)	make	1 quart	(qt.)
8 quarts	"	1 peck	(pk.)
4 pecks	"	1 BUSHEL	(bu.)

The bushel is 8 in. deep, and 18½ in. in diameter, and contains 2150.4 cubic inches.

This table is of but little practical use, as dry articles are now generally bought and sold by weight. See Art. 474.

Oral Exercises.

Read the following, calling the missing numbers:

1. 40 gills = ( ) pt., = ( ) qt.; 9 pt. = ( ) gills.
2. 8 gallons = ( ) qt., = ( ) pt., = ( ) gills.
3. 24 pints = ( ) qt., = ( ) gal.; 18 qt. = ( ) gal.
4. 120 minims = ( ) fʒ; 5 f. ℥ = ( ) f. ʒ = ( ) m.
5. 5 cong. = ( ) O., = ( ) f. ℥; 48 f. ℥ = ( ) O.
6. 252 f. ʒ = ( ) f. ℥ = ( ) O.; 70 O. = ( ) cong.
7. 32 pints = ( ) qt., = ( ) pks.; 32 qt. = ( ) pks.
8. 20 quarts = ( ) pt., = ( ) pks.; 2 bu. = ( ) pks.

MEASURES OF WEIGHT.

249. Weight is the measure of the force of gravity, which draws bodies toward the center of the earth.

250. Troy weight is used in weighing precious stones, gold, silver, etc.

TABLE.

24 grains (gr.)	make	1 pennyweight (pwt.)
20 pennyweights	"	1 ounce (oz.)
12 ounces	"	1 POUND (℔.)

The standard unit of weight in the United States is the troy pound of the Mint, which is identical with the troy pound of Great Britain.

251. Avoirdupois weight is used for all the ordinary purposes of weighing.

TABLE.

16 ounces (oz.)	make	1 POUND (℔.)
100 pounds	"	1 hundredweight (cwt.)
20 hundredweight	"	1 ton (T.)

In weighing some of the coarser articles, as iron and coal at the mines, and goods on which duties are paid at the U. S. Custom Houses, the *long ton* of 2240 pounds is still used.

252. Comparison of weights.

1 ℔. Troy = 5760 gr. Troy. 1 oz. Troy = 480 gr. Troy.
1 ℔. Avoir. = 7000 gr. Troy. 1 oz. Avoir. = $437\frac{1}{2}$ gr. Troy.
175 ℔. Troy = 144 ℔. Avoir.

253. Apothecaries' weight is used in prescribing and in compounding dry medicines.

TABLE.

20 grains (gr.)	make	1 scruple	(℈ or scr.)
3 scruples	"	1 dram	(ʒ or dr.)
8 drams	"	1 ounce	(℥ or oz.)
12 ounces	"	1 POUND	(℔.)

The pound, ounce, and grain of this weight are the same as those of Troy weight.

Oral Exercises.

Read the following, calling the missing numbers:

1. 5 scruples = ( ) grains; 6 ʒ = ( ) ℈, = ( ) gr.
2. 10 ounces = ( ) ʒ, = ( ) ℈, = ( ) gr.
3. 480 drams = ( ) ℥, = ( ) ℔.; 15 scr. = ( ) dr.
4. 28800 grs. = ( ) ℈, = ( ) ʒ = ( ) ℥ = ( ) ℔.
5. 3 pennyweights = ( ) grains; 7 oz. = ( ) pwt.
6. 5 pounds = ( ) oz., = ( ) pwt.; $3\frac{1}{2}$ oz. = ( ) pwt.
7. 1200 pwt. = ( ) oz., = ( ) ℔.; 120 pwt. = ( ) gr.
8. 7 tons = ( ) cwt., = ( ) ℔.; 5 ℔. = ( ) oz.
9. 4800 pounds = ( ) cwt., = ( ) T.; 17 T. = ( ) ℔.
10. $3\frac{1}{4}$ cwt. = ( ) ℔.; $2\frac{1}{5}$ T. = ( ) cwt.; $5\frac{1}{2}$ ℔. = ( ) oz.

MEASURES OF TIME.

254. **Time** is a limited portion of duration.

TABLE.

60 seconds (sec.)	make	1 minute	(min.)
60 minutes	"	1 hour	(hr.)
24 hours	"	1 DAY	(da.)
7 days	"	1 week	(wk.)
4 weeks	"	1 month	(mo.)
365 days	"	1 common year	(yr.)
366 days	"	1 leap year.	
100 years	"	1 century	(cen.)

255. The **solar day** is the interval of time between two successive passages of the sun over the same meridian.

256. The **mean solar day** is the average length of all the solar days in the year. Its duration is 24 hours, and it *is the principal* unit of time measures.

257. The civil year is divided into twelve calendar months, thus:

January (Jan.)	1st mo..31 da.	July (July)	7th mo..31 da.
February (Feb.)	2d mo..28 da.	August (Aug.)	8th mo..31 da.
March (Mar.)	3d mo..31 da.	September (Sep.)	9th mo..30 da.
April (Apr.)	4th mo..30 da.	October (Oct.)	10th mo..31 da.
May (May)	5th mo..31 da.	November (Nov.)	11th mo..30 da.
June (June)	6th mo..30 da.	December (Dec.)	12th mo..31 da.

In most business transactions, 30 days are counted as a month, and 12 months a year.

A *solar year*, or the time of one revolution of the earth around the sun, is 365 da. 5 hr. 48 min. 49.62 sec. If we consider 365 da. as a *year*, the time lost in the calendar in 4 years will amount to 1 day lacking 44 min. 41.52 sec. Hence, 1 day is added to February every 4th year, which is called *leap year*. In 100 years this difference, 44 min. 41.52 sec., amounts to 1 day, lacking 5 hr. 22 min. 42 sec.; hence, every hundredth year we omit to add 1 day, thus losing 5 hr. 22 min. 42 sec., which loss in the calendar amounts to *about* 1 day in 400 years. Hence,

Every year, except those ending with two 0's, *which is exactly divisible by* 4, *is a leap year; as* 1832, 1860, 1888.

Every year ending with two 0's, *which is exactly divisible by* 400, *is a leap year; as* 1600, 2000, 2800.

Every year which is not so divisible is a common year; as 1863, 1900, 2100.

The *civil day* begins and ends at 12 o'clock, midnight. A. M. denotes the time before noon; M., at noon; and P. M., after noon.

Oral Exercises.

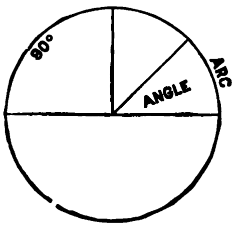
Read the following, calling the missing numbers:

1. 5 da. = ( ) hr.; 8 hr. = ( ) min.; 12 min. = ( ) sec.
2. 56 da. = ( ) wk.; 240 sec. = ( ) min.
3. 8 mo. = ( ) wk.; 12 wk. = ( ) da.; 60 hr. = ( ) da.
4. $2\frac{1}{3}$ da. = ( ) hr.; 100 sec. = ( ) min.; $\frac{3}{4}$ hr. = ( ) min.

Of the following, name the leap years:

5. 1888; 1854; 1860; 1800; 1900; 2000; 2100; 2102; 1916; *1904; 1935; 2600*; *2228*; 1890; 3200.

MEASURES OF ANGLES.



CIRCLE.

258. A **circle** is a plane figure bounded by a curved line called the circumference, every point of which is equally distant from a point within called the center.

259. An **arc** is a part of the circumference.

260. A **degree** is 1 of the 360 equal arcs into which the circumference of a circle may be divided.

261. An **angle of one degree** is 1 of the 360 equal angles that exactly fill the surface about a common point in a plane.

An angle of 1 degree is constant, but the degree itself varies with every change in the size of the circle.

262. The **radius** of a circle is a straight line drawn from the center to the circumference; and the **diameter** is a straight line drawn through the center and terminated both ways by the circumference.

263. **Angular or circular measures** are used in measuring angles, determining latitude and longitude, etc.

TABLE.

60 seconds ($''$)	make	1 minute ($'$)
60 minutes	"	1 DEGREE (°)
30 degrees	"	1 sign (S.)
12 signs, or 360°,	"	1 circumference (cir.)

264. A **quadrant**, or one-fourth of a circumference, is an arc of 90°.

265. MISCELLANEOUS TABLES.

12 things = 1 dozen.	24 sheets = 1 quire.
12 dozen = 1 gross.	20 quires = 1 ream.
12 gross = 1 great gross.	2 reams = 1 bundle.
20 things = 1 score.	5 bundles = 1 bale.

THE METRIC SYSTEM.

266. The **metric system** is a decimal system of measures.

The system originated in France during the French Revolution of 1789, at a time when there was a general disposition to abolish old customs. The object of the metric system was to establish a practical and uniform system of measures in place of the confused and impractical ones which were then in use.

In consequence of its superior merits, it has been adopted, wholly or in part, by nearly all civilized countries. In the United States its use was authorized and recommended by an act of Congress in 1866.

If all the other weights and measures were abolished, and the metric system brought into general use, it would not only secure uniformity, simplicity and brevity in the calculations of Exchange, but would also materially diminish the labor of teaching and learning Arithmetic.

A careful consideration of its chief merits is perhaps the best method of learning the system.

Principal Points of Superiority.

267. I. *A single unit, called the* METER, *is the basis of the entire system.*

Meter is from the Greek *metron*, a measure, and is pronounced mē′ter. It is about the ten-millionth part of the distance from the Equator to the North Pole.

268. II. *There are only two other principal units; the* LITER *and the* GRAM.

Liter is from the Greek *litra*, a pound, and is pronounced lī′ter; and gram is from the Greek *gramma*, a small weight, and is pronounced grăm. The liter and gram each sustain a fixed relation to the meter. See Arts. 276 and 277.

269. III. *The names of the other denominations are formed by* PREFIXES *which immediately show the relation of each to the principal unit.*

270. The prefixes for the higher denominations are the **Greek** numerals, viz.: **deca**, meaning 10; **hecto**, meaning 100; **kilo**, meaning 1000; and **myria**, meaning 10000.

271. The prefixes for the lower denominations are the Latin numerals, viz.: **deci**, meaning .1; **centi**, meaning .01; and **milli**, meaning .001.

These prefixes furnish the key to the whole system, and should be thoroughly committed to memory.

Linear Measures.

272. The legal length of a meter, which is the unit, is 39.37 inches, and its approximate length is $1\frac{1}{12}$ yards.

TABLE.

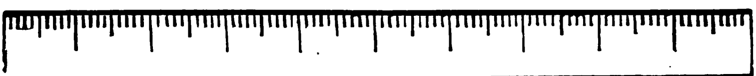
10 mil'li-me'ters (mm)	=	1 cen'ti-me'ter	(cm.)
10 cen'ti-me-ters	=	1 dec'i-me'ter	(dm.)
10 dec'i-me'ters	=	1 METER	(m.)
10 me'ters	=	1 dec'a-me'ter.	(Dm.)
10 dec'a-me'ters.	=	1 hec'to-me'ter	(Hm.)
10 hec'to-me'ters	=	1 kil'o-me'ter	(Km.)
10 kil'o-me'ters	=	1 myr'ia-me'ter	(Mm.)

NOTES.—1. The *accent* of each *unit* and *prefix* is on the *first* syllable, and remains so in the compound words.

2. Abbreviations of the higher denominations begin with a *capital*, those *of the lower* begin with a *small* letter.

The meter, like the yard, is used in measuring short distances; the kilometer, like the mile, in measuring long distances; and the centimeter and millimeter are used for minute measurements.

The following represents the exact length of 10 centimeters:



The approximate length of a millimeter is $\frac{1}{25}$ in.; of a centimeter, $\frac{2}{5}$ in.; of a kilometer, $\frac{5}{8}$ mi.

273. Surface measures are simply the squares of the linear measures. The legal area of a square meter, which *is the unit,* is 1.196 sq. yd., and its approximate area is $1\frac{1}{5}$ *sq. yd.*

TABLE.

100 sq.	millimeters (sq. mm.)	= 1	sq.	centimeter	(sq. cm.)
100 "	centimeters	= 1	"	decimeter	(sq. dm.)
100 "	decimeters	= 1	"	meter	(sq. m.)
100 "	meters	= 1	"	decameter	(sq. Dm.)
100 "	decameters	= 1	"	hectometer	(sq. Hm.)
100 "	hectometers	= 1	"	kilometer	(sq. Km.)
100 "	kilometers	= 1	"	myriameter	(sq. Mm.)

274. The ar (*pro.* är) is the unit of Land Measure; it is a square, each side of which is 10 meters (1 decameter) in length, and hence its area is one square decameter.

100 centars (ca.)	make	1 ar	(a.)
100 ars	"	1 hektar	(Ha.)

The *square meter* and its subdivisions are used for measuring small surfaces.

The approximate area of an ar is $\frac{1}{4}$ sq. chain, or 4 sq. rods; of a hektar $2\frac{1}{2}$ acres.

275. Cubic measures are the cubes of the linear measures. The legal volume of the cubic meter is 1.308 cu. yards, and its approximate value is 1.3 cu. yards.

TABLE.

1000 cubic	millimeters (cu. mm.)	= 1	cubic	centimeter	(cu. cm.)
1000 "	centimeters	= 1	"	decimeter	(cu. dm.)
1000 "	decimeters	= 1	"	meter	(cu. m.)
1000 "	meters	= 1	"	decameter	(cu. Dm.)
1000 "	decameters	= 1	"	hectometer	(cu. Hm.)
1000 "	hectometers	= 1	"	kilometer	(cu. Km.)
1000 "	kilometers	= 1	"	myriameter	(cu. Mm.)

The cubic meter is sometimes called a stere, from the Greek *sterios*, a solid. A stere is about $\frac{1}{4}$ of a cord.

Measures of Capacity.

276. The **liter** (l.), which is the unit of measure, is equal *in volume to a cube* whose edge is a decimeter; that

is, $\frac{1}{10}$ of a meter. Its legal value is 1.0567 liquid quarts, or .908 dry quarts, and its approximate value 1 quart.

Let the student produce the table by writing liter for meter in the preceding table of linear measures.

The liter is used in measuring milk, wine, small fruits, etc.: and the hektoliter, which is about $26\frac{1}{2}$ gals. or $2\frac{5}{8}$ bu., is used in measuring grain, liquids in casks, etc.

Measures of Weight.

277. The **gram** is the *principal unit* of weight, and is equal to a *cubic centimeter* of distilled water at its greatest density, viz.: at 4° Centigrade, or 39.2° Fahrenheit.

The legal weight of a gram is 15.432 grains troy; and its approximate weight is $15\frac{1}{2}$ grains.

Let the student produce the table by writing gram for meter in the preceding table of linear measures.

The gram is used in weighing gold, silver, jewels, letters, etc., and in mixing medicines; the kilogram (often called kilo), which is about $2\frac{1}{5}$ pounds, in weighing common articles, as sugar, butter, etc.; and the metric ton, 100 myriagrams = 1 tonneau or ton (T), which is about 2200 lbs., is used in weighing heavy articles.

The nickel 5-cent piece weighs 5 grams. The silver $\frac{1}{2}$ dollar $12\frac{1}{2}$ grams. The silver dime weighs $2\frac{1}{2}$ grams. The silver $\frac{1}{4}$ dollar $6\frac{1}{4}$ grams.

The weight of a letter for single postage must not exceed 15 grams, or 3 nickels.

278. IV. *Numbers composed of different denominations of the same unit may be written as one number.*

279. Units of length, of capacity, and of weight, form a scale of tens; hence, in writing numbers of these denominations, like \$, d., *ct.*, and m., each order of units will occupy one place.

Thus, 8 m. 5 dm. 3 cm. may be written 853 cm.
9 l. 7 dl. 4 ml. may be written 9704 ml.
6 Dg, 5 g. 2 cg. may be written 6502 cg.

280. Units of square measures form a scale of hundreds; hence, two places must be allowed to each order.

35 sq. m. 3 sq. dm. 5 sq. cm. may be written 350305 sq. cm.
7 hectars (7 Ha.) 5 ars (5a.) 9 centars (9 ca.) may be written 70509 ca.

281. Units of solid measures form a scale of thousands; hence, three places must be allowed each order.

37 cu. m. 124 cu. dm. 8 cu. cm. may be written 37124008 cu. m.

Oral Exercises.

282. Read the following, calling the missing numbers:

1. 5 dm. 7 cm. = ( ) cm.; 8 m. 3 cm. = ( ) cm.
2. 8 Hl. 5 Dl. = ( ) Dl.; 5 Dl. 7 dl. = ( ) dl.
3. 2 Kg. 9 Dg. = ( ) Dg.; 7 Hg. 5 cg. = ( ) cg.
4. 7 Dm. 8 m. 5 dm. 3 cm. = ( ) cm.
5. 7 sq. m. 5 sq. dm. 13 sq. cm. = ( ) sq. cm.
6. 15 cu. Hm. 7 cu. Dm. 123 cu. m. = ( ) cu. m.

283. V. *Changing a number from one denomination to another is performed by simply moving the decimal point.*

284. To perform the operation of reduction with facility, it is only necessary to learn the relative places of the units or prefixes.

	K H D 1 d c m
Thus:	7 6 5 4 . 3 2 1

RULE.—To reduce a number from one denomination to another, the scale being 10, *move the decimal point to the right or left, and as many places as are indicated by the order of the units.*

(1) Change 8.354 Hm to dm.

Solution: Since d. is 3 places to the right of H, I move the point 3 places to the right, and obtain 8354. dm.

(2) Reduce 15.6 cl. to Kl.

Solution: Since k. is 5 places to the left of c., I move the point 5 places to the left, and obtain .000156 Kl.

Oral Exercises.

285. Read the following, calling the missing numbers:

3. 405.32 m. = ( ) dm., = ( ) Dm. = ( ) cm. = ( ) Km.
4. 63.537 l. = ( ) Hl., = ( ) dl., = ( ) ml. = ( ) Dl.
5. 543. g. = ( ) Kg., = ( ) cg., = ( ) Dg., = ( ) mg.
6. 3.87 Hg. = ( ) g., = ( ) Kg., = ( ) cg., = ( ) dg.
7. .0005 Dl. = ( ) dl., = ( ) ml., = ( ) Hl., = ( ) cl.
8. .0237 cm. = ( ) m., = ( ) mm., = ( ) Dm., = ( ) Km.
9. 3425 mg. = ( ) g., = ( ) cg., = ( ) Kg., = ( ) dg.

QUESTIONS.

What is quantity? Name the six kinds of quantity. What is a measure? Standard measures? A denominate number?

What is value? Money? What is the value in U. S. money of a pound sterling? Of a franc? A mark?

What is extension? A line? Surface? Solid? Repeat the table of Long measure. Of surveyors' linear measures. Square measures. Surveyors' square measures. Cubic measures. For what is each of these measures used? What is a square? A cube?

What is capacity? How many kinds of measures of capacity? For what used? Repeat the tables. How many cubic inches in a gallon? In a bushel?

What is weight? How many kinds of measures of weight? For what used? Repeat the tables? How does a pound troy compare with a pound avoirdupois?

What is time? Repeat the table? What is a solar day? Name the number of days in each month? What is the rule for determining the leap and common years?

What is a circle? Arc? Degree? Radius? Diameter? For what are angular measures used? Repeat the table.

What is the metric system? State its origin, etc. Name its principal points of superiority. What are the prefixes for the higher denominations? For the lower denominations? Repeat the table of linear measures. Of surface measures. Cubic measures. Measures of capacity. Measures of weight.

What is the legal value of a meter? A liter? A gram?

COMPOUND NUMBERS.

286. A **compound number** is a denominate number expressed in terms of two or more units of the same kind.

(1) 4 da. 3 hr. 20 min. is a compound number.

REDUCTION.

287. The **reduction** of a denominate number is the process of changing it from one denomination to another without altering its value.

The process of changing a denominate number from a higher to a lower denomination is **reduction descending**; and the process of changing a denominate number from a lower to a higher denomination is **reduction ascending**.

288. ***To perform reduction descending.***

PRINCIPLE—*The value of a denominate number is not altered by decreasing the unit and increasing the number of units in the same ratio.*

(1) 3 gallons = 12 quarts; for, 3 gal. = 4 × 3 × ($\frac{1}{4}$ gal.) = 12 qt.

(1) Reduce 5 bu. 2 pk. 3 qt. to quarts.

Analysis.	Operation.
1 bu. = 4 pk.	5 bu. 2 pk. 3 qt.
5 bu. = 20 pk., + 2 pk. = 22 pk.	4
1 pk. = 8 qt.	22 pk.
22 pk. = 176 qt., + 3 qt. = 179 qt.	8
	179 qt. Ans.

RULE.—I. *Multiply the number of the highest denomination by the number of units of the next lower which equals a unit of the higher, and to the product add the number of the lower denomination, if any.*

II. *Proceed in like manner with this and each successive result thus obtained, until the number is reduced to the required denomination.*

NOTE.—The successive denominations of the compound number should be written in their proper orders, and the vacant denominations, if any, filled with ciphers.

EXERCISE XLIV.

Complete the following:

2. 5 gal. 2 qt. = ( ) qt.
3. 9 yd. 2 ft. 5 in. = ( ) in.
4. 7 bu. 3 pk. = ( ) pk.
5. 5 A. 11 sq. rd. = ( ) sq. rd.
6. \$5 7ct. = ( ) ct.
7. 3 mi. 5 yd. 2 ft. = ( ) in.
8. 8 dm. 3 mm. = ( ) mm.
9. 7 gal. 2 qt. 1 pt. = ( ) pt.
10. 5 yd. 7 in. = ( ) in.
11. 2 oz. 7 pwt. 19 gr. = ( ) gr.
12. £2 5d. = ( ) d.
13. 5 cwt. 80 ℔. 12 oz. = ( ) oz.
14. 2 ℔. 5 oz. = ( ) oz.
15. 3 ℔. 10 ℥. 5 ʒ. 1 ℈ = ( ) ℈.
16. 3 oz. 7 pwt. = ( ) pwt.
17. 1 ℔. 7 oz. = ( ) scr.
18. 4 hr. 17 min. = ( ) min.
19. 3 wk. 4 da. 5 hr. = ( ) sec.
20. 3 S. 15° = ( )°.
21. 5° 17′ 23″ = ( )″.

22. Reduce 14 mi. 256 rd. 3 yd. 2 ft. 5 in. to inches.

23. A grocer sold 75 bu. 3 pk. 7 qt. of berries at 15 cts. a quart; what did he get for them?

24. How many panels of fence 12 ft. long will it take to enclose a field 66 rd. long and 42 rd. wide?

25. Reduce 3 Dm. 5 dm. to millimeters.

26. Reduce 4 Kl. 2 Hl. 3 l. to centiliters.

27. Express 5 sq. Hm. 3 sq. m. 5 sq. dm. as sq. decimeters.

28. Express 4 cu. Dm. 15 cu. m. 125 cu. cm. in terms of cu. centimeters.

29. How many times will a wheel 11 feet in circumference turn in going 11 mi. 132 rd?

30. Find the cost of 9 T. 17 cwt. 93 ℔. of sugar at $5\frac{1}{4}$ cts. a pound.

31. Find the cost of 12.875 Kg. of quinine at 14 cts. per gram.

32. How many seconds old is a man who has lived 64 yr., 120 da., 10 hr., allowing $365\frac{1}{4}$ da. to the yr.?

33. If the length of a man's step is 2 ft. 9 in., how many steps will he make in walking 275 mi.

34. Find the cost of 2 ℔. 3 ℥ 4 ʒ of calomel, at $2\frac{1}{2}$ cts. a grain.

35. What is the value of 12 ℔. 5 oz. 6 pwt. of gold, at 87 cts. a pwt?

36. One penny is about $2\frac{1}{4}$ cts.; what is the value of £68 3 s. 6 d. in U. S. money?

289. ***To perform reduction ascending.***

PRINCIPLE.—*The value of a denominate number is not altered by increasing the unit and decreasing the number of units in the same ratio.*

(1) 6 pints = 3 quarts; for 6 pt. = $\frac{1}{2}$ of 6 × (2 pt.) = 3 qt.

(1) Reduce 575 ft. to rods, etc.

Analysis.

575 ft. = 573 ft. + 2 ft.
573 ft. = $\frac{1}{3}$ of 573 yd. = 191 yd.
191 yd. = 187 yd. + 4 yd.
187 yd. = $\frac{1}{5\frac{1}{2}}$ of 187 rd. = $\frac{2}{11}$ of 187 rd. = 34 rd.
Hence, 575 ft. = 34 rd. 4 yd. 2 ft.

Operation.

3	575 ft.
$5\frac{1}{2}$	191 yd. + 2 ft.
2	2
11	382 half-yards,
	34 rd. + 8 half-yd.

Ans, 34 rd. 4 yd. 2 ft.

RULE.—I. *Divide the given denominate number by the number of units of its denomination which equals one unit of the next higher, and place the remainder, if any, at the right.*

II. *Proceed in like manner with this and each successive quotient thus obtained, until the number is reduced to the required denomination.*

III. *The last quotient, with the several remainders annexed in proper order, will be the answer required.*

EXERCISE XLV.

Complete the following:

2. 45s. = £ ( ), etc.

3. 5375 far. = £ ( ), etc.

4. 56 in. = ( ) ft., etc

5. 443 in. = ( ) rd., etc.

6. 65 sq. ch. = ( ) A.

7. 12607 sq.ch = ( ) sq.mi.,etc

8. 28 gi. = ( ) qt., etc.

9. 5007 gi. = ( ) gal., etc.

10. 60 pt. = ( ) pk., etc.

11. 1891 pt. = ( ) bu., etc.

12. 65 pwt. = ( ) oz., etc.

13. 46364 gr. = ℔., etc.

14. 120 ℥ = ( ) ℔., etc.

15. 1120 ℈ = ( ) ℔., etc.

16. 320 min. = ( ) hr., etc.

17. 648370 sec. = ( ) wk., etc.

18. 645″ = ( )′ etc.

19. 465301″ = ( ) S., etc.

20. 68 sq. ft. = ( ) sq. yd., etc.

21. 3340 cu. ft. = ( ) c., etc.

22. Reduce 426508 gr. to pounds, ounces, etc.

23. Reduce 8420724 oz. to tons, cwts., etc.

24. Reduce 85264 sq. ft. to sq. rods, etc.

25. Reduce 48400 ft. to miles, etc.

26. Reduce 40335 ml. to l., dl., cl., etc.

27. Reduce 370400 sq. m. to sq. Hm., sq. Dm.

28. Express 321 cg. in terms of Hg.

29. At 4 cts. a pint, how many bushels, etc., of cranberries can I buy for \$117.44?

30. A jeweler paid \$701.52 for a quantity of silverware at $\frac{1}{2}$ ct. per grain; how many pounds, etc., did he buy?

31. A priced his land at 4 cts. per sq. foot; but deducted $\frac{3}{8}$ of the price, and received \$34888.83$\frac{3}{4}$ for all; how many acres, etc., had he?

32. A wheel, 1 ft. 4 in. in circumference, revolved 1858 times in going a certain distance; how many rods, etc, did it pass over?

33. How long would it take a cannon ball, flying at the rate of 704 ft. per second, to go from the earth to the moon, a distance of 240,000 miles?

Denominate Fractions.

290. A **denominate fraction** is one which expresses one or more of the equal parts of a denominate unit.

(1) $\frac{2}{3}$ of an hour, .9 of a pk., etc., are denominate fractions.

291. ***To reduce a denominate fraction from a greater to a less denomination.***

(1) Reduce $\frac{3}{160}$ of a bushel to a fraction of a pint.

Explanation and rule are essentially the same as in Art. 288.

Operation.

$\frac{3}{160} \times \frac{4}{1} \times \frac{8}{1} \times \frac{2}{1} = \frac{6}{5}$.

EXERCISE XLVI.

Complete the following:

2. $\frac{5}{64}$ bu. = (   ) pt.
3. $\frac{7}{52800}$ mi. = (   ) ft.
4. $\frac{1}{40}$ qt. = (   ) gi.
5. $\frac{3}{1600}$ gal. = (   ) gi.
6. $\frac{1}{96}$ s. = (   ) far.
7. $\frac{7}{4356000}$ A. = (   ) sq ft.

292. ***To reduce a denominate fraction from a less to a greater denomination.***

(1) Reduce $\frac{12}{13}$ of a scruple to the fraction of a pound.

Explanation and rule are essentially the same as in Art. 289.

Operation.

$\frac{12}{13} \times \frac{1}{3} \times \frac{1}{8} \times \frac{1}{12} = \frac{1}{312}$.

EXERCISE XLVII.

Complete the following:

2. $\frac{3}{4}$ in. = (   ) ft.
3. $\frac{12000}{13}$ oz. = (   ) T.
4. $\frac{3}{5}$ sq. ft. = (   ) sq. yd.
5. $\frac{187280}{7}$ gr. = (   ) ℔.
6. $\frac{8}{11}$ min. = (   ) hr.
7. $\frac{2}{5}$ min. = (   ) yr.
8. $\frac{4}{5}$ cm. = (   ) m.
9. $\frac{80}{11}$ dg. = (   )Kg.

293. ***To change a denominate fraction to integers of lower denominations.***

(1) Reduce $\frac{4}{5}$ yd. to integers of lower denominations.

Analysis.

$\frac{4}{5}$ yd. = $\frac{12}{5}$ ft. = $2\frac{2}{5}$ ft.
$\frac{2}{5}$ ft. = $\frac{24}{5}$ in. = $4\frac{4}{5}$ in.
Hence, $\frac{4}{5}$ yd. = 2 ft. $4\frac{4}{5}$ in.

Operation.

$\frac{4}{5} \times 3 = \frac{12}{5} = 2\frac{2}{5}$.
$\frac{2}{5} \times 12 = \frac{24}{5} = 4\frac{4}{5}$.
2 ft. $4\frac{4}{5}$ in., Ans.

(2) Reduce .725 qt. to integers of lower denominations.

Analysis.	Operation.
.725 qt. = 2 × .725 pt. = 1.55 pt.	.725 qt.
.55 pt. = 4 × .55 gi. = 2.2 gi.	2
Hence,	1.550 pt.
.725 qt. = 1 pt. 2.2 gi., Ans.	4
	2.200 gi.

RULE.—I. *Multiply the fraction by that number which will reduce it to the next lower denomination, and reduce the result, if possible, to a whole or mixed number.*

II. *Proceed with the fractional part, if any, as before, until reduced to the denominations required.*

III. *The integral parts of the several products, including all of the last product, will be the required result.*

EXERCISE XLVIII.

Complete the following:

3. $\frac{3}{4}$ yd. = ( ) ft. ( ) in.	**4.** $\frac{4}{9}$ wk. = ( ) da., etc.
5. $\frac{5}{8}$ s. = ( ) d. ( ) far.	**6.** $\frac{3}{5}$ ℔.(Troy) = ( ) oz., etc.
7. $\frac{5}{6}$ ℥ = ( ) ʒ ( ) ℈.	**8.** $\frac{3}{7}$ bu. = ( ) pk., etc.
9. £.75 = ( ) s. ( ) d.	**10.** .6254 da. = ( ) hr., etc.
11. .625 gal. = ( ) qt. ( ) pt.	**12.** .3167 Tp. = ( ) sq. mi., etc.

294. ***To change integers of lower denominations to fractions of a higher.***

(1) What part of £1 is 12 s. 6 d.?

Operations.

12 s. 6 d. = 151 d.	6 d. ÷ 12 = .5 s.
£1 = 240 d.	12 s. 6 d. = 12.5 s.
$\frac{150}{240} = \frac{5}{8}$, or .625.	12.5 s. ÷ 20 = £.625, or £$\frac{5}{8}$.
Hence,	Hence,
12 s. 6 d. = £$\frac{5}{8}$, or £.625.	12 s. .6 d. = £.625, or £$\frac{5}{8}$.

RULE.—*Reduce the number which is a part, and the number which is the whole, to the same denomination, and divide the former by the latter, expressing the quotient under the form of a fraction or decimal.*

NOTE.—The process may often be shortened, as indicated by the second of the preceding operations.

EXERCISE XLIX.

2. Reduce 3 qt. 1 pt. 3 gi. to the fraction of a gal.
3. Reduce 2 pk. 3 qt. 1 pt. to the decimal of a bu.
4. Change 8 oz. 7 pwt. 12 gr. to the decimal of a ℔.
5. Change 18 rd. 1 yd. to the fraction of a mi.

Complete the following:

6. 3 yr. 3 mo. 18 da. = ( ) mo.; = ( ) yr.
7. 7 yr. 7 mo. 6 da. = ( ) mo.; = ( ) yr.
8. 3 pk. 2 qt. 1 pt. = ( ) qt.; = ( ) pk.; = ( ) bu.
9. 7 oz. 14 pwt. 19.2 gr. = ( ) pwt.; = ( ) oz., = ( ) ℔.

(10) What part of 5 yd. 1 ft. is 2 yd. 2 ft.?

Solution: 5 yd. 1 ft. = 16 ft.; 2 yd. 2 ft. = 8 ft.; $\frac{8}{16} = \frac{1}{2} = .5$, Ans.

What part:

11. Of 1 da. is 10 hr. 40 min.?
12. Of 5 yd. 2 in. is 2 yd. 1 ft.?
13. Of 3 bu. is 3 pk. 7 qt. $1\frac{1}{2}$ pt.?
14. Of 4 ℔. 2 oz. 12 pwt. is 6 oz. 8 pwt.?
15. A father divided £1 6 d. among his three sons, giving the first 8 s. 8 d., the second 11 s. 2 far., and the third, 9 d. 2 far.; what part of the whole did each receive?

295. ***To reduce metric to common measures.***

(1) Reduce 375 centimeters to feet.

Explanation.—375 cm. = 3.75 m. Since 1 m. = 39.37 in., 3.75 m. = 3.75 × 39.37 in., = 147.6375 in., ÷ 12 = 12.3031 ft., Ans.

Operation.

```
     39.37
      3.75
12|147.6375
   12.3031 ft. Ans.
```

RULE.—*Multiply the value of the principal metric unit of the table by the given metric number expressed in the same unit, and reduce the product to the denomination required.*

NOTE.—Under this and the next article let the approximate values of the metric *numbers be used* in solving the oral problems.

EXERCISE L.

Complete the following:

2. 6 meters = ( ) yd.
3. 95 m. = ( ) rd.
4. 8 centimeters = ( ) in.
5. 642 dm. = ( ) ft.
6. 12 kilometers = ( ) mi.
7. 80 Km. = ( ) mi.
8. 10 sq. meters = ( ) sq. yd.
9. 4 sq. Hm. = ( ) sq. ft.
10. $5\frac{1}{2}$ ars = ( ) sq. rd.
11. 125 ars = ( ) A.
12. 15 sters = ( ) cords.
13. 24 cu. Dm. = ( ) cu. yd.
14. .7 liters = ( ) pt.
15. 120 Hl. = ( ) bu.
16. 4 grams = ( ) gr.
17. 45 Kg. = ( ) lb.

18. What will it cost to repair a road 45 Km. long, at $150 per mile?

19. How much will 250 hectoliters of wheat cost, at $.60 a bushel?

20. What will be the cost of 45 liters of syrup at $.50 per gallon?

21. How much money will it take to buy 150 kilos of butter, at 40 cts. a pound?

22. What is the cost of 40 sters of wood, at $3 a cord?

23. If a pipe discharges 6.5 liters of water in a minute, how many gallons will it discharge in an hour?

24. How many pills, each weighing 1.2 grams, can be made from 3 ounces of quinine?

296. ***To reduce common to metric measures.***

(1) Reduce 5 miles to kilometers.

Explanation.—I reduce 5 mi. to in. and obtain 316800 in. Dividing this number by 39.37, the number of in. in *a meter,* I have 8046 + m. I now *move the decimal* point 3 places to the left, and obtain 8.046.

Operation.

$5 \times 320 \times 16\frac{1}{2} \times 12 = 316800$

$316800 \div 39.37 = 8046 +$.

8.046 Km., Ans.

RULE.—*Divide the given number by the value of the principal metric unit of the table, and reduce the quotient to the denomination required.*

NOTE.—Before dividing, reduce the given number, if not already so, to the denomination in which the value of the principal unit is expressed.

EXERCISE LI.

Complete the following:

2. 1 in. = ( ) centimeters. 3. 5 ft. = ( ) dm.
4. 10 in. = ( ) millimeters. 5. 7 in. = ( ) cm.
6. 3 mi. = ( ) kilometers. 7. 150 mi. = ( ) Km.
8. 10 acres = ( ) hectars. 9. 100 A. = ( ) sq. Dm.
10. 31 gr. = ( ) grams. 11. 64 ℥ = ( ) Dg.
12. $2\frac{1}{2}$ gal. = ( ) liters. 13. 5 hhd. = ( ) Hl.
14. 22 cords = ( ) sters. 15. 40 cu. ft. = ( ) cu. m.
16. Cloth, at 8 cts. per yard, is how much per meter?
17. Calomel, at 4 cts. per grain, is how much per centigram?

18. What part of a decameter is 5 yards?

19. How many decagrams in 2 ℔. 5 ℥ 4 ʒ?

20. I sold 250 qts. of milk, at 10 cts. a liter; what did I receive for it?

21. How many times will a wheel, 5 meters in circumference, revolve in rolling 2 mi. 120 rd.?

22. What is the value of 3 cwt. 15 ℔. 12 oz. of sugar, at 20 cts. per kilo?

297. The method of adding, subtracting, multiplying and dividing compound numbers is the same as the corresponding operations in integers, and special rules are unnecessary.

In integers the scale is decimal and uniform, and in compound numbers *it is variable and irregular.*

ADDITION.

298. (1) Find the sum of 5 gal. 3 qt. 1 pt., 2 qt. 1 pt. 3 gi., 4 gal. 1 pt. 3 gi., and 1 qt. 1 pt. 1 gi.

Explanation.—The sum of the right-hand column is 7 gi. = 1 pt. 3 gi. I write the 3 gi. beneath, and adding the 1 pt. to the col. of pt., the sum is 5 pt., = 2 qt. 1 pt. I write 1 pt. beneath, and adding 2 qt. to the col. of qt., the sum is 8 qt. = 2 gal., which, added to the col. of gal., gives 11 gal.

Operation.

5 gal.	3 qt.	1 pt.	0 gi.
	2	1	3
4		1	3
	1	1	1
11 gal.		1 pt.	3 gi.

EXERCISE LII.

2. Find the sum of £6 19 s. 11 d. 3 far., £9 6 s. 2 far., £3 10 s. 6 d. 2 far., and £7 8 s. 8 d. 1 far.

3. A person in London paid £7 13 s. 6 d. for a coat, £2 17 s. 9 d. 1 qr, for a vest, £3 8 s. 3 d. for a pair of pantaloons, and £9 11 s. 8 d. 3 qr. for a trunk; what was the amount of his bill?

4. A goldsmith bought 7 ingots of silver, three of which weighed 9 ℔. 7 oz. 14 pwt., and each of the others 8 ℔. 5 oz. 15 pwt. 16 gr.; how much did the whole weigh?

5. My granary contains 59 bu. 4 qt., and I put into it at one time 14 bu. 2 pk. 5 qt.; at another, 23 bu. 3 pk.; at another, 8 bu. 7 qt.; at another, 19 bu. 1 pk.; how many bushels does it now contain?

6. A druggist mixed together 3 ℥ 4 ʒ 1 ℈, 4 ℥ 3 ʒ 2 ℈, 1 ℈ 18 gr., and 6 ℥ 5 ʒ 2 ℈ 16 gr.; how much did it all weigh?

7. A boy served as an apprentice to one man 2 yr. 6 mo. 3 wk. 21 da., to another 1 yr. 8 mo. 1 wk. 5 da., to a third 3 yr. 4 mo. 6 da.; how long did he serve altogether?

(8) Find the sum of $\frac{5}{9}$ ℔., $\frac{1}{6}$ oz. and $4\frac{1}{2}$ pwt.

Rule.—*Reduce each fraction to integers of lower denominations, and add the results.*

Operation.

$\frac{5}{9}$ ℔.	=	6 oz.	13 pwt.	8 gr.
$\frac{1}{6}$ oz.	=		3	8
$4\frac{1}{2}$ pwt.	=		4	12
Sum	=	7 oz.	1 pwt.	4 gr.

Find the sum of:

9. £$\frac{5}{6}$, $\frac{3}{8}$ s., and $5\frac{1}{2}$ d.

10. $2\frac{3}{4}$ rd. and $1\frac{3}{4}$ ft.

11. $3\frac{3}{8}$ T., $4\frac{4}{5}$ cwt., $\frac{3}{4}$ ℔.

12. $\frac{3}{8}$ mi. and $17\frac{1}{6}$ rd.

13. $\frac{5}{6}$ wk., $\frac{7}{8}$ da., and $\frac{3}{4}$ hr.

14. $7\frac{1}{6}$ wk. and $\frac{5}{8}$ da.

15. $\frac{3}{5}$ C., $\frac{3}{8}$ S., and $\frac{5}{6}°$.

16. .365 da. and .75 hr.

17. Find the sum of 5 m. 6 dm. 5 mm., 4 Dm. 7 dm. 8 cm., and 375 dm.

18. Add $\frac{3}{4}$ Dl. $3\frac{7}{8}$ dl. and $42\frac{2}{5}$ cl.

SUBTRACTION.

299. (1) From 9 ℔. 7 oz. 11 pwt. take 3 ℔. 10 oz. 5 pwt.

Explanation.—I write the units of the subtrahend under like units of the minuend, and begin at the right to subtract.

Operation.

℔.	oz.	pwt.
9	7	11
3	10	5
5	9	6

5 pwt. from 11 pwt. leave 6 pwt., which I write beneath.

10 oz. cannot be subtracted from 7 oz.; but the 9 ℔. 7 oz. of the minuend are 8 ℔. 19 oz.; and 10 oz. from 19 oz. leave 9 oz., which I write beneath.

3 ℔. from the remaining 8 ℔. leave 5 ℔., which I write beneath.

Hence, the required difference is 5 ℔. 9 oz. 6 pwt.

EXERCISE LIII.

2. From 60 bu. 2 pk. 3 qt. take 37 bu. 3 pk. 5 qt.

3. From 16 mi. 135 rd. 11 ft. take 9 mi. 46 rd. $13\frac{1}{3}$ ft.

4. From 5 T. 13 cwt. take 2 T. 15 cwt. $46\frac{1}{4}$ ℔.

5. From 3 wk. 5 hr. take 2 wk. 3 da. 35 min.

6. A merchant tailor sold cloth that cost £115 8 s. $7\frac{1}{2}$ d. for £140 6 s.; what was his profit?

7. From a piece of land containing 83 A. a piece was sold, containing 46 A. 130 sq. rd. 25 sq. yd. 8 sq. ft.; how much remained?

8. From a bin containing 125 bu. 7 qt. of corn, 56 bu. 2 pk. 3 qt. were sold at one time, and 38 bu. 4 qt. 1 pt. at another time; how much remained unsold?

9. From $7\frac{2}{3}$ rods take $4\frac{5}{6}$ yards.

10. From $\frac{5}{24}$ lb. troy take $\frac{6}{35}$ lb. avoirdupois.

11. Subtract .0625 bu. from 3 pk. 5 qt. 1 pt.

12. From $2\frac{5}{6}$ cu. Hm. take $37\frac{3}{4}$ cu. m.

13. From 9 Kg. 7 Dg. 3 dg. take $8.63\frac{5}{8}$ Hg.

300. ***To find the time between two dates.****

(14) How many years, months and days from June 16, 1844, to March 15, 1889?

Explanation.—I write the later of the two dates for a minuend, and the earlier for the subtrahend, writing the number of the year, month and day in order.

Operation.

1889 yr.	3 mo.	15 da
1844	6	16
44 yr.	8 mo.	29 da.

I then subtract as in other compound numbers, allowing 30 da. to the mo. and 12 mo. to the yr.

Find the time:

15. From May 2, 1881, to Sept. 5, 1889.

16. From July 7, 1885, to Dec. 4, 1887.

17. From Oct. 15, 1864, to Feb. 6, 1888.

18. How long has a note to run that is dated April 7, 1889, and made payable Feb. 23, 1892?

19. What is the age of a person who was born Dec. 13, 1859?

20. Thomas Jefferson was born April 2, 1743, and died July 4, 1826; how old was he when he died?

21. Gen. Robert E. Lee died Oct. 12, 1870, at the age of 63 yr. 8 mo. 23 da.; what was the date of his birth?

MULTIPLICATION.

301. (1) Multiply 6 mi. 90 rd. 5 ft. by 8.

Explanation.—I write the multiplier under the lowest denomination of the multiplicand, and begin at the right to multiply.

8 times 5 ft. are 40 ft., or 2 rd. 7 ft.; I write *the 7 ft.* beneath.

Operation.

6 mi.	90 rd.	5 ft.
		8
50 mi.	82 rd.	7 ft.

*For other methods, see Appendix, Art. 475.

8 times 90 rd., plus the 2 rd. of the first partial product, are 722 rd., or 2 mi. 82 rd.; I write the 82 rd. beneath.

8 times 6 mi., plus the 2 mi. of the second partial product are 50 mi., which I write beneath. The required product is 50 mi. 82 rd. 7 ft.

EXERCISE LIV.

2. Multiply 7 bu. 3 pk. 6 qt. 1 pt. by 6

3. Multiply 43 rd. 14 ft. $1\frac{3}{4}$ in. by 12.

4. Multiply 5 ℔. 8 oz. 10 pwt. by 7.

5. If each of 10 sacks contains 2 bu. 2 pk. $3\frac{3}{8}$ qt. of corn, how much do all together contain?

6. How much water will a family use in a week, at the rate of 27 gal. 3 qt. 1 pt. 3 gi. per day?

7. How far will a man travel in 9 days, at the rate of 26 mi. 87 rd. 4 yd. 2 ft. per day?

DIVISION.

302. (1) Divide £19 10 s. 4 d. by 8.

Explanation.—I write the dividend and divisor as in integers, and begin at the left to divide.

Operation.

```
8|£19 10 s. 4 d.
  £2   8 s. 9 d. 2 far.
```

$\frac{1}{8}$ of £19 is £2, and £3 remainder; I write the £2 beneath.

The £3 remainder plus the 10 s. of the dividend are 70 s.; $\frac{1}{8}$ of 70 s. is 8 s., and 6 s. remainder; I write the 8 s. beneath.

The 6 s. remainder plus the 4 d. of the dividend are 76 d.; $\frac{1}{8}$ of 76 d. is $9\frac{1}{2}$ d., or 9 d, 2 far., which I write beneath.

Hence, the required quotient is £2 8 s. 9 d. 2 far.

EXERCISE LV.

2. Divide £11 6 s. 3 d. by 5.

3. Divide 39 ℔. 4 ℥ 1 ʒ 2 ℈ by 7.

4. Divide 30 cd. 94 cu. ft. 88 cu. in. by 5.

5. If a garden, containing 72 sq. rd. $16\frac{1}{4}$ sq. yd., be laid off into 11 equal parts, how much will each part contain?

6. A silversmith melted 8 ℔. 10 oz. 7 pwt. of silver, which he made into 12 cups; what did each cup weigh?

7. A man traveled 70 mi. 40 rd. 3 yd. in 6 hours; at what rate per hour did he travel?

8. How much is $\frac{1}{8}$ of 7 Hm. 3 m. 2 cm.?

9. If 9 Dl. 3 l. 6 dl. of chloroform be put equally into 15 bottles, how much will each bottle contain?

10. If 6 Hg. 3 Dg. 14$\frac{2}{5}$ dg. of quinine be made into 1800 pills, how much quinine will each pill contain?

LONGITUDE.

303. The **meridian** of any place is a line on the surface of the earth, passing from the North to the South Pole through that place.

304. The **longitude** of any place is the number of degrees, minutes, and seconds, reckoned on the equator, between a standard meridian (marked 0°) and the meridian of the given place.

All places are in east or west longitude, according as they are east or west of the standard meridian, 180°, or half the circumference of the earth, being the limit.

The meridian of Greenwich is usually taken as the standard meridian by the Americans and English. The longitude from Greenwich of each of the places mentioned in the following problems, is given in the Appendix, Art. 476.

305. ***To find the difference in longitude of two places.***

RULE.—I. When the longitudes are both east or both west, *subtract the less from the greater.*

II. When one longitude is east and the other west, *add them.*

EXERCISE LVI.

1. Find the difference in longitude between Boston and New Orleans; Washington and San Francisco; Cincinnati and Rome; Paris and New York; Berlin and Pekin; Boston and Constantinople; Austin and Vienna.

2. If a town is east of San Francisco and north of Austin, how far is it in longitude from San Francisco?

3. How many degrees of longitude does a person pass over who leaves Paris and goes west until he is north of Baton Rouge?

4. A vessel leaves Charleston for San Francisco; after going 28° 42′ 35″, how many degrees has she still to go?

5. A man left New Orleans and went east until he reached a point south of Washington; how many miles did he travel, a degree of longitude in the latitude of New Orleans being about 59.9 miles?

Relation of Longitude and Time.

306. The earth rotates east on its axis once in 24 hours; hence, in 1 hour it rotates $\frac{1}{24}$ of 360° = 15°; in 1 minute $\frac{1}{60}$ of 15° = 15′; and in 1 second, $\frac{1}{60}$ of 15′ = 15″. That is,

A difference in longitude of		A difference in time of
15°	makes	1 hr.
15′	"	1 min.
15″	"	1 sec.

PRINCIPLE.—*There are 15 times as many degrees, minutes, and seconds in the difference of longitude as there are hours, minutes, and seconds in the difference of time, respectively.*

As the sun appears to move from east to west, places east of a given meridian have earlier or more time, and places west have later or less time.

307. ***Difference in time given, to find the difference in longitude.***

(1) The difference in time between two places is 54 min. 19 sec.; *what is the difference in longitude?*

Explanation.—In accordance with the preceding principle I multiply the difference in time by 15 and obtain the difference in longitude.

Operation

	54 min.	19 sec.
		15
13°	34′	45″, Ans.

RULE.—*Multiply the difference in time by* 15.

EXERCISE LVII.

Find the difference in longitude between two places whose difference in time is:

2. 49 min. 20 sec. **3.** 1 hr. 6 min. 17 sec.

4. 4 hr. 46 min. **5.** 3 hr. 25 min. 26 sec.

Find the longitude of the place whose time:

6. Is 1 hr. 25 min. earlier than Boston time.

7. Is 2 hr. 25 min. 30 sec. later than St. Louis time.

8. Is 3 hr. 2 sec. earlier than Paris time.

9. Is 2 hr. 7 min. 12 sec. later than Rome time.

10. Is 7 hr. 16 min. 47 sec. later than Berlin time.

11. What is the longitude of Detroit, whose time is 46 min. 58 sec. earlier than that of Galveston?

308. ***Difference in longitude given to find the difference in time.***

(1) The difference in longitude of two places is 64° 36′ 26″; find their difference of time.

Explanation.—According to the preceding principle, I divide the difference in longitude by 15 and the quotient is the difference in time.

Operation.

15	64°	36′	30″
	4 hr.	18 min.	26 sec. Ans.

RULE.—*Divide the difference in longitude by* 15.

EXERCISE LVIII.

Find the difference in time:

2. Between New York city and Denver.

3. Between Baton Rouge and Washington.

4. Between Boston and Berlin.

5. Between Rome and St. Petersburg.

6. When it is 8 o'clock P. M. at Nashville, what is the time at Sitka?

7. When it is 12 o'clock M. at Baltimore, what is the time at Paris?

8. When it is 20 min. past 10 o'clock A. M. at London, what is the time at Sydney?

9. When it is 35 min. past 1 o'clock P. M. at Constantinople, what is the time at Austin?

MISCELLANEOUS PROBLEMS.

EXERCISE LIX.

309. 1. How many pint bottles will 3 gal. 1 qt. of water fill?

2. What is the value of 3 bu. 1 pk. 5 qt. of berries at 15 cts. a quart?

3. If I use 3 pt. of milk per day, what will my milk cost per week, at 10 cts. a quart.

4. My horse eats 1 bu. 3 pk. $4\frac{4}{5}$ qt. of corn in a week; what does it cost to feed him 5 weeks, when corn is worth \$.60 per bushel?

5. A crock of butter weighed 43 lb. 7 oz., and the crock weighed 6 lb. 9 oz.; what did the butter weigh?

6. A farmer gathered 30 T. 6 cwt. 35 lb. of hay, and after reserving 8 T. 11 cwt. 85 lb. for his own use, sold the remainder at \$.40 per cwt.; what did he receive for it?

7. All of a 10-acre lot but 3 A. 40 sq. rd. is meadow; what is the meadow worth at \$12 per acre?

8. From a 20-gallon can 8 gal. 3 qt. 1 pt. of kerosene were drawn; what is the remainder worth at \$.16 per gal.?

9. How many barrels, each holding 3 bu. 1 pk., will be required to hold 22 bu. 3 pk. of apples?

10. A farmer put 125 bu. 3 pk. 6 qt. of wheat into sacks holding 1 bu. 3 pk. 6 qt. each; how many sacks did he fill?

11. How long is it from 15 min. past 9 o'clock A. M. to 40 min. past 5 o'clock P. M.?

12. How much shorter are the days when the sun rises at 6 o'clock 45 min. and sets at 5 o'clock 15 min., than when it rises at 5 o'clock 15 min. and sets at 6 o'clock 45 min.?

13. A stationer, by selling paper at a profit of 8 cts. a quire, gained \$4.80; how many reams did he sell?

14. What is 1 ℥ 2 ʒ 2 ℈ of calomel worth when made into pills of 5 gr. each, and priced at 2 cts. per pill?

15. How many ounces of quinine will make 480 pills, each weighing 5 grains?

16. How many rings, each weighing $4\frac{5}{8}$ pwt., can be made from a bar of gold weighing $1\frac{5}{6}$ ℔?

17. Reduce $\frac{3}{5}$ of a pound to oz., pwt., etc.

18. Reduce $\frac{5}{9}$ of a mile to rd., yd., etc.

19. From £6 3 s. 5 d. take £3.275.

20. From 9.69687 ℔. take 3 ℔. 5 oz. 7 pwt. 11 gr.

21. At the rate of 3 d. 3 far. per pencil, how many pencils can be bought for £$\frac{3}{4}$?

22. How many times will a buggy-wheel 14 ft. 8 in. in circumference revolve in going $\frac{13}{18}$ of a mile?

23. Into how many lots of 2 A. 1 sq. ch. each can a piece of land be divided which contains 7 lots of 1 A. 8 sq. ch. each?

24. The circumference of the fore wheel of a carriage is 12 ft. 6 in., and of the hind wheel 15 ft. 9 in.; how many times does the latter revolve while the former is rotating 63 times?

25. Express $\frac{2}{3}$ rd. in yards, feet, etc.

26. Find the value of .3975 of a mile?

27. $\frac{1}{3}$ of the width of a street exceeds $\frac{1}{5}$ of it by 5 yd. 1 ft.; what is the width of the street?

28. A merchant sold a saddle at a profit of £1 3 s. $4\frac{1}{2}$d., *which was* $\frac{3}{8}$ of the cost; what was the cost?

29. Express 3 qt. 1 pt. as the fraction of a gallon.

30. Express 7 hr. 30 min. 45 sec. as the fraction of a day.

31. What cost 2 bu. 3 pk. 4 qt. of corn, at 20 cts. a peck?

32. If $4.50 per month is charged for the use of a sum of money, to what will the use amount in 5 yr. 3 mo. 18 da.?

33. A boy bought $\frac{3}{4}$ of a bushel of chestnuts for $2.50, and sold them at $12\frac{1}{2}$ cts. a quart; how much did he gain?

34. A grocer bought 14.375 bu. of cranberries for $43.25, and sold $\frac{3}{5}$ of them at 12 cts. a quart and the remainder at 10 cts. a quart; how much did he gain?

35. If 1 pt. 3 gi. of water is put into a gallon measure, and the measure filled with milk, what part of the mixture will be milk?

36. A dairyman bought $5\frac{3}{4}$ gallons of milk, to which he added water at the rate of $1\frac{1}{2}$ gills to each quart, and sold the mixture at 5 cts. a pint; what did he receive for it?

37. To what will the services of a man amount in 2 yr. 9 mo. 15 da., at $450 per year?

38. If 5.83\frac{3}{4}$ will buy 11 gal. 3 qt. 1 pt. of syrup, how much will 17.51\frac{1}{4}$ buy?

39. What is cloth worth per meter, at 30 cts. per yard?

40. What will it cost to build a fence $2\frac{1}{2}$ miles long, at the rate of $75 per kilometer?

41. I bought land at $$\frac{5}{8}$ per ar; how much is that per acre?

42. What will be the cost of 165 Ha. of land, at 12\frac{1}{2}$ per ar? At $10 per acre?

43. What is sugar worth per pound, at 1.2 cts. per hectogram?

44. What will be the cost of 18 Hl. of milk, at 10 cts. a liter? At 8 cts. a quart?

45. The difference in time in the observations of an eclipse, on two vessels at sea, is 1 hr. 35 min. 12 sec.; what is their difference in *longitude*?

46. A gentleman left Savannah and traveled until his watch was 6 hr. 12 min. 24 sec. too fast; in what longitude was he?

47. If a man weigh 160 ℔. avoirdupois, what will he weigh by troy weight?

48. A silversmith has 5 ℔. 4 oz. 7 pwt. of silver, and 3 ℔. 5 oz. 5 pwt. of gold, and desires to manufacture each into rings of the largest equal weight; required the weight of each ring.

49. Required the least amount of calomel that may be put in bottles containing 6 ℥ 3 ʒ 1 ℈, or 5 ℥ 4 ʒ.

50. A dealer bought a quantity of pure whisky, to which he added water at the rate of 1 gill to each pint, and sold the mixture for $303.75, at 12½ cts. a gill; how much whisky did he buy?

51. A gentleman leaves Washington and travels west, at the rate of 36 min. of longitude per hour; how much should his watch gain or lose per minute to indicate continually the correct time?

QUESTIONS.

What is a compound number? Reduction descending? How performed? Reduction ascending? How performed?

How do you reduce metric to common measures? Common to metric measures?

What is a meridian? Longitude? Explain the relation of longitude and time. How find the difference of longitude when the difference of time is known? The difference of time when the difference of longitude is known?

ORDINARY MENSURATION.

310. Ordinary mensuration treats of the measurements of quantities having rectangular forms.

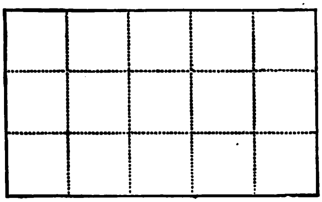
RECTANGULAR SURFACES.

311. A **plane figure** is a figure all parts of which are in the same plane.

312. The **area** of a plane figure is the quantity of surface it contains.

313. A **rectangle** is a plane figure having four sides and four right angles.

The area of a rectangle 5 in. long and 3 in. wide is 5 sq. in. taken 3 times, or 15 sq. in.



A RECTANGLE, 5 BY 3.

314. ***To find the area of rectangular surfaces.***

1. How many square yards in a garden 120 yd. long and 80 yd. wide?

Analysis.—A rectangle 120 yd. long and 1 yd. wide contains 120 sq. yd.; hence, a rectangle 120 yd. long and 80 yd. wide contains 80×120 sq. yd. $= 9600$ sq. yd.

Operation.

$120 \times 80 = 9600$

9600 sq. yd., Ans.

RULE.—*Multiply the length by the breadth.*

NOTES.—1. Both *dimensions* should be reduced to the *same denominations* before they are multiplied.

2. One *line* is said to be multiplied by another when the *number* of units in the former is taken as many times as there are *like units* in the latter.

3. The *area* and *one side* of a rectangular surface being given, the *other* side is found by *dividing the area by the given side.*

EXERCISE LX.

Find the area:

2. Of a board 2 ft. long and 5 in. wide.
3. Of a city lot 60 ft. front and $150\frac{1}{2}$ ft. deep.

4. Of a floor 20 ft. long and $15\frac{1}{2}$ feet wide.

5. Of a field 32 ch. by 117 rd.

How many acres:

6. In a garden 5 ch. long and 3 ch. wide?

7. In a meadow 363 yd. long and 175 yd. wide?

8. In a pasture 160 rd. long and 56 rd. wide?

9. What cost a farm 240 rd. long and 88 rd. wide, at $15 per acre?

10. A board is 9 in. wide, and its area is 1 sq. ft.; what is its length?

11. A building lot is 50 ft. front, and contains half an acre; how far back does it extend?

12. How many yards of carpeting 1 yd. wide will it take to cover a floor 21 ft. long and 15 ft. wide.

13. How many hektars in a field 475.5 meters long and 246 meters wide?

14. What is the cost of paving a street 628 ft. long and $60\frac{1}{2}$ ft. wide, at $2.25 per sq. yard?

15. How many planks 9 ft. by 7 in. will floor a room 21 ft. by 15 ft.?

16. What will be the cost of paving a street 650 yd. by 60 ft., with marble slabs 2 ft. 6 in. by 1 ft. 4 in., at $12 per hundred?

17. What will it cost to plaster the ceiling of a room that is 8.5 m. square, at 24 cts. per sq. meter?

18. One side of the roof of a house is 32 ft. by 15 ft.: how many boards, 5 in. wide, will it take to cover both sides, if 8 in. of the length of each board is exposed?

19. How many boards are required to cover a house, if one side of the roof is 14 m. by 6.3 m. and the part of the board exposed is $2\frac{1}{4}$ dm. by $1\frac{3}{4}$ dm.?

Shingling.

As shingles are usually made and laid 1000 are estimated to cover 100 sq. ft., called a square, or 10 shingles to the sq. foot.

20. One side of the roof of a house is 40 ft. by $18\frac{1}{2}$ ft.; how many shingles will it take to cover it?

21. How much will it cost to shingle a church of which the length is 80 ft., the rafters being 26 ft. long, the cost of shingles $\$5\frac{1}{2}$ per M., and the cost of the work $1.25 per square?

Carpeting.

The number of yards of carpeting required to carpet a room is found by multiplying the length of the strips by the number of strips.

If the strips run lengthwise, the length of the strips is equal to the length of the room, provided there is no loss in matching, in which case the loss must be added, to obtain the length of the strips; and the number of strips is found by dividing the width of the room by the width of the carpet, provided that the quotient is not a mixed number, in which case the fractional part is to be considered as 1.

If the strips run crosswise, proceed in a similar manner, substituting "width of room" for "length of room," and vice versa.

22. How many yards of carpeting 1 yard wide will be required for a floor 20 ft. long, 17 ft. wide, if the strips run lengthwise, and there be a waste of 4 in. in matching?

Solution: Length of strips = 20 ft. 4 in. = $20\frac{1}{3}$ ft.; and since $17 \div 3 = 5\frac{2}{3}$, 6 strips will be required. $6 \times 20\frac{1}{3}$ ft. = 122 ft. = $40\frac{2}{3}$ yd., Ans.

23. Find the answer to the preceding problem after changing "lengthwise" to "crosswise.".

24. A floor is 21 ft. by 16 ft.; how many yards of carpeting 27 in. wide will be required to cover it: (1) If the strips run lengthwise, and there is a waste of 6 in. in matching? (2) If the strips run crosswise, and there is a waste of 9 in. in matching?

25. If Brussels carpeting is $\frac{3}{4}$ of a yard wide, and cost $\$1\frac{5}{8}$ per yd., *what will it* cost to use it in carpeting a hall

45 ft. by 32 ft., the strips running lengthwise, and there being a waste of $\frac{1}{4}$ of a yard in matching the pattern?

26. I wish to carpet a room 24 ft. by 20 ft. with carpeting 2 ft. 8 in. wide, and worth $1 a yard; what will be the difference between the cost of running the strips lengthwise and crosswise, there being no waste in matching in the first case, and a waste of 4 in. in the second?

Plastering, Papering, Painting, Etc.

Plastering, painting, and kalsomining are measured by the square yard. There is no uniform rule respecting the allowance to be made for doors and windows.

The surface of the walls of a room may be found *by multiplying the sum of the lengths of the four sides by the height.*

27. How much will it cost to plaster the walls and ceiling of a room 18 ft. long, 16 ft. wide, and 11 ft. high, at $33\frac{1}{3}$ cts. per sq. yd., no allowance being made for doors and windows?

Solution.

Area of both sides,	$2 \times (18 \times 11)$,	396 sq. ft.
" " " ends,	$2 \times (16 \times 11)$,	352 "
" " ceiling,	18×16,	288 "
" " all,		1036 sq. ft.

1036 sq. ft. $= 115\frac{1}{9}$ sq. yd., @ $\$.33\frac{1}{3} = \$38.37$, Ans.

28. What will be the cost of plastering a room 21 ft. 6 in. by 16 ft., and 9 ft. high, at $.35 a square yard, allowing 225 sq. ft. for doors, windows, etc.?

29. What will it cost to paint the walls of a parlor $18\frac{1}{2}$ ft. long, $16\frac{1}{2}$ ft. wide, and 12 ft. high, at 27 ct. per sq. yd., allowing for 2 doors each $6\frac{1}{4}$ ft. by $3\frac{1}{5}$ ft.?

30. What will be the cost of papering the walls and ceiling of a room 23 ft. 3 in. long, 17 ft. 4 in. wide, and 12 feet high, at 12 cts. per sq. yd., allowing for 3 doors each 6 ft. 8 in. by 3 ft. 9 in., and 6 windows each 5 ft. 3 in. by 2 ft. 8 in.?

31. At 6 cts. a sq. meter, what will it cost to kalsomine the walls and ceiling of a diningroom 12 m. long, 6 m. wide

and 45 dm. high, allowing for 3 doors each $2\frac{1}{2}$, m. by 12 dm., and a wainscot, 11 dm. high, extending around the room?

Dividing and Designating Land.

315. The governmental method of surveying and designating land is as follows:

The first thing done is to run a line north and south, called the **Principal Meridian (P. M.)**, marking its course by stones, posts, etc.

Next, lines are run 6 miles apart, and parallel to the **P. M.**; thus dividing the country into *strips* called **Ranges (R.)**, which are numbered in order, both east and west, from the **P. M.**

Thus, **R.** 3 **E.** is the third strip or range east of the **P. M.**, and **R.** 4 W. is the 4th range west of the **P. M.**

Next, a line is run east and west, called the **Base Line (B. L.)**.

Next, lines are run 6 miles apart, and parallel to the **B. L.**, thus dividing the ranges into squares of 36 sq. mi. each, called **Townships**, which are numbered in order, both north and south, from the **B. L.**

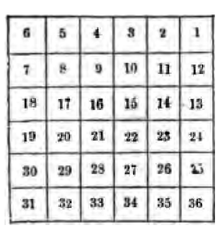
Thus, **Tp.** 18 **N. R.** 5 **W.** is the 18th township north of the **B. L.** and in the 5th range west of the **P. M.**

Locate the following: Tp. 15 S. R. 4 W.; Tp. 3 S. R. 7 E.; Tp. 11 N. R. 9 E.

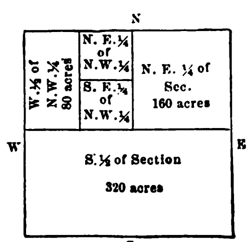
A **township** is divided into 36 square miles, called **sections (Sec.)**.

The U. S. Land Office recognizes the following divisions of a section:

Half Section	= 1 mi. by $\frac{1}{2}$ mi.	= $\frac{1}{2}$ sq. mi.	= 320 acres.
Quarter Section	= $\frac{1}{2}$ mi. by $\frac{1}{2}$ mi.	= $\frac{1}{4}$ sq. mi.	= 160 acres.
Half-Quarter Section	= $\frac{1}{2}$ mi. by $\frac{1}{4}$ mi.	= $\frac{1}{8}$ sq. mi.	= 80 acres.
Quarter-Quarter Section	= $\frac{1}{4}$ mi. by $\frac{1}{4}$ mi.	= $\frac{1}{16}$ sq. mi.	= 40 acres.



SECTIONS OF A TOWNSHIP.



A SECTION ENLARGED.

NOTE.—In solving the following, the student should draw a diagram or plot of the land designated.

How many acres:

32. In the W. ½ of N. E. ¼ and N. ½ of S. E. ¼ of Sec. 14?

33. In the S. ½ of N. W. ¼ and E. ½ of S. W. ¼ of Sec. 20?

34. In the E. ½ of N. E. ¼ and N. E. ¼ of S. E. ¼ of Sec. 25?

35. In the N ½, and N. E. ¼ of S. W. ¼ of Sec. 3?

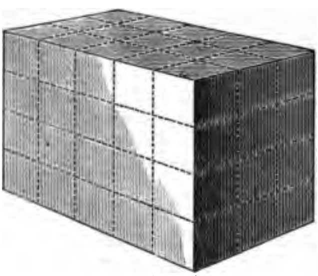
36. In the W. ½, N. ½ of S. E. ¼ and S. W. ¼ of N. E. ¼ of Sec. 2?

37. In the E. ½ of N. W. ¼, N. ½ of S. W. ¼, W. ½ of S. E. ¼ and S. ½ of N. E. ¼ of Sec. 11, Tp. 3, N. R. 5 W.?

38. In the S. E. ¼ of N. W. ¼, S. W. ¼ of N. E. ¼, N. ½ of S. E. ¼ and N. E. ¼ of S. W. ¼ of Sec. 31, Tp. 7, S. R. 4 E.?

39. A owns E. ½, E. ½ of N. W. ¼, and S. E. ¼ of S. W. ¼ of Sec. 22; B owns E. ½ of W. ½, S. ½ of S. E. ¼, and N. ½ and S. W. ¼ of N. E. ¼ of Sec, 21; and C owns all the land between A and B. (1) How many acres has each? (2) What is the description of C's land?

RECTANGULAR SOLIDS.



A SOLID, 5 BY 4 BY 3.

316. A **rectangular solid** is one bounded by six rectangular sides or faces.

317. The **contents** or **volume** of a solid is the quantity of matter or space it contains.

Thus, the volume of a rectangular solid, 5 in. long, 3 in. wide and 4 in. high, is $5 \times 3 \times 4$ cu. in., or 60 cu. in.

318. ***To find the volume of rectangular solids.***

1. Find the volume of a box 5 ft. long, 4 ft. wide and 3½ ft. deep.

Analysis.—A box 5 ft. by 4 ft. and 1 ft. deep, contains 20 cu. ft.; hence a box 5 ft. by 4 ft. and $3\frac{1}{2}$ feet deep contains $3\frac{1}{2} \times 20$ cu. ft. $= 70$ cu. ft.

Operation.
$5 \times 4 \times 3\frac{1}{2} = 70.$
70 cu. ft., Ans.

RULE.—*Multiply the length, breadth, and thickness together.*

NOTES.—1. When the contents and two dimensions are given, the other dimension may be found by dividing the contents by the product of the two given dimensions.

2. *Excavations* and *embankments* are estimated by the cubic yard. In removing earth, a cubic yard is called a *load*.

EXERCISE LXI.

Find the volume of a rectangular solid:

2. 2 ft. by 5 in. by 3 in.
3. $3\frac{1}{3}$ yd. by $8\frac{1}{2}$ ft. by $2\frac{4}{5}$ ft.
4. 5 ft. by 2 ft. by $3\frac{1}{2}$ ft.
5. $8\frac{2}{3}$ yd. by $4\frac{1}{2}$ ft. by 3.25 ft.

Find the volume:

6. Of a cube each of whose edges is 1 ft. 8 in.

7. Of a cube each of whose edges is 1 yd. 2 ft. 6 in.

8. How many cu. feet of air in a room 20 ft. by 15 ft. by 12 ft. 9 in.?

9. Find the volume of a solid 12.5 ft. long, the end of which is 3 ft. 9 in. square.

10. The volume of a box is 50 cu. ft., its length 5 ft. and its width 4 ft.; find its height.

11. A bin is 4 ft. 6 in. wide, 3 ft. deep, and contains $111\frac{3}{8}$ cu. ft.; find its length.

12. Find the cost of digging a trench 200 yd. long, $2\frac{2}{5}$ ft. wide and $4\frac{1}{2}$ ft. deep, at $\$\frac{3}{4}$ per cu. yd.

13. How many loads of earth must be removed in digging a cellar 45 ft. by 21 ft. by 8 ft. 3 in.?

14. What will be the cost of elevating the surface of a street 60 ft. wide, 627 ft. long, and $4\frac{1}{2}$ ft. below grade, at 51 cts. *per cu. yard?*

15. I wish to construct a box 7 in. long and 6 in. wide that shall hold exactly one gallon; what must be its depth?

16. How many cu. meters in a rectangular solid 2 Dm. by 6 m. by 15 dm.?

17. How many cords of wood in a pile 36 ft. by 6 ft. by 10 ft.?

18. What is the worth of a pile of wood 8 ft. long, 3 ft. high and $3\frac{1}{2}$ ft. wide, at $4.50 per cord?

19. What must be the height of a pile of wood 20 ft. long and 6 ft. wide, to contain 10 cords?

20. How many sters in a pile of wood $7\frac{1}{2}$ m. by $3\frac{1}{3}$ m. by $1\frac{1}{5}$ m.?

21. How many liters of milk will a tin can hold which is $\frac{1}{2}$ m. by 3 dm. by 14 cm.?

Bricks.

Bricks are usually 8 in. long, 4 in. wide and 2 in. thick, and are generally sold by the thousand.

A brick contains $8 \times 4 \times 2$ cu. in. $= 64$ cu. in. Hence, a cu. ft. contains 27 bricks, for 1728 cu. in. $\div$ 64 cu. in. $= 27$. In walls, as the mortar occupies a part of the space, 20 bricks are usually estimated to the cu. ft.

In measuring the walls of cellars and buildings, masons usually take the outside dimensions, and make no allowances for corners, doors and windows.

22. How many bricks in a pile 12 ft. long, 4 ft. wide and 5 feet high?

23. How many bricks in a wall 20 ft. long, $1\frac{1}{2}$ ft. thick and $8\frac{1}{2}$ ft. high?

24. What is the value of a pile of brick sufficient to construct the walls of a house 30 ft. long, $16\frac{1}{2}$ ft. wide and $11\frac{1}{6}$ ft. high, the walls being 1 ft. thick, at $$8\frac{1}{2}$ per M.?

Capacity of Tanks, Etc., in Gallons.

Since 1728, the number of cu. in. in a ft., contains 231, the number of cu. in. in a gal., about $7\frac{1}{2}$ times, $7\frac{1}{2}$ gallons are usually estimated to the cu. foot.

(25) How many gallons of water will a box hold which is 5 ft. long, 3 ft. wide and 2 ft. deep?

Solutions.

Exact method, $5 \times 3 \times 2 \times \frac{1728}{231} = 224.4 +$
Practical method, $5 \times 3 \times 2 \times 7\frac{1}{2} = 225.$

RULE.—*Divide the number of cubic inches by* 231, *or multiply the number of cubic feet by* $7\frac{1}{2}$.

Solve the following by the practical method.

How many gallons will a box hold:

26. Which is 3 ft. long, 2 ft. wide and 8 in. deep?

27. Which is 5 ft. long, 3 ft. wide and 1 ft. 9 in. deep?

28. A cistern is 6 ft. by 6 ft. by 10 ft., and is $\frac{2}{3}$ full of water; how long will it supply a family that uses 25 gallons per day?

29. A tin box, 2 ft. 8 in. by 10 in. by 1 ft. 6 in., is $\frac{4}{5}$ full of honey; what is the honey worth at $\$\frac{5}{8}$ per gallon?

30. I wish to construct a box 11 in. long and 7 in. wide that shall contain *exactly* 2 gal.; how deep must it be?

Capacity of Bins, Etc., in Bushels.

Since 1728, the number of cu. in. in a cu. ft., contains 2150.4, the number of cu. inches in a bu., about .8 times, .8 of a bushel is usually estimated to the cu. ft.

(31) How many bushels of wheat will a box hold which is 6 ft. long, 5 ft. wide and 2 ft. deep?

Solutions.

Exact method, $6 \times 5 \times 2 \times \frac{1728}{2150.4} = 48.21 +.$
Practical method, $6 \times 5 \times 2 \times .8 = 48.$

RULE.—*Divide the number of cubic inches by* 2150.4, *or multiply the number of cubic feet by* .8.

Solve the following by the practical method.

How many bushels will a box hold which is:

32. 3 ft. by 2 ft. by 9 in.? **33.** 6 ft. by 4 ft. by 3 ft. 9 in.?

34. 5 ft. by 4 ft. by 15 in.? **35.** 7 ft. by 3 ft. by 2 ft. 8 in.?

36. A granary 9 ft. by $6\frac{1}{3}$ ft. by $4\frac{5}{8}$ ft. is $\frac{3}{4}$ full of wheat; what is the value of the wheat at 65 ct. per bushel?

37. I wish to make a bushel measure 16 in. long, and 12 in. wide; how deep *exactly* must it be?

Board Measure.

319. **A board foot** is 1 foot long, 1 foot wide, and 1 inch thick, and is used for measuring boards, planks, and sawed timber generally.

(38) How many board feet in a plank 18 feet long, 10 in. wide and $1\frac{1}{2}$ in. thick?

Explanation.—I multiply the length in feet by the width and thickness expressed in inches, and divide the product by 12; the quotient is the number of board feet.

Operation.

$$\frac{18 \times 10 \times 1\frac{1}{2}}{12} = 22\frac{1}{2}.$$

NOTES.—1. If a board is less than 1 inch thick, it is disregarded; that is, the calculation is made as if the thickness were 1 inch; and no deductions are made in the price except when the thickness is $\frac{3}{8}$ of an inch.

2. If a board is tapering, the width of the broader end only is considered.

3. When no thickness is mentioned, 1 inch is understood.

How many board feet are there:

39. In a plank 15 ft. long and 10 in. wide?

40. In a plank 14 ft. long, 9 in. wide, and $1\frac{1}{4}$ in. thick?

41. In a plank 10 ft. long, 9 in. wide, and $\frac{3}{4}$ in. thick?

42. In 25 boards 16 ft. long and 6 in. wide?

43. In 15 beams 12 ft. long, 8 in. wide and 5 in. thick?

44. What cost 36 joists whose dimensions are 4 in. by 3 in. and 10 ft. long, at $2\frac{1}{2}$ ct. per board foot?

45. Find the cost of 200 tapering boards 15 ft. long, 11 in. wide at one end and 9 in. at the other, at $17 per M. *board ft.*

46. A room is 44 ft. long and 30 ft. wide; what will it cost to floor it with flooring $1\frac{1}{4}$ in. thick, at $18 per M. board feet?

47. I wish to build a fence 120 yd. long and $5\frac{1}{2}$ ft. high, with plank 6 in. wide and 4 in. apart, and posts 5 in. square and 10 ft. apart; what will the lumber cost at $15 per M. board ft., supposing the post to be sunk $1\frac{1}{2}$ ft. in the ground?

MISCELLANEOUS PROBLEMS.

EXERCISE LXII.

320. 1. A garden containing $3\frac{1}{2}$ acres is 7 chains long; how wide is it?

2. How many bricks, 8 in. by 4 in., will pave a walk 60 feet long and $10\frac{2}{3}$ ft. wide, with no allowance for edges or waste?

3. A bin is 10 ft. long, $4\frac{1}{2}$ ft. wide and 6 ft. deep; how many bushels of grain will it hold?

4. What will it cost to excavate a cellar 7.6 m. long, 5.4 m. wide and 3.5 m. deep, at $\$\frac{1}{4}$ per cubic meter?

5. How many cubical blocks whose edges are 3 inches each, can be packed in a box whose inside edges are 15, 12, and 9 inches?

6. How many yards of carpeting $\frac{3}{4}$ of a yard wide will carpet a room 22 ft. long and 20 ft. wide, if the strips run lengthwise, with an allowance of 6 inches for matching figures?

7. Find the cost of the E. $\frac{1}{2}$ of N. E. $\frac{1}{4}$ of Sec. 11, at $\$4\frac{1}{2}$ per acre?

8. Find the cost of the S. $\frac{1}{2}$ of N. W. $\frac{1}{4}$, and N. E. $\frac{1}{4}$ of S. W. $\frac{1}{4}$ of Sec. 22, at $15 per acre.

9. A box 5 ft. long, 4 ft. wide and 3 ft. deep is $\frac{2}{3}$ full of water; how many gallons of water are in the box?

10. What will it cost to plaster the walls and ceiling of a room 8.2 m. long, 5.6 m. wide, and 3.8 m. high, at 45 cts. per sq. *meter?*

What is the cost of a rectangular field:

11. 60 rd. long and 40 rd. wide, at \$6 per acre?

12. 605 yd. long and 280 yd. wide, at $\$5\frac{2}{7}$ per acre?

Find the cost of a pile of wood:

13. 8 ft. long, 8 ft. wide, and 4 ft. high, at \$5 per cord.

14. 120 ft. long, $6\frac{1}{2}$ ft. wide, and $8\frac{3}{4}$ ft. high, at $\$3\frac{1}{5}$ per cord.

15. How many square meters of matting are required to cover a stage 5 m. 4 dm. long and 1 m. 2 dm. wide?

16. What is the value of a pile of wood 16 m. 1 dm. 5 cm. long, 1 m. 2 dm. 2 cm. wide and 1 m. 6 dm. 8 cm. high, at \$2.30 a ster?

17. How many yards of silesia, $\frac{5}{8}$ yd. wide, will line 21 yd. of silk, $\frac{5}{8}$ yd. wide?

18. The length of a room is $21\frac{1}{2}$ ft., the height 13 ft., and the four walls contain 988 sq. ft.; find the width of the room?

19. Allowing 8 shingles to the square foot, how many shingles will be required to cover a barn 50 ft. long, and 15 ft. from the comb to the eaves?

20. A cubic yard of lead weighs 19128 lb.; what is the weight of a rectangular block of lead 2 m. 8 cm. long, 1 m. 2 dm. 5 cm. wide, and 7 dm. 5 cm. thick?

21. How many board feet in a plank 14 ft. long, 12 in. wide, and $2\frac{1}{2}$ in. thick?

22. Find the cost of 36 scantling 9 ft. long and 4 in. by 3 in., at $\$2\frac{1}{4}$ per C.?

23. A bin 10 ft. by 8 ft. by 5 ft. is full of wheat; how many bushels of flour will it yield, allowing $\frac{1}{5}$ for waste?

24. A cubic foot of water weighs $62\frac{1}{2}$ pounds, which is about $1\frac{2}{23}$ times the weight of that much ice; what is the weight of a block of ice 8 ft. by $5\frac{1}{3}$ ft. by $2\frac{1}{4}$ ft.?

25. A piece of land, 12 chains long, is worth \$33 at \$11 per acre; what would it be worth if it were $7\frac{1}{2}$ chains *wider?*

26. A box, 4 ft. 8 in. long and 4 ft. wide, holds 128 gallons; how many bushels would it hold if it were $5\frac{1}{5}$ inches leeper?

27. How may 140 bricks be put in a box whose inside limensions are 32, 14, and 20 inches?

28. A farmer has a section of land, and after reserving he S. E. quarter for himself, desires to divide the remaining ; quarters equally among his 4 sons, so that each of the 4 arts shall have the same shape. Required the shape, mount and description of each son's part?

29. A garden is 12 ch. long and 5 ch. wide; how many cres larger would it be if it were 3 ch. longer and 1 ch. vider?

30. A rectangular field is 72 rd. long and contains 18 cres; if its width were 24 rd. greater, how much greater nust its length be that its area may be doubled?

31. A pile of bricks is 3 ft. long, 2 ft. wide and 1 ft. high; ow many more bricks are required to make the pile 1 ft. onger, 1 ft. wider and 1 ft. higher?

32. Around a garden 18 rd. long by 10 rd. wide is a ditch ft. wide and 3 ft. deep. This ditch being insufficient it was ut 1 ft. 6 in. wider from the outer edge, and the whole ditch ft. deeper, how many cu. ft. of dirt were thrown out in naking this change?

QUESTIONS.

What is mensuration? A plane figure? Area? Rectangle? How nd the area of rectangular surfaces?

What is said about shingling? Carpeting? Plastering, etc.? Diiding and designating land? Draw and number the sections of a ownship.

What is a rectangular solid? Contents or volume? How find the olume of a rectangular solid?

What is said about bricks? Capacity of tanks? Of bins? Board neasure?

CHAPTER IV.

PERCENTAGE.

321. A **per cent** is a number of hundredths.

(1) 5 hundredths, $\frac{7}{100}$ and $.06\frac{2}{3}$ are per cents.

Per cent is a contraction of *per centum*, which means by the hundred. Thus, when we say that 7 per cent of a number of apples are rotten, we mean 7 of every 100 are decayed.

322. The **sign** of per cent is %.

(1) 7 per cent, $\frac{7}{100}$, .07 and 7% all have the same meaning.

Oral Exercises.

Reduce 60 per cent to an equivalent common fraction.

Solution: 60 per cent $= \frac{60}{100} = \frac{6}{10} = \frac{3}{5}$, Ans.

Reduce the following to equivalent fractions:

1. 4 per cent.	**2.** 5%.	**3.** $6\frac{1}{4}$%.
4. $8\frac{1}{3}$ per cent.	**5.** 10%.	**6.** $12\frac{1}{2}$%.
7. $16\frac{2}{3}$ per cent.	**8.** 20%.	**9.** 25%.
10. $33\frac{1}{3}$ per cent.	**11.** 40%.	**12.** 50%.
13. $66\frac{2}{3}$ per cent.	**14.** 75%.	**15.** 100%.
16. $133\frac{1}{3}$ per cent.	**17.** 125%.	**18.** 120%.

(19) Reduce $\frac{3}{8}$ to a per cent.

Solution: The process is the same as that of reducing $\frac{3}{8}$ to a fraction whose denominator is 100, Art. 168. It may be reduced thus: since 100% [illegible] of 100% = $37\frac{1}{2}$%.

Reduce the following to equivalent per cents:

20. $\frac{1}{2}$.	**21.** $\frac{1}{3}$.	**22.** $\frac{1}{4}$.	**23.** $\frac{1}{5}$.	**24.** $\frac{1}{6}$.
25. $\frac{2}{3}$.	**26.** $\frac{3}{4}$.	**27.** $\frac{2}{5}$.	**28.** $\frac{5}{6}$.	**29.** $\frac{6}{5}$.
30. $\frac{5}{4}$.	**31.** $\frac{3}{5}$.	**32.** $\frac{4}{5}$.	**33.** $\frac{7}{12}$.	**34.** $\frac{11}{11}$.
35. $\frac{4}{3}$.	**36.** $\frac{5}{8}$.	**37.** $\frac{9}{8}$.	**38.** $\frac{3}{4}$.	**39.** $\frac{7}{8}$.

40. What part of a number is 5% of it? 25% of it? 50%? 75%? $12\frac{1}{2}$%? $33\frac{1}{3}$%? 100%?

41. What per cent of a number is $\frac{1}{4}$ of it? $\frac{1}{6}$ of it? $\frac{1}{8}$? $\frac{1}{10}$? $\frac{1}{20}$? $\frac{1}{30}$? All of it?

42. What is the value of 5 per cent + 8 per cent? 9% + 8%? 16% — 7%? 12% + 15% — 20%? 1 — 60%?

43. A boy spent 75% of his money for apples; how much of it had he left?

44. A lad gave 60% of his money for a saddle and 25% of it for a bridle; how much had he left? How much more did the saddle cost than the bridle?

45. How much is $\frac{1}{2}$ of 38%? $\frac{4}{5}$ of 40%? $\frac{3}{8}$ of 56%? 25% of 80%? $33\frac{1}{3}$% of 24%? $12\frac{1}{2}$% of 32%?

323. Percentage is the process of computing by per cents.

In this process there are primarily three elements, viz.: the base, the rate, and the percentage.

324. The **base** is the number of which the per cent is taken; the **rate** is the given per cent; and the **percentage** is the result of taking that part of the base expressed by the rate.

Illustration.—20% of \$35 = \$35 × .20 = \$7.

Here, \$35 is the base, 20% is the rate, and \$7 is the percentage.

PRINCIPLE.—*The base, rate, and percentage sustain to each other the relation of multiplicand, multiplier, and product.*

Hence, when either two of the elements [illegible] the third may be found.

325. ***Base and rate given, to find the percentage.***

(1) What is 40% of \$180?

Solution: 40% of a number is $\frac{40}{100}$ or $\frac{2}{5}$ of the number; $\frac{1}{5}$ of \$180 is \$36, and $\frac{2}{5}$ *is 2 times \$36,* = *\$72,* **Ans.**

PRINCIPLE.—*The percentage of any nnmber is the same part of the base that the given rate is of* 100%.

2. What is 20% of \$35? 25% of \$32? $12\frac{1}{2}$% of \$24? $33\frac{1}{3}$% of 48 days? 75% of 60 sheep? 60% of 350 men?

When the given rate is not a simple part of 100%, it is better to proceed as in the solution of the next problem.

(3) In a box are 160 pears, of which $7\frac{1}{2}$ per cent are rotten; how many of the pears are rotten?

Explanation.—Since $7\frac{1}{2}$% of a number is $.07\frac{1}{2}$ of that number, I multiply the base, 160 pears, by the rate, expressed decimally, $.07\frac{1}{2}$, and obtain the percentage, 12 pears.

Operation.

$$\begin{array}{r} 160 \\ \underline{.07\frac{1}{2}} \\ 80 \\ \underline{11\ 20} \\ 12.00 \end{array}$$

12 pears, Ans.

Formula.—**Percentage** = **Base** × **Rate**......(a)

RULE.—*Multiply the base by the rate.*

EXERCISE LXIII.

4. Find 6% of 50.
5. Find 7% of \$3875.
6. Find 4% of 325.
7. Find 16% of 2275 sheep.
8. Find 20% of 635.
9. Find $15\frac{5}{8}$% of 576 days.
10. Find 25% of 60%.
11. Find $16\frac{2}{3}$% of 81%.
12. Find $12\frac{1}{2}$% of $\frac{16}{19}$.
13. Find $17\frac{1}{2}$% of $3\frac{1}{5}$.
14. Find $8\frac{1}{3}$% of \$384.
15. Find $39\frac{1}{2}$% of \$1867.25.

16. A farmer had 575 sheep, of which he soid 4 per cent; how many did he sell?

17. I rented a house worth \$1825 and paid 7% of its value for the use of it; what was the rent?

18. A merchant borrowed \$900 and paid $16\frac{2}{3}$% for the use of it; what did the use of it cost him?

19. A cotton buyer invested \$4275 in cotton, and sold it at a profit of 12%; how much did he gain?

20. Find 10% of 40% of $625.

21. Find 60% of 40% of 75% of $43.40.

22. A man owning 75% of a ship, sold $33\frac{1}{3}$% of his share; what part of the ship did he sell?

23. A father owned 65% of an estate, of which he gave his son 16%; what part of the estate did the son receive?

24. At a school of 150 pupils the average daily attendance is 90% of the whole number; what is the daily attendance?

25. A merchant collected $15675 and deposited 35% of it in bank; how much did he deposit?

26. What is the value of 70% of 60 oranges, at 5 cts. apiece?

27. If a man ride $16\frac{2}{3}$% of 1896 rods in 1 hour, how far will he ride in 175 hours?

28. From a cask of 40 gal. of oil, 15% leaked out; how much oil leaked out?

29. A man owes $65375, and is able to pay only 68% of his debts; how much money has he?

30. A farmer cultivated 240 acres of land, of which he planted $33\frac{1}{3}$% in corn, 25% in cotton, 20% in oats, and the balance in wheat; how many acres did he plant in each?

31. A has an income of $1280 a year; he pays $24\frac{1}{2}$% of it for board, $11\frac{7}{8}$% for clothing, $\frac{5}{8}$% for servant's hire, and $15\frac{3}{4}$% for other expenses; how much does he pay for each item, and how much does he save?

32. A good cotton seed meal shows on analysis 3.3% phosphoric acid, 7.3% nitrogen and 1.7% potash; how much of each of these ingredients is contained in 1560 pounds of the meal?

326. ***Percentage and rate given, to find the base.***

(1) 20% of a number is 15; what is the number?

1°. **Solution.**—20% of a number is $\frac{1}{5}$ of it. If $\frac{1}{5}$ of a number is 15, $\frac{5}{5}$, or the whole number, is 5 times 15, equal to 75.

2°. **Solution.**—If 20% of a number is 15, then
1% of the number is $\frac{1}{20}$ of 15, and
100% of the number is $\frac{100}{20}$ of 15, or $5 \times 15 = 75$.

PRINCIPLE.—*The base is the same multiple of the percentage that* 100% *is of the rate.*

2. What is the number of which 15 is 25%? Of which $40 is 5%? Of which 12 bushels is 60%? Of which 21 days is 30%? Of which 45 men is $62\frac{1}{2}$%?

When 100% is not a simple multiple of the rate, it is better to proceed as in the solution of the next problem.

(3) What is the weight of a bale of cotton, if 16% of it weighs 74 pounds?

Explanation.—Since the percentage is the product of the base by the rate, I divide the percentage, 74 pounds, by the rate, .16, and obtain the base, $462\frac{1}{2}$ pounds.

Operation.
$74. \div .16 = 462\frac{1}{2}$.
$462\frac{1}{2}$ pounds, Ans.

Formula.—**Base = Percentage ÷ Rate......(b)**

RULE.—*Divide the percentage by the rate.*

EXERCISE LXIV.

4. $60 is 25% of what amount?

5. $70 is 20% of what amount?

6. 35 horses is $33\frac{1}{3}$% of what number of horses?

7. 56 sheep is 35% of what number of sheep?

8. A lad lost $5, which was $12\frac{1}{2}$% of all he had; how much had he?

9. A farmer sold 81 cattle, which was $56\frac{1}{4}$% of all he had; how many had he?

10. James is 7 years old, and his age is $16\frac{2}{3}$% of his father's; *required* the age of his father.

11. 9% of a stone weighs 231.0606 pounds; what is the weight of the entire stone?

12. Your watch is worth $45, which is only 90% of the value of my watch; what is my watch worth?

13. To-day coffee is 18.98 cts. a pound, which is 104% of what it was yesterday; what was it worth per pound yesterday?

14. A boy by selling a ball for 5 cts. more than it cost him gained 25%; what did it cost him?

15. A merchant, by selling a lot of goods for $1402.80 more than cost, gained 8%; what did the lot cost?

16. Henry gave 30% of his money for apples, 36% of it for oranges, and his oranges cost 30 cts. more than his apples; how much money had he?

17. A farmer has $27\frac{1}{4}$% of his farm planted in wheat, and $19\frac{2}{3}$% of it in oats; what is the size of his farm, there being $18\frac{1}{5}$ acres more in wheat than in oats?

18. 564 is 25% of what number?

19. 4578 men is 84% of what number?

20. $5\frac{1}{8}$ is $33\frac{1}{3}$% of what number?

21. $16\frac{1}{4}$ bushels is $2\frac{1}{2}$% of what number?

22. Henry gave 28% of his money for a pony, 4% of it for a saddle, and $1\frac{1}{3}$% of it for a bridle, all of which cost him $80; how much money had he?

23. A man drew $9\frac{1}{2}$% of his bank deposits to buy a carriage, $8\frac{1}{3}$% to buy a span of horses, and $1\frac{3}{4}$% to pay a driver, after which he had $1034 less in bank than before; how much had he in bank at first?

24. A boy, after selling 60% of his berries, received 80 cts. for the remainder, at 10 cts. a quart; how many had he at first?

25. From a hogshead of molasses 16% leaked out; how much did it contain at first, if $45.15 was received for the remainder, at $.86 per gallon?

26. William gave 60% of his money for a pony, and 20% of the remainder for a saddle; how much money had he if the saddle cost $4?

27. After selling 8% of a hogshead of sugar, and 50% of the remainder, there were 414 pounds left; how much did the hogshead contain at first?

28. After paying 18% of a debt, and 20% of the remainder, there was still an unpaid balance of $492; what was the amount of the debt?

29. In an orchard 10% of the trees died the first year, $11\frac{1}{9}$% of the remainder died the second year, and 25% of the number left died the third year, after which there were 198 trees in the orchard; how many trees were there at first?

30. A man traveling from Benton to Sparta went 10% of the distance the first week, $16\frac{2}{3}$% of the remaining distance the second week, and 20% of what still remained the third week, and was then 432 miles from Sparta; required the distance from Benton to Sparta.

327. ***Base and percentage given, to find the rate.***

(1) What per cent of 24 is 6?

Solution.—6 is $\frac{1}{4}$ of 24; $\frac{1}{4}$ of a number is equal to $\frac{25}{100}$, or 25% of the number. Hence, 6 is 25% of 24.

PRINCIPLE.—*The rate is the part that the percentage is of the base, expressed in hundredths.*

2. What per cent of 16 is 4? Of 15 is 3? Of 30 is 15? Of $12 is $4? Of 50 hours is 2 hours? Of 30 is 21? Of 50 is 15?

When the percentage is not a simple part of the base, it is better *to proceed* as in the solution of the next problem.

(3) What per cent of 576 is 90?

Explanation.—Since the percentage is the product of the base by the rate, I divide the percentage, 90, by the base, 576, and obtain the per cent, or rate, $15\frac{5}{8}\%$.

Operation.
$90.00 \div 576 = .15\frac{5}{8}$
$15\frac{5}{8}\%$, Ans.

Formula.—**Rate = Percentage ÷ Base......(c).**

RULE.—*Divide the percentage by the base; the quotient, expressed in hundredths, is the rate.*

EXERCISE LXV.

What per cent:

4. Of 9 is 3?	5. Of 12 is 2?	6. Is 7 of 7?
7. Of 10 is 2?	8. Of 20 is 7?	9. Is $\$2\frac{1}{2}$ of $10?
10. Of 5 is 4?	11. Of 25 is 13?	12. Is $\$2\frac{1}{3}$ of $\$3\frac{1}{2}$?

13. What per cent of $125 is $25?

14. What per cent of $3875 is $271.25?

15. A boy had 15 marbles and lost 3; what per cent did he lose? What per cent had he left?

16. A merchant, who had $4275, invested $3762 in goods, and deposited the remainder in bank; what per cent of his money did he invest? What per cent did he deposit?

17. A miller grinds 15 bushels of corn for $1\frac{1}{2}$ bushels; what per cent toll does he charge?

18. A real estate owner received $510.75 for the use of a house valued at $5675; what per cent rent did he receive?

19. A father divided 40 apples among his three sons; to the first he gave 16, and to the second 13; what per cent did he give each?

20. Five men, A, B, C, D, E, bought a school-house for $1875, of which A paid $425; B, $375; C, $500; D, $15, and E the remainder; what per cent did each pay?

21. I bought a pig that weighed 25 pounds gross, and the waste in dressing was $5\frac{1}{4}$ lb.; what per cent was the waste?

22. A merchant owes \$6296 and his property is worth only \$5509; what per cent of his debts can he pay?

23. James has 20 marbles; by what per cent would the number be increased should he buy 6 more?

24. If \$278.30 were subtracted from \$3478.75, by what per cent would it be diminished?

25. A nurseryman planted 500 trees, of which only 122 lived; what per cent of them died?

26. When a ship has sailed 1745.01 miles of a voyage of 1896.75 miles, what per cent of the distance remains to be sailed?

27. In an examination 120 questions were asked, of which A answered 96, B 66 and C 54; what per cent did each make?

28. A paid \$280 for a horse, buggy and harness; the buggy cost \$91, which was 52% of the cost of the horse; what per cent of the entire cost did he pay for the harness?

Amount and Difference.

328. The **amount** is the base plus the percentage, and the **difference** is the base less the percentage.

329. ***Base and rate given, to find the amount or difference.***

(1) What is the result of increasing 75 by 4% of itself?

Explanation.—75 is 100% of 75; hence, 75 increased by 4% is 104% of 75. Therefore, I multiply the base, 75, by 1 plus the rate, 1.04, and obtain the amount, 78.

Operation.

$1 + .04 = 1.04$

$75 \times 1.04 = 78.$

Formula.—**Amount** = **Base** $\times$ (1 + **Rate**)......(d).

RULE.—To find the amount, *multiply the base by 1 plus the rate.*

(2) How much is 80 less 5% of 80.

Explanation.—80 is 100% of 80; hence, 80 less 5% is 95% of 80. Therefore, I multiply the base, 80, by 1 minus the rate, .95, and obtain the difference, 76.

Operation.

$1 - .05 = .95$

$80 \times .95 = 76.$

Formula.—Difference = Base $\times$ (1 — Rate) .... (e).

RULE.—To find the difference, *multiply the base by* 1 *minus the rate.*

EXERCISE LXVI.

Find the result:

3. Of increasing \$35 by 20% or $\frac{1}{5}$ of itself.

4. Of increasing \$1450 by 18% of itself.

5. Of diminishing \$48 by $33\frac{1}{3}$% or $\frac{1}{3}$ of itself.

6. Of diminishing 125 days by 28% of itself.

7. A clerk receiving \$60 a month had his salary raised 30%; what did he then receive?

8. A merchant bought goods for \$1825 and sold them at a profit of 7%; what did he receive for them?

9. A shepherd having 400 sheep lost 20% of them; how many had he left?

10. A merchant bought goods to the amount of \$9325 on a credit of 6 months, but in consequence of his paying cash he received a discount of 5%; what did the goods cost him?

11. A lad had 60 marbles of which he sold 20%, then gave his brother 50% of the remainder, and then lost $12\frac{1}{2}$% of what he had; how many had he left?

12. A man owing debts amounting to \$18620, paid 20% of what he owed, and afterward 35% of what remained unpaid; how much did he still owe?

13. A is $33\frac{1}{3}$% older than B, B is 25% older than C, C is 20% older than D, and D is 20 years old; how old is A?

14. A man invested \$300 in trade, which was increased *by 6% each year for 3 years*; to what did it then amount?

330. ***Rate and amount or difference given, to find the base.***

(1) What number increased by 15% of itself is equal to 46?

Explanation.—If the required number or base were equal to 46, I should have

100% of base, or $\frac{20}{20}$ of base, $=$ 46.

But since 46 is 15% or $\frac{3}{20}$ more than the required number or base, I have

115% of base, or $\frac{23}{20}$ of base, $=$ 46.

Therefore, I divide the amount, 46, by 1 + the rate, 1.15 or $\frac{23}{20}$, and obtain the base, 40, Ans.

Operation.

$1 + .15 = 1.15$

$46 \div 1.15 = 40$, Ans.

Or,

$1 + \frac{3}{20} = \frac{23}{20}$

$46 \div \frac{23}{20} = 40$, Ans.

Formula.—**Base = Amount $\div$ (1 + Rate)......(f).**

RULE.—*Divide the amount by* 1 + *the rate.*

(2) What number diminished by 25% of itself is equal to 45?

Explanation.—If the required number or base were equal to 45, I should have

100%, or $\frac{4}{4}$ of the base, $=$ 45.

But since 45 is 25% or $\frac{1}{4}$ less than the base I have

75% or $\frac{3}{4}$ of the base $=$ 45.

Therefore, I divide the difference, 45, by 1 — the rate, .75 or $\frac{3}{4}$, and obtain the base, 60, Ans.

Operation.

$1 - .25 = .75$

$45 \div .75 = 60$, Ans.

Or,

$1 - \frac{1}{4} = \frac{3}{4}$

$45 \div \frac{3}{4} = 60$, Ans.

Formula.—**Base = Difference $\div$ (1 — Rate)......(g).**

RULE.—*Divide the difference by* 1 *minus the rate.*

EXERCISE LXVII.

What number:

3. Increased by 25% of itself is equal to 90?
4. Increased by $2\frac{1}{3}$% of itself is equal to 767.5?
5. *Diminished* by 20% of itself is equal to 48?
6. *Diminished* by $71\frac{2}{3}$% of itself is equal to .17?

7. I sold a cow for \$44 and thereby made 10%; what did the cow cost me?

8. A man sold a town lot for \$378 at a profit of 5%; what did the lot cost him?

9. After $12\frac{1}{2}$% of the syrup in a barrel had leaked out, there were 35 gallons left; how many gallons were in the barrel at first?

10. A merchant sold a lot of goods at a loss of $4\frac{1}{2}$% and received \$551.99 for them; what did the goods cost?

11. The sale is \$160, the loss $16\frac{2}{3}$%; find the cost.

12. The sale is \$250, the loss $10\frac{1}{2}$%; find the cost.

13. The sale is \$1, the gain $33\frac{1}{3}$%; find the cost.

14. The sale is \$137.50, the gain $52\frac{7}{8}$%; find the cost.

15. A boy received \$18 per month after his wages had been raised 20%; what were his wages at first?

16. A man, after drawing out $22\frac{2}{9}$% of his money, had \$1575 left in bank; how much had he at first?

17. James is 15 years old, and is 25% older than John; how old is John?

18. In 1880 the population of a city was 237300, and had increased 13% in 10 years; what was the population in 1870?

19. A boy bought a hat, a knife and a bat, paying 20% more for the knife than the bat, and 50% more for the hat than the knife; find the cost of each, if the hat cost 90 cts.

20. In a company of men, women, and children there are 28% less of women than of children, and 15% less of men than women; the number of men is 153; how many women and children are there?

What per cent:

21. Is 6 more than 5? 22. Is 162 more than 150?

23. Is 5 less than 6? 24. Is 150 less than 162?

25. A bought 3 mules; the first cost $161, which was 15% more than the cost of the second, and $4\frac{1}{6}$% less than the cost of the third; find the cost of all.

26. A man owes three debts; the second is 8% more than the first, and $13\frac{3}{5}$% less than the third, and the third is $16.75 more than the first; find the entire debt.

27. A dealer sold one buggy for $82, at a profit of 25%, and another, which cost 5% more, at a profit of 15%; what per cent did he make on both together?

QUESTIONS.

What does per cent mean? How expressed? How change a common fraction to a per cent? What is the base? Rate? Percentage? Amount? Difference?

How find the percentage when base and rate are given? How find amount? Difference? How find rate from base and percentage? How find base from the rate and amount or difference?

APPLICATIONS OF PERCENTAGE.

331. The principal applications of Percentage, in which ..e is not an element, are Trade Discount, Profit and Loss, ..ommission, Insurance, Taxes, Duties, Quantitative Chem..cal Analyses, etc.

Those in which time is an element are Interest, True and ..nk Discount, Exchange, Equations of Payments, etc.

..e the problems in the above subjects may be solved by ..e appli..tion of one or more of the formulas of Percent..age, designa.. ..d (a), (b), (c), (d), (e), (f), and (g).

TRADE DISCOUNT.

332. Trade discou..nt is a deduction from the list price of goods.

Merchants and manufacturers generally have fixed price lists of ... goods, and when the market varies, instead of changing their

prices, they change the rate of discount. The fixed price is called the **list price.**

A "discount of 30% and 10%" means, not 40%, but that 30% is first deducted, leaving 70% of the price, from which a deduction of 10% is made, leaving a remainder of 63% of the price. Hence, "30% and 10% off" is equivalent to a single deduction of 37%.

The term "$\frac{1}{4}$ and 12% off" means a discount of $\frac{1}{4}$ or 25% from the price, and then another deduction of 12% from the remainder.

In comparison with Percentage, the **list price** is the Base; the **rate of discount**, the Rate; and the **discount**, the Percentage.

(1) The list price of a bill of goods on a credit of 60 days is \$525; what is the cash value of the goods at 20% discount, and 5% off for cash?

Explanation.—20% of \$525 is \$105, which, being deducted from \$525, leaves \$420.

Again, 5% of \$420, or \$21, deducted from \$420, leaves \$399, Ans.

Operation.

$\$525 \times .20 = \$105;$
$\$525 - \$105 = \$420.$
$\$420 \times .05 = \$\ 21;$
$\$420 - \$\ 21 = \$399.$

EXERCISE LXVIII.

2. The list price of a bill of goods on a credit of 3 months is \$435; what is the cash value of the goods at 8% discount and 5% off for cash?

3. The gross amount of a bill is \$4250; the rates of discount are 10% and 12%; what is the net amount?

4. I bought a piano for \$675, at a discount of $\frac{2}{5}$ and 15%; what did it cost me?

5. From a bill of \$1644.60 a deduction of $\frac{1}{6}$ and 10% was made; what was the amount of the discount?

6. Find a direct discount equal to a discount of 8% and 10%.

Explanation.—From the sum of the two rates, .18, I take their product, .008, and obtain .172, which is equal *to the per cent, 17⅕%*, **Ans.**

Operation.

$.08 + .10 = .18;$
$.08 \times .10 = .008$
$.18 - .008 = .172 = 17\frac{1}{5}\%.$

What direct discount:

7. Is equal to a discount of 6% and $4\frac{1}{2}$%?

8. Is equal to a discount of $\frac{1}{4}$ and 12%?

9. On a bill of $800, what is the difference between a discount of 20% and a discount of 15% and 5%?

10. On a bill of $660, what is the difference between a discount of $\frac{1}{3}$ and $16\frac{2}{3}$% and a discount of $\frac{1}{4}$ and 20%?

PROFIT AND LOSS.

333. Profit is the sum above cost, and **loss** the sum below cost, for which goods are sold.

In comparison with Percentage, the cost or capital invested is the Base; the rate per cent of profit or loss the Rate; the profit or loss the Percentage; and the selling price in case of a gain, the Amount, and in case of a loss, the Difference.

EXERCISE LXIX.

(1) What is gained by selling a farm which cost $1260 at a profit of 25%?

Formula.—**Profit or Loss = Cost × Rate.** See (a).

2. A dealer invested $8375 in shoes, and sold them at a profit of 9%; how much did he make?

3. What does a butcher lose who pays $4320 for cattle and sells them at a loss of $16\frac{2}{3}$%?

4. A owes B $125800 due in 2 years, which he can settle for the cash at a discount of 15%; what deduction will be allowed for the cash?

5. A house and lot which cost $5920 were sold at a profit of $12\frac{1}{2}$%; what was the gain?

6. Bought a bill of goods amounting to $675.50, at 20% discount, and 3% off for cash; what was the total deduction?

7. *A lady* sold a cow for $8 more than cost, and thereby *gained 20%*; what did the cow cost her?

Formula.—**Cost = Profit or Loss ÷ Rate.** See (b)

8. If a merchant can sell his goods at a profit of $3870 he will realize a gain of 15% of his entire stock in trade; find the value of his stock.

9. What is the cost of cloth which sells at a profit of $.75 per yd., at 15% gain?

10. A merchant sells silk at a gain of $.245 per yard, and makes a profit of 14%; find the cost.

11. A boy lost 75 cts. in selling 60 apples at a loss of 25%; what did the apples cost apiece?

12. A merchant sold a bill of goods at a discount of $\frac{1}{4}$ and 10%, and thereby made a deduction of $146.25; find the list price of the goods.

13. What per cent will be gained by selling a gun which cost $12 at a profit of $3?

Formula.—**Rate = Profit or Loss ÷ Cost.** See (c).

14. A dealer paid $480 for a piano and sold it at a profit of $210; what per cent did he make?

15. Paid $50 for a cow and sold her for $40; what per cent was lost?

16. If flour cost $6.80 a barrel, and 25 barrels be sold at a loss of $51, what per cent is lost?

17. Henry bought a pony for $60 and sold him to Albert for $80, and Albert sold him for $70; what per cent did Henry make? What per cent did Albert lose?

18. The cost of manufacturing a quantity of goods is $450. The manufacturer sold them to the jobber at a profit of $72, the jobber sold them to the retail merchant at a profit of $87, and he sold them to the consumer at a gain of $152.25. (1) What per cent did each make? (2) What per cent of the price paid by the consumer is profit?

19. A man sold a horse for $100 and thereby gained 25%; what did he cost?

Formula.—**Cost = Sale ÷ (1 + Rate).** See (f).

20. Sold silk for $4.13 a yard, at a profit of 18%; find the cost.

21. A lot of gloves, being damaged, were priced at $1.40 per pair, which was 20% less than cost; what was the cost?

Formula.—**Cost = Sale ÷ (1 — Rate).** See (g).

22. What cost a horse that sold for $240, at a loss of 15%?

23. A farmer sold 25 hogs for $110, which was 30% more than the cost of raising them; how much did it cost per head to raise them?

24. A dealer sold 120 bbl. of flour for $792, which was 12% less than he paid for it; what did he pay per barrel?

25. A drover sold one horse for $168, at a profit of 12%, and another for $161, at a loss of 8%; how much did he make or lose?

(26) A merchant paid $10 for a saddle; how much must he ask for it that he may deduct 25% of the price and still make a profit of 20% on the cost?

Solution: $\begin{cases} \text{Selling Price} = \$10 \times (1 + .20) = \$12. \\ \text{Asking Price} = \$12 \div (1 - .25) = \$16. \end{cases}$

27. Bought cloaks at $38; what price must they be marked that 20% may be deducted and leave 16% profit?

28. A trunk cost $6; how must I mark the price that after abating 10% the profit may be 33⅓%?

29. A merchant was in business 3 years. At the close of the first year his capital amounted to $24650, and at the close of the second year it was 32% greater than it was the previous year, and 42% less than it was at the close of the succeeding year; what was he worth on retiring from the business?

30. A drover bought hogs at $6 a head, and sold them at $7½ a head; what was the gain per cent?

31. A drover bought horses at $131.50 a head; expended $4.50 each in taking them to market, and sold them at $153.50 a head; what was the gain per cent?

32. If a man fails to pay his taxes until he is charged 12% for delay; how much will he lose if his taxes are $18.25?

33. I bought a carriage for $178.50, and sold it at a loss of 8%; how much did I lose?

34. What per cent do I make by selling apples at $\frac{5}{4}$ of their cost?

35. I sell for $6 what cost me $4.60; what per cent do I gain?

36. What per cent do I lose by selling gloves at $\frac{7}{8}$ of their cost?

37. I sell for $8 what cost me $9.50; what per cent do I lose?

38. A newsboy sold papers for 5 cts., which was 25% advance on the cost; what did they cost?

39. A drover sold a horse for $112.50, and thereby made 15%; what did the horse cost?

40. What is the price of gold pens which cost $1.30 and are marked 25% below cost?

41. A grocer sold tea that cost $.78 a pound, at a loss of 15%; at what price per pound did he sell it?

42. What must be the price of cassimere that it may be sold at $1.98 a yard, after a deduction of 10% from the price has been made?

43. How must cigars, which cost $3.50 a box, be marked that a dealer may deduct 15% from the marked price, and still make 15% *profit?*

44. Potatoes were bought at $1.60 and sold at a profit of 20%; what was the selling price?

45. I bought 95 sheep at $3.78 each, and sold them at a profit of 36%; what did I get for them?

46. One bale of cotton cost $48 and was sold at 25% gain; another bale cost $42 and was sold at $16\frac{2}{3}$% loss; what was received for both bales?

47. A merchant bought hats at $3.40, gloves at $1.65, and ties at $.46, and desires to sell the hats at 12% gain, the ties at 85% gain, and the gloves (being damaged) at 18% loss; how should he price them?

48. A merchant sold some damaged goods which cost $120 for $100; what per cent was lost?

49. A man failing in business owes $3283.47, of which he is able to pay only $2918.64; what per cent will the creditors lose?

50. If $\frac{4}{5}$ of an investment is sold for what $\frac{1}{2}$ of it cost, what is the loss per cent?

51. A merchant sold a quantity of merchandise at a profit of $375, and the cost was 80% of the sale; find the cost.

52. A lad invested 70 cts. in oranges and 50 cts. in apples. He sold the oranges at a profit of 30%, and the apples at a loss of 20%; what per cent did he make on the entire investment?

53. A person owns two estates worth respectively $4275 and $6325. If the first rise in value 24%, and the second fall $13\frac{3}{5}$%, find the rise or fall per cent in the value of his whole property.

54. One-half of a melon was sold for 40 cts., at a gain of $33\frac{1}{3}$%; what per cent would have been made had the entire *melon* been sold for 75 cts.?

55. A farmer made $12\frac{1}{2}\%$ in selling 15% of a drove of sheep for \$162; what per cent would he have made had he sold the entire drove for \$1320?

56. The price of a hat is \$6, and if 10% is deducted for cash a profit of 8% will be made; what did the hat cost?

57. The price of a crate of crockery is \$360. If it is sold at a discount of $\frac{1}{4}$ and 10%, a profit of $21\frac{1}{2}\%$ will be made; find its cost.

COMMISSION.

334. An **agent** is a person who transacts business for another.

335. Commission is the fee or compensation allowed an agent for his services.

An agent who buys and sells merchandise is called a **commission merchant**, and one who collects debts, rents, etc., a **collector**.

The **consignor** or **shipper** is the person who sends goods to be sold; a **consignment**, the goods sent; and the **consignee** or **correspondent**, the person to whom the goods are sent.

The **net proceeds** of a transaction is the sum of money due the principal or consignor after all expenses of commission, etc., are paid.

An **account sales** is a written statement made by a commission merchant to his principal, containing an account of goods sold, their price, the expenses, and the net proceeds.

In comparison with Percentage, the amount of sales or money invested or collected is the Base; the per cent of commission, the Rate; and the commission, the Percentage.

EXERCISE LXX.

1. What is the fee of an agent who charges 8% for collecting a debt of \$450?

Formula.—**Com.** $= \left\{ \begin{array}{c} \textbf{Amt. Collected or} \\ \textbf{Amt. of Sales.} \end{array} \right\} \times$ **Rate.**

2. A collector collects for me \$375, and charges 10% commission; what is his commission?

3. A commission merchant in Mobile sold for a merchant in Havana, on a commission of 4%, 360 bags of coffee at $[illegible].25 per bag: find the commission.

4. My attorney collected 90% of a debt of $[illegible], and charged 5% commission: what amount should he pay me?

5. A book agent sold 144 copies of a book at $3.25 per copy, on a commission of [illegible]%: what amount should be remit to the publishers?

6. How much will an auctioneer receive for selling a piano for $450 and furniture for $[illegible], at [illegible]% commission?

7. A correspondent in New Orleans received a consignment of 36 hhd. of sugar, averaging 1130 lb. to the hhd., which he sold at 7¾ cts. a pound, on a commission of 1¼%; what amount should be remit to the principal after paying an expense of $31.25 for storage, insurance and drayage?

8. An agent charges $30 for purchasing goods at 2% commission: what did he pay for the goods?

Formula.—**Amt. of Sales = Com. ÷ Rate.** See (b).

9. Required the sale of sugar on which a commission of $275.87 was charged, at 1¾%.

10. My city agent bought a horse for me on a commission of 1½%: what did he pay for the horse if his commission was $2.25?

11. A tax collector, whose commission for collecting was 5½%, received $937.20: how much did he collect?

12. An agent sold cotton on a commission of 2%, and the net proceeds were $490: find the amount of sales.

Formula.—**Amt. Sales = Net Pro. ÷ (1 — Rate)**...(g)

13. Find the amount of sales when the net proceeds are $2422.50, and the rate 5%?

14. An agent sold for me a horse on a commission of 10%, and paid me \$135; what did he receive for the horse?

15. A consignor received \$933.18$\frac{3}{4}$ for 30 bales of cotton, each weighing 450 pounds, which were sold on a commission of 1$\frac{1}{4}$%; at what price per pound did the cotton sell?

16. What amount of goods can an agent purchase with \$2846.25, after deducting a commission of 3$\frac{1}{2}$%?

Analysis.—For each \$1 invested in goods the agent charges 3$\frac{1}{2}$ cents, making the expense to the consignor \$1.03$\frac{1}{2}$. Hence, the agent will invest in all as many dollars as \$1.03$\frac{1}{2}$ is contained times in \$2846.25, or \$2750.

Formula.—Sum Inv. = Entire Amt. ÷ (1 + Rate). See (f)

17. A commission merchant received \$582.40, with instructions to invest it in flour at \$7 a barrel, after deducting his commission of 4%; how many barrels did he buy?

18. A dealer in New Orleans sent to his agent in Texas \$5541.90 for the purchase of cattle; after deducting a commission of 1$\frac{1}{2}$%, how many cattle could be bought at \$15 a head?

19. A manufacturer in Liverpool sent his correspondent in Mobile \$9578.40 with which to buy cotton; how many bales at \$45 each can be bought after deducting a commission of 2$\frac{1}{3}$%?

INSURANCE.

336. Insurance is an agreement by one party to indemnify another in case of loss.

The parties who agree to make good the loss are called insurance companies or underwriters. Insurance companies are of two kinds, viz.: a stock company is one which has a paid-up capital, and divides the profit and loss among its stockholders; and a mutual company is one in which the losses are shared by the parties insured.

Insurance is of two kinds, viz.: property insurance includes *fire insurance, marine insurance,* and *live stock insurance*; and personal

insurance includes *life insurance*, *health insurance*, and *accident insurance*.

The **premium** is the sum paid for insurance; and the **policy** is the written contract between the insurers and the insured.

In comparison with Percentage the sum insured, or the face of the policy, is the Base; the rate of the premium is the Rate, and the premium is the Percentage.

EXERCISE LXXI.

(1) What must I pay to insure my house for \$4000 against loss by fire for 2 years, at 2%?

Formula.—**Prem. = Valuation × Rate.** See (a).

2. What is the premium for insuring a store and goods, valued at \$6000, at $1\frac{1}{2}$%?

3. What is the annual premium for insuring a steam saw-mill for \$2250, at $3\frac{3}{4}$%?

4. At $\frac{1}{4}$% per year what premium do I pay yearly for an insurance of \$3000 on my house?

5. A ship was insured for \$35000, and its cargo for \$85000; if the rate on the ship was $1\frac{1}{2}$%, and on the cargo $2\frac{1}{2}$%, what was the total premium?

6. What must I pay for a policy of \$2500 on the furniture in my house, at $\frac{3}{5}$%?

7. What will it cost to insure a drove of 45 horses, valued at $\$96\frac{1}{2}$ each, at $\frac{4}{5}$% premium?

(8) I paid \$54 for insuring my house, at $\frac{3}{4}$%; what was the face of the policy?

Solution: Since $\frac{3}{4}$% of the face of the policy is \$54, the face of the policy is \$54 ÷ $\frac{3}{4}$%, or \$7200, Ans.

Formula.—**Valuation = Prem. ÷ Rate.** See (b).

9. If \$8 are paid for insuring a barn at 2%, what is the value of the barn?

10. A ship and its cargo was insured at $1\frac{3}{8}\%$, and the premium was \$5445; what was the face of the policy?

11. At $\frac{3}{4}\%$, \$9 is the premium on what valuation?

12. A manufacturer wishes to pay \$924.60 to have his mill insured at $2\frac{7}{8}\%$; at what must he value the mill?

13. A cow is insured at $\frac{4}{5}$ of her value for \$.60, at $1\frac{1}{2}\%$ premium; what is the value of the cow?

14. If 88% of the value of a ship is insured for $\$271\frac{1}{3}$, at $\frac{5}{8}\%$ premium, what is the value of the ship?

(15) At what rate is a house insured if the house is valued at \$1800 and the premium is \$27?

Formula.—**Rate = Prem. ÷ Valuation.** See (c)

What is the rate of insurance when:

16. A premium of \$5 is paid for an insurance of \$400?

17. A premium of \$50.50 is paid for an insurance of \$3700?

18. My life is insured for \$5000, at an annual cost of \$40; what is the rate of premium?

19. A sugarhouse was insured for \$15000 at a cost of $\$262\frac{1}{2}$; what was the rate?

20. At what rate per cent is \$4, the premium on \$600 valuation?

21. A shipper paid \$37.50 for an insurance of \$9600 on a cargo of produce; what per cent premium did he pay?

(22) Goods bought in New Orleans for \$492.50 were insured at $1\frac{1}{2}\%$; what sum will cover the value of the goods and the premium?

Solution: The sum insured less $1\frac{1}{2}\%$ of itself = \$492.50, or $98\frac{1}{2}\%$ of the sum insured = \$492.50; hence, the sum insured = \$492.50 ÷ .985 = \$500, Ans.

23. What sum must be insured, at 5%, on a consignment of *cotton worth \$950, to* cover property and premium?

TAXES.

337. A **tax** is a sum of money raised for public use.

In general, there are three kinds of taxes, viz.: **a property tax** is a tax on property; **a personal tax** is a tax on the *person*, and is called a *poll* or *capitation tax;* a **license** or **excise tax** is a tax on business or business avocation.

Property is of two kinds, viz.: **real property** or **real estate** is immovable property, as houses and lands; and **personal property** is movable property, as furniture, horses, books, etc.

An **assessor** is a person appointed to estimate the value of the property to be taxed, and apportion the taxes accordingly.

In comparison with Percentage the **assessed value of the property** is the Base, the **tax rate** or **tax on $1** is the Rate, and the **tax** levied on the property is the percentage.

EXERCISE LXXII.

1. The assessed property of a village amounted to $474000, and was taxed 8 mills on the dollar for public improvements; what amount of tax was raised?

Formula.—**Tax** $=$ **Valuation** $\times$ **Tax Rate.** See (a).

2. What amount of tax must a man pay who is assessed $4500 for real estate, and $2300 for personal property, if he pays $\frac{4}{5}$% city tax, and $\frac{1}{2}$% state tax?

3. At what rate must property valued at $12500 be assessed to raise a tax of $600?

Formula.—**Rate of Tax** $= \frac{\textbf{Sum to be Raised}}{\textbf{Valuation.}}$. See (c).

4. The tax levied in a certain city, for all purposes, was $259776, and the taxable property was listed at $21648000; what was the rate per cent of tax?

5. A courthouse, costing $7380, was built in a county whose taxable property was valued at $2460000; what was the rate, and what was the tax of A, whose property was assessed at $5000?

6. What is the valuation of a piece of property that pays a tax of $38.50, at the rate of $5\frac{1}{2}$ mills on the dollar?

$$\text{Formula.—Valuation} = \frac{\text{Sum Raised}}{\text{Rate of Tax}}. \quad \text{See (b)}$$

7. A's property is assessed at $1\frac{2}{3}\%$, and he pays a tax of $23.50; what is the value of his property?

8. A is worth $4860, and pays a tax of $85.05; what is the rate, and what is B worth if his tax is $65.80?

9. What sum must be assessed to build a bridge, at a cost of $1029, and pay 2% for collection?

$$\text{Formula.—Sum to be Ass'd} = \frac{\text{Sum to be Raised}}{(1 - \text{Rate of Collection})}. \quad \text{(g)}$$

10. A city park, which cost $15207.50, was built by a tax on the property of the town. The tax rate was 7 mills on the dollar, and the cost of collection $1\frac{1}{4}\%$; what was the value of the city property?

RULE FOR GENERAL TAXES.—I. *From the sum to be raised deduct the poll-tax, if any, and divide the remainder by the assessed value of the taxable property, real and personal; the quotient will be the rate of tax.*

II. *Multiply the assessed value of each man's property by the rate, and to the product add his poll-tax, if any; the sum will be the whole tax.*

11. A tax of $44475 is assessed upon a certain parish containing 4850 taxable polls, at $1.50 each; the real estate is valued at $4,436,500, and the personal property at $213,500; what will be the rate of tax?

In the above parish:

12. What was A's tax, whose property was assessed at $4850, and who paid for 3 polls.

13. What was B's tax, whose property was assessed at $8327, and who paid for 4 polls?

CUSTOMS OR DUTIES.

338. Customs or **duties** are taxes levied on imported goods and other property, for the support of the general government, or for the protection of home industry.

Imports are goods or other property brought into a country; a **customhouse** is a place at which duties are collected, and a **port of entry** is a seaport at which a customhouse is located.

There are two kinds of duties, viz.: an **ad valorem duty** is a certain per cent assessed on the actual cost of the goods, in the country from which they were imported, as shown by the invoice, or fixed by appraisement; a **specific duty** is a tax assessed upon the number, weight, or measure of the goods, per bale, ton, pound, hhd., gal., etc., without regard to their value.

Tare, leakage and **breakage** are allowances for the weight of whatever contains the goods, leakage of liquids, and the breakage of bottles. These terms are now but seldom used in the calculation of duties.

Gross weight is the weight without deductions, and **net weight** is the weight less the deductions.

An **invoice** is a written list of merchandise, with prices and charges annexed; and a **manifest** is a complete invoice of a ship's cargo.

The long ton of 2240 lb. is used in the U. S. Customhouses; and in weights, less than ½ lb. is not regarded, and more than ½ lb. is considered a pound.

EXERCISE LXXIII.

NOTE.—In the following problems, the given rates of duty are the actual charges by the government for the articles named.

(1) What is the duty at 50% ad valorem on 1450 yards of silk that cost in France $1.25 a yard?

Solution: The cost of 1450 yd. @ $1.25 was $1812.50; and 50% of *$1812.50* = $906.25, Ans.

Formula.—**Ad Val. Duty = Net Inv. Price × Rate %.**

2. Find the duty on 56000 lb. of railroad iron, specific duty being \$17 per ton?

NOTE.—The computation of specific duties does not involve the principles of percentage.

3. What is the duty on 50 casks of wine, each containing 36 gallons, at 50 cts. per gallon, allowing 2% for leakage?

4. A merchant imported 120 hhd. of sugar, 950 lb. each, and 42 boxes of raisins, 25 lb. each; what was the duty at \$.012 a pound for sugar, and \$.02 a pound for raisins?

5. Find the duty on 150 bags of rice, 180 lb. each, at $2\frac{1}{4}$ cts. a pound, $\frac{3}{4}$% being allowed for tare.

6. Find the duty on 4375 lb. of woolen goods, invoiced at \$2560, specific duty being 35 cts. a pound, and ad valorem 40%.

7. Duty on cigars is \$2.50 per pound specific, and 25% ad valorem; find the duty on 20500 cigars, weighing 12 lb. per M., and invoiced at \$3.50 per C.

8. One dozen bottles of porter contains 2.17 gal., and the bottles themselves are worth 18 cts. per dozen; find the duty on 45 cases of porter, 12 dozen bottles each, invoiced at \$.54 a gallon, duty being 35 cts. per gal. for the porter and 30% ad valorem on the glass, allowing 2% for breakage.

9. Find the rate of duty on table cutlery, the duty on an invoice valuation of \$1240 being \$434.

10. A hardware merchant bought Rodger knives at \$.75 apiece; after paying 4% for transportation and 50% duty, at what price must he sell them to make a clear profit of $33\frac{1}{3}$%?

11. A merchant imported woolen suits weighing 5 lb. each, which he sold at \$17 apiece at a profit of 25%, after paying 5% for transportation, and a duty of 40 cts. a pound specific and 35% ad valorem; find the cost per suit.

QUANTITATIVE CHEMICAL ANALYSIS.

339. Quantitative chemical analysis consists in the separation, and the determination of the relative proportions, of the elements or constituents of which a substance is composed.

The relative proportions of constituents are expressed in per cents of the whole.

In the analysis of cane juice, syrup, molasses, etc., the constituents of importance ordinarily determined are **sucrose, glucose, and total solids.**

The term **coefficient of purity** is used to designate the per cent that the sucrose is of the total solids; and the per cent which the glucose is of the sucrose, is called **glucose ratio.**

Thus, a cane juice having 15% total solids, 12.5% sucrose and 1.1% glucose would have a coefficient of purity of $12.5 \div 15 = 83\frac{1}{3}\%$; and a glucose ratio of $1.1 \div 12.5 = 8.8\%$.

In **fertilizers** the constituents ordinarily estimated are **phosphoric acid, nitrogen,** and **potash.**

In comparison with Percentage the **amount of the substance** analyzed is the Base, the **per cent of a constituent** is the Rate, and the **quantity of the constituent** is the Percentage.

NOTE.—The proportion of constituents, as presented in these problems, is copied from analyses made at the stations of the Louisiana State University and A. and M. College.

EXERCISE LXXIV.

1. A sample of cane juice has 14.2% sucrose; how many pounds of sucrose are there in 2250 lb. of juice?

Formula.—**Amt. of Const. = Amt. of Subst. × Rate.** See (a)

2. A certain lot of cane has 89% juice, and the juice contains 11.4% sucrose; how much sucrose is there in $3\frac{1}{5}$ tons of cane?

3. A sample of cane juice contains $16\frac{2}{3}\%$ total solids, and the coefficient of purity is $86\frac{1}{4}\%$; how many pounds of sucrose are there in 220 gal. of juice, allowing 8.9 lb. to the gal.?

4. In the last example, if the glucose ratio is 9.3%, how much glucose is there?

5. A certain commercial fertilizer contains 9.7% available phosphoric acid, 2% nitrogen, and 1.4% potash; how many pounds of each of these ingredients is there in 640 pounds of the fertilizer?

6. In example (5), what will be the commercial value of a ton of the fertilizer, available phosphoric acid being estimated at $7\frac{1}{2}$ cts. per ℔., nitrogen at $19\frac{1}{2}$ cts., and potash at 5 cts.?

7. The analysis of a sample of kainite shows 11.9% potash; what is the value of 820 ℔s. of it, potash being valued at 5 cts. per ℔.?

8. A superphosphate contains 12% phosphoric acid; how much of it must be used to obtain 78 ℔. of phosphoric acid to an acre of land?

Formula.—**Amt. Substance = Amt. Constit. ÷ Rate**....(b)

9. A sample of nitrate of soda contains 15.8% nitrogen; how much of it will yield 173.8 ℔. of nitrogen?

10. From a lot of cane there were extracted 6278.4 ℔. of juice; how many tons of cane were there, the rate of extraction being 65.4%?

11. In a tank of cane juice there are 93.5 ℔. of available sugar, which is 8.5% of the juice; what is the weight of the juice?

12. In 1640 ℔. of cane juice there are 188.6 ℔. sucrose; what per cent of sucrose does the juice contain?

Formula.—**Rate = Amt. of Const. ÷ Amt. of Subst**.....(c).

13. $4\frac{7}{8}$ tons of cane were passed through the mill, and the bagasse was found to weigh 3250 pounds; what per cent of juice was *extracted?*

14. In example (13) the juice contained 942.5 lb. of total solids and 812.5 lb. of sucrose; find the per cent of each, and the coefficient of purity.

15. In 1750 lb. of bone meal are $80\frac{1}{2}$ lb. of nitrogen and $400\frac{3}{4}$ lb. of phosphoric acid; what is the per cent of each of these ingredients?

16. 1130 lb. of cotton seed meal contain 73.5 lb. of nitrogen, 35.1 lb. of phosphoric acid and 20.3 lb. of potash; find the per cents of these several constituents.

NOTE.—In the following problem the given yields and prices are fair averages.

17. One acre of land produces 660 pounds of seed cotton, of which 33% is lint and 67% is seed. The lint is worth $9\frac{1}{4}$ cts. per pound, and the seed at the oil mill yields $1\frac{1}{4}$% of lint, which is worth 82% of the price of the other lint. The seed also yields $34\frac{1}{2}$% of cotton seed meal, worth $19.50 per ton; 14% of oil, worth $31\frac{1}{2}$ cts. per gallon (8 lb. to the gal.), and $50\frac{1}{4}$% of hulls, which give, on burning, 3% of ash, worth $1\frac{1}{2}$ cts. per pound; find the total value of the cotton grown on 10 acres of land.

QUESTIONS.

What are the principal applications of percentage in which time is not an element? In which time is an element?

What is trade discount? Profit and loss? How find the profit or loss? The cost? Rate?

What is commission? How calculated?

What is insurance? The premium? Policy? How find sum to be insured to cover loss and premium?

What are taxes? How computed?

What are duties or customs? Ad valorem duty? Specific duty?

What is quantitative chemical analysis? What constituents are ordinarily determined in sugar analysis? In the analysis of fertilizers? What the coefficient of purity? Glucose ratio?

INTEREST.

340. Interest is money paid for the use of money.

341. The **principal** is the sum for the use of which interest is paid.

342. The **amount** is the sum of the principal and interest.

343. The **rate** is the per cent of the principal paid for its use for a given time. The time is one year, unless another period of time is specified.

Legal Interest is the interest at the rate fixed by law. Usury is interest at a higher rate than Legal Interest.

In some of the States a higher rate than the legal is allowed, if specified in writing. When no rate is specified in accounts, notes, etc., the legal rate is always understood. For a table of legal rates in the different States, see Appendix, Art. 477.

In comparison with Percentage, the principal is the Base; the rate multiplied by the number of years, the Rate; and the interest, the Percentage.

344. Hence, while there are several methods of computing interest, all are but modifications of the following general formula:

Interest = Principal × Rate × Number of Years. See (a).

NOTE.—It is not necessary for the student to learn all the following methods of computing interest. In general, the method by months, in the opinion of the author, is preferable. See also Art. 478.

345. ***To compute interest for any given time and rate.***

I. General Method—By Years.

(1) What is the interest of $420, for 3 yr. 3 mo., at 7%?

Explanation.—Since the interest of any sum at 7%, for 1 yr., is .07 of the principal, I multiply $420 by .07, and obtain $29.40, which is the interest for 1 yr. I next multiply the interest for one yr. by the number of years, 3 yr. 3 mo. = $3\frac{1}{4}$ yr., and obtain the required interest, $95.55.

Operation.

$420	Principal.
.07	Rate.
$29.40	Int. for 1 yr.
$3\frac{1}{4}$	No. of yr.
7.35	Int. for $\frac{1}{4}$ yr.
88.20	Int. for 3 yr.
$95.55	Int. for $3\frac{1}{4}$ yr.

(2) Find the interest of \$320, for 3 yr. 5 mo. 18 da., at 4%.

Explanation.—3 yr. 5 mo. 18 da. = 3 yr. $5\frac{18}{30}$ mo. = 3 yr. 5.6 mo. = $3\frac{5.6}{12}$ yr. = $3\frac{7}{15}$ years.

Operation.
$\$320 \times .04 \times 3\frac{7}{15} = \$44.37\frac{1}{3}$,

The interest of \$320 for 1 yr., at 4% is $\$320 \times .04 = \$12.80$; and for $3\frac{7}{15}$ yr., it is $3\frac{7}{15}$ times as much as it is for 1 yr., or $\$12.80 \times 3\frac{7}{15} = \$44.37\frac{1}{3}$. Ans.

Thirty days are usually considered as making a month.

RULE.—I. *Multiply the principal by the rate, and that product by the time expressed in years; the product is the interest.*

II. *Add the principal and interest to find the amount.*

The work may often be shortened by taking such a per cent of the principal as is expressed by the product of the rate by the time, as in the next example.

(3) Find the interest of \$180 for 2 yr. 6 mo. at 8%.

Solution: 2 yr. 6 mo. = $2\frac{1}{2}$ yr. 8% for 1 yr. = $2\frac{1}{2} \times 8\%$, or 20% for $2\frac{1}{2}$ yr. 20% = $\frac{1}{5}$, and $\frac{1}{5}$ of \$180 = \$36, Ans.

EXERCISE LXXV.

Find the interest of:

4. \$240, for 1 yr. 3 mo., at 8%; for 3 yr. 4 mo., at 6%.
5. \$135.70, for 2 yr. 3 mo., at 5%; for 1 yr. 9 mo., at $6\frac{1}{4}$%
6. \$500, for 3 yr. 4 mo., at 9%; for 5 yr. 10 mo., at 12%
7. \$2684.80, for 6 yr. 6 mo., at 7%; for 9 mo., at 8%.
8. \$800, for 2 mo. 12 da., at 5%; for 1 mo. 6 da., at 7%
9. \$512.60, for 7 mo. 9 da., at 6%; for 8 mo. 18 da., at 7%
10. \$360, for 18 da., at 4%; for 24 da., at 5%.
11. \$92.50, for 21 da., at 6%; for 112 da., at 5%.
12. \$378, for 4 yr. 5 mo. 20 da., at 6%.
13. \$1020.96, for 3 yr. 7 mo. 18 da., at 5%.
14. \$583.20, for 6 yr. 11 mo. 13 da., at $4\frac{1}{2}$%.

Find the amount of:

15. \$500, for 1 yr. 8 mo. 6 da., at 12%.

16. \$656.84, for 4 yr. 10 mo. 15 da., at 6%.

17. \$531.70, for 3 yr. 3 mo. 23 da., at 6%.

II. Special Method — By Months.

NOTES.—1. Since 30 days are reckoned as a month, 3 days is $\frac{1}{10}$, or .1 of a month; hence, any number of days may be reduced to tenths of a month by dividing by 3. Thus, 6 da. = .2 mo.; 18 da. = .6 mo.; 22 da. = .7$\frac{1}{3}$ mo.; 5 mo. 12 da. = 5.4 mo.; 2 yr. 7 mo. 27 da. = 31.9 mo.

2. Rate per year may be reduced to rate per month by dividing by 12. Thus, 7% per yr. = $\frac{7}{12}$% per month; 8% per annum = $\frac{2}{3}$% per month; 6% yearly = $\frac{1}{2}$% monthly.

(18) Find the interest of \$516.60 for 2 yr. 3 mo. 18 da., at 8%.

Explanation.—Rate per mo. = 8% ÷ 12 = .00$\frac{2}{3}$. Hence, the interest for 1 mo. is .00$\frac{2}{3}$ of the principal, which is \$3.444, and for 2 yr. 3 mo. 18 da., or 27.6 mo., it is 27.6 times \$3.444, which is \$95.0544, the required interest.

Operation.

\$516.60	Principal.
.00$\frac{2}{3}$	Rate per mo.
\$3.4440	Int. for 1 mo.
27.6	No. mo.
\$95.05440	Int. for 27.6 mo.

RULE.—I. *Divide the rate by* 12; *the quotient is the monthly rate.*

II. *Multiply the principal by the rate per month; the product is the interest for* 1 *month.*

III. *Multiply the interest for* 1 *month by the time in months, and tenths of a month; the product is the required interest.*

Find the interest of:

19. \$2250 for 2 yr. 7 mo., at 9%.

20. \$504.72 for 7 mo. 26 da., at 6%.

21. \$927 for 3 yr. 5 mo. 6 da., at 8%.

22. \$4642.68 for 5 mo. 17 da., at 15%.

23. \$580 for 4 yr. 1 mo. 24 da., at 10%.

Find the amount of:

24. $1883 for 1 yr. 4 mo. 21 da., at 6%.
25. $1248.72 for 4 yr. 11 mo. 15 da., at 8%.
26. $583.20 for 6 yr. 11 mo. 13 da., at $4\frac{1}{2}$%.
27. $1000 for 1 yr. 1 mo. 1 da., at $4\frac{4}{5}$%.
28. $200.35 for 3 yr. 4 mo. 24 da., at 4%.

III. Special Method — By Aliquot Parts.

29. Find the interest of $250.64 for 2 yr. 7 mo. 19 da., at 5%.

Solution.

		$250.64	
		.05	
		$12.5320	Int. for 1 yr.
2 × (int. for 1 yr.)	2	$25.064	" " 2 yr.
$\frac{1}{2}$ of int. for 1 yr.	$\frac{1}{2}$	6.266	" " 6 mo.
$\frac{1}{6}$ of int. for 6 mo.	$\frac{1}{6}$	1.044	" " 1 mo.
$\frac{1}{2}$ of int. for 1 mo.	$\frac{1}{2}$	.522	" " 15 da.
$\frac{1}{5}$ of int. for 15 da.	$\frac{1}{5}$	.104	" " 3 da.
$\frac{1}{3}$ of int. for 3 da.	$\frac{1}{3}$	.035	" " 1 da.
Adding,		$33.04	" " 2 yr. 7 mo. 19 da.

Each item of interest is carried no lower than mills, the next figure being neglected if less than 5; but if 5 or more, it is counted 1 mill.

Find the interest of:

30. $625, for 2 yr. 5 mo. 27 da., at 8%.
31. $580, for 4 yr. 1 mo. 24 da., at 10%.
32. $515.80, for 2 yr. 3 mo. 3 da., at $3\frac{1}{2}$%.
33. $156.25, for 3 yr. 10 mo. 28 da., at $4\frac{1}{2}$%.

IV. Special Method — By 6 per cent.

34. Find the interest of $80 for 4 yr. 7 mo. 9 da., at 6%.

Solution: At 6%, the interest

Of $1, for 4 yr.	= 4 × 6 cents	= $.24
Of $1, for 7 mo.	= 7 × 5 mills	= $.035
Of $1, for 9 da.	= 9 × $\frac{1}{6}$ of a mill	= $.0015
Of $1, for 4 yr. 7 mo. 9 da. =		$.2765
Of $80, for 4 yr. 7 mo. 9 da.	= 80 × $.2765	= $22.12, **Ans.**

RULE.—*Take* 6 *cts. for each year, five mills for each* *ionth,* $\frac{1}{6}$ *of a mill for each day, and their sum for each* *'ollar in the principal; the result will be the interest at* 6%.

When the rate is greater or less than 6%, find the interest of the rincipal at 6% for the given time, then add or subtract from it such part of itself as the given rate exceeds or falls short of 6%.

NOTE.—For examples, the student may solve the problems given nder the other methods.

V. Special Method—Banker's Rule.

35. What is the interest of $726.60 for 78 da., at 6%?

Explanation.—The interest of any sum ›r 1 yr. at 1% is $\frac{1}{100}$ of the principal; ut 1 yr. at 1% = $\frac{1}{6}$ yr. (60 da.) at 6%. Ience, from the right of the dollars I ut off two figures, this gives the interest ›r 60 da., = $7.266. Since 15 da. is $\frac{1}{4}$ of 60 da., $7.266 ÷ 4 = 1.816, is the interest for 15 da., and $1.816 ÷ 5 = $.363, is the in- :rest for 3 da.

Operation.

7	266	Int. for 60 da.
1	816	" " 15 da.
	363	" " 3 da.
9	45	Int. for 78 da.

RULE.—*Move the decimal point in the principal two* *laces to the left, to obtain the interest for* 60 *days, at* 6%; *nd take multiples or aliquot parts of the result, to find the* *iterest for any number of days.*

To find the interest at any other rate, take such a part or multiple : the interest at 6%, as the given rate is of 6%.

Find the interest, at 6%:

3. Of $6253 for 96 days.
3. Of $94.60 for 80 days.
). Of $750 for 48 days.
2. Of $324.25 for 93 days.

Find the interest of:

37. $640 for 33 da. at 8%.
39. $560 for 70 da. at 9%.
41. $480 for 100 da. at 7%.
43. $520 for 85 da. at 5%.

44. Find the interest of $600, from June 10, 1884, to .ugust 28, 1887, at 5%.

Solution.

yr.	mo.	da.
1887	8	28
1884	6	10
3	2	18

3 yr. 2 mo. 18 da. = 3.2$\frac{1}{6}$ yr.

$600 × .05 × 3.2$\frac{1}{6}$ = $96.50, Ans.

In finding the time, I take the difference between the two dates, as denominate numbers. For other methods, see Appendix, Art. 475.

Find the interest of:

45. $520, from March 21, 1880, to Dec. 30, 1882, at 7%.
46. $1630, from April 1, 1878, to Oct. 10, 1882, at 6%.
47. $10.40, from Feb. 4, 1885, to Nov. 1, 1888, at 8%.
48. $63.45, from Sept. 23, 1883, to May 5, 1889, at 9%.

Find the amount of:

49. $2150, from May 10, 1877, to Jan. 1, 1881, at 6%.
50. $153.80, from June 10 to Dec. 31, at 5%.
51. $350, from Oct. 17, 1875, to Apr. 11, 1878, at 7%.
52. $125.40, from March 1, 1882, to Jan. 10, 1883, at $6\frac{1}{2}$%.

EXACT INTEREST.

346. The methods based on the supposition that 360 days make a year are not strictly accurate. As a year contains 365 days, the interest found by these methods is $\frac{5}{365}$, or $\frac{1}{73}$ part of itself too large, except when the time is expressed in years. Hence,

347. To compute exact interest for months and days, *find the interest by the ordinary method, and from it subtract $\frac{1}{73}$ part of itself.*

(1) Find the exact interest of $2568 for 93 da., at 6%.

Solution.—2568 × 15.5 mills = $39.804, ordinary interest. $39.804 ÷ 73 = $.545; $39.804 — $.545 = $39.26, Ans.

EXERCISE LXXVI.

Find the exact interest of:

2. $642.72 for 25 da., at 6%.
3. $144.50 for 144 da., at 5%.
4. $567.50 for 73 da., at 4%.
5. $5200 for 123 da., at 7%.

PROBLEMS IN INTEREST.

347. ***The interest, rate and time given, to find the principal.***

(1) What principal in 3 yr. 9 mo. 24 da., at 8%, will give *$114.50* interest?

Explanation.—I first find the interest of $1 for the given time and rate, which is $.305⅓. Then I say, since $1 for the given time and rate will produce $.305⅓ interest, it will take as many dollars to produce $114.50 as $.305⅓ is contained times in $114.50, equal to $375.

Operation.

$\$1 \times .08 \times 3.8\frac{1}{6} = \$.305\frac{1}{3}$.

$\$114.50 \div .305\frac{1}{3} = \$375$, Ans.

Formula.—**Principal = Interest ÷ (Rate × No. of Years).**

RULE.—*Divide the given interest by the interest of* $1 *for the given time and rate; the quotient will express the number of dollars in the principal.*

If the amount is given instead of the interest, *divide the given amount by the amount of* $1 *for the given time and rate.*

EXERCISE LXXVII.

What principal:

2. In 2 yr. 6 mo., at 8%, will give $7 interest?

3. In 6 yr. 10 mo. 18 da., at 7%, will give $178.0856 interest?

4. In 6 yr. 8 mo., at 9%, will give an amount of $480?

5. In 2 yr. 2 mo. 25 da., at 6%, will gain an amount of $762.4322?

6. What sum in 108 da., at 6%, will yield an interest of $80?

7. I rented a house at 7% of its value, and in 9 mo. 18 da. I received $122.08 rent; find the value of the house.

8. What sum must be loaned at 10% to amount to $60 in 2 yr. 6 mo.?

9. What sum must a father invest, at 8%, that his son, now 15 yr. 3 mo. old, may have $3650 when he is 21?

348. ***The principal, interest and time given, to find the Rate.***

(1) At what rate will $1350 yield $321.30 interest in 3 yr. 4 mo. 24 da.?

Explanation.—I first find the interest of $1350 for the given time at 1%, which is $45.90. Then I say, if $1350 produces $45.90 at 1%, to produce $321.30 in the same time will require as many times 1% as $45.90 is contained times in $321.30, equal to 7%.

Operation.

$\$1350 \times .01 \times 3.4 = \$45.90$

$\$321.30 \div \$45.90 = 7.$

Formula.—**Rate = Interest ÷ (Principal × No. of Years).**

RULE.—*Divide the given interest by the interest of the given principal at 1% for the given time; the quotient will express the number of per cents in the rate.*

EXERCISE LXXVIII.

At what rate:

2. Will $120 give $24 interest in 3 yr. 4 mo.?

3. Will $972 give $211.41 interest in 3 yr. 7 mo. 15 da.?

4. Will $60 amount to $68 in 2 yr. 8 mo.?

5. Will $844.75 amount to $910.22 in 93 da.?

6. At what rate per month will $50 gain $1 in 90 da.?

7. At what rate per month must I invest $618.75 so that it may yield me $271.2875 in 5 yr. 5 mo. 23 da.?

8. A debt of $516.60 amounted to $611.6544 in 2 yr. 3 mo. 18 da.; what was the rate of interest?

9. At what rate per year will $100 gain $100? $200? $300? $400? $500?

10. At what rate per annum will a principal double itself? Triple itself?

11. At what rate per annum will a principal double itself in 2 yr.? 3 yr.? 4 yr.? 5 yr.? 10 yr.?

NOTES—1. At 1% a principal will double itself in 100 yr.; hence, to double itself in 2 yr., the rate must be as many times greater than 1% as 100 yr. is times greater than 2 yr., etc.

2. At 1% a principal will triple itself in 200 yr.; hence, to triple *itself in 2 yr.* the rate must be 1% multiplied by 200 yr. ÷ 2 yr., etc.

12. At what rate per annum will a principal triple itself in 2 yr.? 3 yr.? 4 yr.? 5 yr.? 10 yr.?

13. At what rate will $136510 gain $3635.28, exact interest, in 216 days?

349. ***The principal, rate and interest given, to find the time.***

(1) In what time will $760 give $169.29 interest, at 9%.?

Explanation.—I first find the interest of $760 for 1 yr., which is $68.40. Then I say, if $760 produces $68.40 in 1 yr. it will take it as many years to produce $169.20 as $68.40 is contained times in $169.20, equal to $2\frac{19}{40}$ yr. Reducing $\frac{19}{40}$ of a yr. to integers of lower denominations gives 5 mo. 21 da.

Operation.

$\$760 \times .09 = \$68.40$

$\$169.29 \div \$68.40 = 2\frac{19}{40}$.

$2\frac{19}{40}$ yr. = 2 yr. 5 mo. 21 da., Ans.

Formula.—**No. of Years = Interest ÷ (Principal × Rate).**

RULE.—*Find the interest of the given principal for one year at the given rate; divide the given interest by this interest, and the quotient will be the required time in years.*

EXERCISE LXXIX.

In what time:

2. Will $40 gain $2.50, at 5%?

3. Will $560 gain $124.32, at 6%?

4. Will $75 give an amount of $91, at 8%?

5. Will $245.60 amount to $284.4048, at 6%?

6. The interest of $56.78 for a certain time, at 10%, was $22.24; what was the time?

7. If $456 is put at interest, at 5%, how long before it will be sufficient to cancel a debt of $535.04?

8. How long will it take $100 to gain $100, at 2%? 4%? 5%? 10%?

9. How long will it take any principal to double itself, at 2%? 3%? 4%? 5%? 10%? $12\frac{1}{2}$%?

NOTES.—1. At 100% any principal doubles itself in 1 yr.; hence, to double itself, at 2%, it will require as many years as 2 is contained in 100, etc.

2. At 200% any principal will triple itself in 1 yr.; hence, to triple itself, at 4%, will require (200 ÷ 4) years.

10. How long will it take any sum to triple itself, at 3%? 7%? 10%? 20%?

11. The use of $243, at 8%, for 2 yr. 4 mo. 24 da. is equivalent to the use of $324, at 9%, for what length of time?

ANNUAL INTEREST.

350. Annual interest is simple interest on the principal, and on each year's interest of the principal, to the time of settlement.

351. ***To compute annual interest.***

(1) Find the annual interest and amount of $500 for 4 yr. at 8%.

Solution: The annual interest is $40, the first of which will draw interest 3 yr.; the 2d, 2 yr.; the 3d, 1 yr.

Int. of $500 for 4 yr., at 8%,	$160.
Int. of $40 for 3 yr., at 8%,	9.60
Int. of $40 for 2 yr., at 8%,	6.40
Int. of $40 for 1 yr., at 8%,	3.20
Total int.	$179.20

Amount, $500 + $179.20 = $679.20.

RULE.—*Compute the interest on the principal for the given time and rate; to which add the interest on each year's interest for the time it has remained unpaid.*

EXERCISE LXXX.

2. What is the amount of $640 for 6 yr., at 5% annual interest?

3. Find the annual interest of $750 for 5 yr. at 6%.

4. Find the amount of a note for $8560 for 10 yr., at $7\frac{1}{2}$% annual interest.

5. Find the amount of the following note, May 10, '86:

$850. HOMER, La., May 10, 1877.

On demand, value received, I promise to pay G. G. Gill, or order, eight hundred fifty dollars, with interest at 6%, payable annually. L. R. LAY.

PROMISSORY NOTES.

352. A **promissory note** is a written promise to pay a specified sum of money.

The **face** of a note is the sum promised to be paid.

The **maker** or **drawer** of a note is the party who promises to pay, and who signs the note. The **payee** is the party to whom, or to whose order, the money is to be paid.

An **indorser** is a person who signs his name on the back of a note, and thereby makes himself responsible for its payment.

A **negotiable note** is one made payable to bearer, or to the order of the payee. When so made, it can be sold or transferred.

Without the words, "or order," or the words, "or bearer," inserted, the note is not negotiable, and is payable only to the payee.

If the note is payable to "bearer" it may be transferred from one person to another without the indorsement of the payee; but if it is payable to "order" the payee must indorse it before he can transfer it.

The **holder** of a note is the person who has it in his possession.

The **maturity** of a note is the day it becomes legally due. In most States a note does not mature until 3 days after the time named for its payment. These 3 days are called **days of grace.**

The words "value received" should be inserted in every note, and the sum to be paid should be written in words.

There are several varieties of promissory notes, as a demand note, time note, joint and several note, etc.

A **demand note** is one that must be paid whenever demanded by the holder; a **time note** is one in which the time of payment is specified; and a **joint and several note** is one in which the makers are jointly and singly responsible for the payment.

The following are the usual forms of such notes:

Demand Note.

$1000.00. NEW ORLEANS, Jan. 1, 1889.

On demand, we promise to pay F. F. Hansell & Bro., or order, one thousand dollars, with interest at 8%, *value received.* L. GRAHAM & SON.

Time Note.

$125.50. BATON ROUGE, La., Oct. 2, 1888.

Sixty days after date, I promise to pay Wm. Garig & Co., or bearer, one hundred twenty-five $\frac{50}{100}$ dollars, with interest at 7%, value received. ANDREW JACKSON.

Joint and Several Note.

$270.35. ARCADIA, La., Feb. 3, 1887.

Three months after date we jointly and severally promise to pay Atkins and Marsalis, or order, two hundred seventy $\frac{35}{100}$ dollars, with interest at 9%, value received.

R. A. SMITH,
W. M. BAKER.

EXERCISE LXXXI.

Find the amounts due on the following:

1. A note for $584.48, due in 133 days, and paid at maturity, with interest at $7\frac{1}{2}$%.

2. A note for $4603.15, bearing interest at 7%, dated July 17, 1881, and paid March 8, 1883.

3. A note for $640, dated Feb. 2, 1884, due in 6 mo., paid March 26, 1886, rate of interest being 6%.

4. A note for $960.45, dated July 7, 1886, due in 1 yr. 3 mo., paid Jan. 2, 1889, with interest at 8% from maturity.

PARTIAL PAYMENTS.

352. Partial payments are part payments of notes, bonds or other obligations.

Indorsements are acknowledgments of such payments written on the back of the note, bond, etc., stating the time and amount of the same.

The time between the date of a note and the date of the **first** payment is called the **first period**; the time between the **dates of the first** *and second* payments, second period; between the **dates of the** *second and* **third** payments, third period, etc.

For computing the balance due on a note, bond, etc., on which partial payments have been made, two rules are in common use, viz.: the mercantile rule, and the United States rule.

353. ***To find the balance on a note when partial payments have been made.***

United States Rule.

RULE.— I. *Find the periods.*

II. *Find the amount of the principal for the first period, and from it subtract the first payment for a second principal.*

III. *Find the amount of the second principal for the second period; from it subtract the second payment for a third principal; and so continue to the settlement, unless the interest for any period exceeds that payment, in which case that period and payment are added to the next period and payment respectively, for a single one of each.*

1. On a note for \$375, dated June 13, 1885, were the following indorsements: Aug. 25, 1886, \$102; Feb. 4, 1888, \$273.50; what was due April 24, 1889, interest at 8%?

Solution.

First Period.	Second Period.	Third Period.
1886 — 8 — 25	1888 — 2 — 4	1889 — 4 — 24
1885 — 6 — 13	1886 — 8 — 25	1888 — 2 — 4
1 — 2 — 12	1 — 5 — 9	1 — 2 — 20
14.4 mo.	17.3 mo.	$14\frac{2}{3}$ mo.

Principal	\$375.
Interest for first period, $\$375 \times .00\frac{2}{3} \times 14.4 =$	36.
Amount	411.
First payment	102.
Second principal	309.
Interest for second period, $\$309 \times .00\frac{2}{3} \times 17.3 =$	35.638
Amount	344.638
Second payment	273.50
Third principal	71.138
Interest for third period, $\$71.138 \times .00\frac{2}{3} \times 14\frac{2}{3} =$	6.956
Balance due April 24, 1889 . . .	\$78.09

EXERCISE LXXXII.

2. On a note for $1500, dated July 1, 1883, and bearing 6% interest, were the following indorsments: July 1, 1884, $50; Jan. 1, 1886, $1000; what was due July 1, 1886?

NOTE.—In this example the interest for the first period, $90, is greater than the first payment, $50. Hence, we take from July 1, 1883, to Jan. 1, 1886, for the first period, and $1000 + $50, or $1050 for the first payment.

3. A note for $1824 was given Oct. 10, 1881, bearing 9% interest from date; on the back of the note was the following credit: "Jan. 28, 1883, nine hundred twenty-five $\frac{56}{100}$ dollars." Find the balance due Sept. 13, 1887.

4. On a note for $4000, dated May 1, 1882, bearing interest at 6%, were the following indorsements: May 21, 1883, $800; June 10, 1884, $1200; Aug. 10, 1885, $1500. What was due May 1, 1886?

5. Memoranda: Face of a note, $1000; date, June 20, 1878; payments, Jan. 10, 1879, $125; June 16, 1879, $93; Feb. 20, 1880, $200; rate, 5%. Find the balance to be paid Aug. 1, 1880.

(6) $1000. MOBILE, Ala., April 10, 1884.

On demand, I promise to pay J. W. Bush, or order, one thousand dollars, with interest at 10%. D. P. HILL.

Indorsements: July 28, 1884, $500; Dec. 13, 1884, $8; Feb. 25, 1885, $12; July 7, 1885, $125; Oct. 3, 1885, $200; March 15, 1886, $50. What was due on this note June 3, 1886?

Mercantile Rule.

354. When partial payments are made on short notes or interest accounts, business men usually employ the following method:

Find the amount of the whole debt to the time of settlement; also find the amount of each payment from the time

it was made to the time of settlement, and subtract the sum of these amounts from the amount of the debt.

(7) On a note for $600, at 6%, dated Jan. 1, 1888, were the following indorsements: May 13, 1888, $150; Aug. 22, 1888, $275. What was due Dec. 28, 1888?

Solution: Amount at 6%:

Ot $600, from Jan. 1 to Dec. 28 . . .		$635.70
Of $150, from May 13 to Dec. 28 .	$155.625	
Of $275, from Aug. 22 to Dec. 28 .	280.775	
Sum of amounts	$436.40 =	436.40
Balance due Dec. 28, 1888		$199.30

8. A bill of goods, amounting to $1008, was to be paid May 5, 1888. Received July 5, 1888, $360; Sept. 20, 1888, $450; Dec. 15, 1888, $180. What was due May 5, 1889, at 8% interest?

9. An account of $750, due Jan. 1, 1887, received the following payments that year: June 10, $145; Sept. 23, $465; Oct. 3, $23. What was due on the 31st of the next December, allowing 6% interest?

COMPOUND INTEREST.

355. Compound interest is interest on both principal and unpaid interest.

Interest may be compounded annnally, semi-annually, or quarterly. When compounded semi-annually, the rate of interest is one-half the yearly rate; and when compounded quarterly, the rate is one-fourth of the yearly rate.

Annual interest is the gain of a principal whose yearly interests have become debts at simple interest; but, distinct from this, *compound interest is the whole gain of a principal, increased at the end of each interval by all the interest drawn during that interval.*

The final amount in compound interest is called the **compound amount.**

356. ***Given the principal, rate, and time, to find the compound interest and amount.***

(1) Find the compound amount and interest of $360 for 3 years and 3 months, at 5%.

Solution.

$360	1st princ.	19.845	3d yr's int.
.05		396.90	
18.00	1st yr's int.	416.745	Amt. at the end of the 3d year.
360.		.01¼	
378	2d princ.	1.04186	
.05		4.16745	
18.90	2d yr's int.	5.209	Int. for 3 mo.
378		416.745	
396.90	3d princ.	421.954	Required amt.
.05		360.	1st princ.
19.8450	3d yr's int.	61.954	Comp. int.

RULE.—*Find the amount of the given principal for one interval of time; then, taking the amount as a new principal, find the amount of the second interval, and so continue for the entire time. The difference between the last amount and the principal is the compound interest for the time.*

EXERCISE LXXXIII.

2. What is the compound interest of $1500 for 3 years at 8%?

3. What is the compound amount of $100 for 2 yr. 6 mo. at 21%?

4. What is the compound amount of $350 for 5 years at 2%?

5. What is the compound interest of $450 for 3 yr. 6 mo. at 6%?

6. What is the amount of $148.50 at compound interest for 4 yr. 7 mo. 12 da., at 7%?

7. Find the interest of $500 for 2 yr. 8 mo., at **6%**, *interest* compounded semi-annually.

Suggestion: Compute the interest at 3% for 5⅓ yr.

8. Find the compound interest of $13062.50, for 1 yr. 10 mo. 12 da, at 8% payable quarterly.

Suggestion: Compute the interest at 2% for $7\frac{7}{15}$ yr.

9. Find the amount of $735.60 for 2 yr. 5 mo. 24 da., at 8%, interest compounded quarterly.

NOTE.—In actual business, compound interest is found by means of a table showing the amount of $1 for various times at different rates. For such a table, see Appendix, Art. 479.

TRUE DISCOUNT.

357. True discount is the difference between the face of a debt and its present worth.

358. The **present worth** of a debt is that sum of money which, at a given rate of interest, will amount to the same as the debt at its maturity.

NOTES.—1. True discount is the simple interest on the *present worth*, from the day of discount until the day of maturity.

2. Discount on a debt must be carefully distinguished from *Commercial* Discount, which is simply a deduction from the regular price or value of an article; the latter is usually expressed as such a "per cent off."

The different cases of true discount may be solved like those of simple interest: the **present worth** corresponding to the *Principal*; the **discount**, to the *Interest*; and the **face of the debt**, to the *Amount*. The following case is the only one much used:

359. ***Given the face, time, and rate, to find the present worth and true discount.***

(1) Find the present worth and true discount of $304.25 due in 3 yr. 1 mo. 6 da., at 7%.

Operation.

$37.2 \times .\frac{07}{12} = .217$

$304.25 \div 1.217 = 250.$

$304.25 - 250 = 54.25.$

Explanation.—The interest of $1 at 7%, for 37.2 mo. is $.217, and the amount $1.217; that is, the present worth of $1.217 for the given time and rate is $1. Hence, the present worth of $304.25, for the same time and rate, is as many dollars as $1.217 is contained times in $304.25, equal to $250, present worth; and *$304.25 — $250* gives the true discount, $54.25.

RULE.—I. *Divide the debt by the amount of $1 at the given rate for the given time, and the quotient will express the number of dollars in the true present worth.*

II. *Subtract the present worth from the entire debt, and the remainder will be the true discount.*

NOTE.—When the obligation bears interest, the amount is first found, and that sum is discounted.

EXERCISE LXXXIV.

2. What is the cash value of a note for $6780 due in 1 yr. 9 mo., without interest, money being worth 6%?

3. How much should I pay for a claim for $1380, due in 2 yr. 6 mo., without interest, when money is worth 6%?

4. A man owes me $1272.50, due in 1 yr. 3 mo., without interest, and desires to settle it now; how much should I deduct for the cash, money being worth 7%?

5. Bought a farm for $26990.541, payable in 1 yr. 3 mo. 15 da., without interest; how much cash would have bought it, money being worth 6%?

6. A offers me $4100 cash for a city lot, and B offers me $5034.15, payable in 3 yr. 5 mo. 20 da.; how much more in cash is A's offer than B's, if money is worth 7%?

7. A debt of $53.95 was due May 21, 1880, but was settled Nov. 9, 1874, at a discount of 6%; how much money was paid?

8. What is the present worth of a note for $7500, bearing interest at 6%, and due in 3 yr. 4 mo., discounted at 5%?

9. A merchant buys a bill of dry goods amounting to $2513.79 on a credit of 4 mo.; a bill of crockery for $469.68, payable in 6 mo.; and a bill of groceries for $1691.75, due in 2 mo.; how much ready cash will pay all the bills, discounting at 6%?

BANK DISCOUNT.

360. A **bank** is a chartered institution that receives and loans money, or issues bank-bills that circulate as money.

A **bank of issue** is one that issues notes and bank-bills; a **bank of discount** is one that loans money by discounting notes, drafts, etc.; and a **savings-bank** is one that receives money on deposit, paying interest on the sums deposited, and loans its deposits, by discounting notes.

Some banks combine two or all of these kinds of business.

361. Bank discount is a deduction made by a bank for loaning money.

When a bank loans money the borrower gives his note payable at a specified time, *without interest*. This note is then discounted by taking from its face the interest for the actual number of days plus *three days of grace;* and the difference, called the **proceeds**, is paid to the borrower. The interest thus deducted is bank discount.

A **protest** is a statement made by a notary public, giving legal notice to the maker and indorsers of a note of its non-payment. If a note is not protested on the third day of grace, the indorsers are released from all obligation to pay it.

362. ***To find the bank discount and proceeds of a note.***

(1) Required the bank discount and proceeds of a note for $1260, due in 60 days, discounted at 6%.

Explanation.—I find the interest of $1260 for 60 da., + 3 da. of grace, which is the bank discount, $13.23. Subtracting the bank discount from the face of the note, I obtain $1246.77, the proceeds.

Operation.

$12.60 = Int. for 60 da.
.63 = " " 3 da.
$13.23 = Bank discount.
$1260 — $13.23 = $1246.77.

RULE.—I. *Find the interest on the sum discounted for three days more than the given time, at the given rate; it is the discount.*

II. *Subtract the discount from the sum discounted, and the remainder is the proceeds.*

NOTE.—In the case of an interest-bearing note, treat the amount of the note at maturity as the sum to be discounted. Some banks employ the method of exact interest.

[illegible]

Find the bank discount and proceeds:

1. [illegible] due in [illegible] da., discounted at 6%.
2. [illegible] note for $[illegible], discounted at [illegible]%.
3. [illegible] note for $[illegible], discounted at [illegible]%.
4. [illegible] payable in [illegible] days, discounted at [illegible]%.
5. [illegible] payable in [illegible] days, discounted at [illegible]%.
6. [illegible] payable in [illegible] days, discounted at 6%.
7. What will be the bank discount on my note for $195, payable in [illegible] mo., at [illegible]?

8. A merchant sold a bill of goods for $342.15, for which he took a note payable in [illegible] mo., without interest. Find the cash value of the note discounted at a bank at 6%.

10. Find the difference between the true and bank discount of $[illegible] for [illegible] mo. 15 da., at 6%.

11. Find the proceeds of the following note:

$737.[illegible]. New Orleans, Feb. 14, 1889.

Value received, two months after date, I promise to pay F. F. Hansell & Bro., or order, seven hundred thirty-seven [illegible] dollars, at the Louisiana National Bank.

Discounted Feb. 23, at [illegible]%. Warren Easton.

Note.—In finding the time in notes like the preceding one, bankers often include the day from which the counting begins. Thus, 2 months after Feb. 14 is April 14. Now, the number of days from February 23 to April 14, including both these days and 3 days of grace, is 54 days, the required time.

12. Find the proceeds of the following note, if discounted April 11, 1889, at 8%:

$1860. Baton Rouge, Dec. 23, 1888.

Nine months after date, for value received, I promise to pay Andrew Jackson, or order, eighteen hundred sixty dollars, with interest at 6%, at the First National Bank.

A. Rosenfield.

363. ***To find the face of the note, when the proceeds, rate, and time are given.***

(13) For what amount must a note be drawn, payable in 7 mo. 15 da., so as to yield $1924 proceeds when discounted at 6%?

Explanation.—The interest of $1 for 7 mo. 18 da., is $.038; hence, the bank proceeds are $.962. Now, if $1, for the given time and rate, yields $.962 proceeds, it will take as many dollars to yield $1924 proceeds as $.962 is contained times in $1924, or $2000.

Operation.

$7.6 \times .00\frac{1}{2} = .038$
$1 - .038 = .962$
$1924 \div .962 = 2000.$

RULE.—*Divide the given proceeds by the proceeds of* $1 *for* 3 *days more than the given time, and the quotient will express the number of dollars in the face of the note.*

14. Find the face of a note due in 6 mo., such that if discounted at 6% the proceeds shall be $775.60.

15. What must be the face of a note due in 4 mo., whose proceeds when discounted at 8% shall be $4350.90.

16. I owe a debt of $500, which I wish to pay with the proceeds of a note for 6 mo., discounted at 7%; for what sum must I give the note?

17. I borrowed $758.065 from a bank for 90 days, at 6%, for which I promised to pay a certain sum; find the sum.

18. Copy and complete the following note, so that Mr. S. I. Reymond, on presenting it at the bank, may draw $1000 cash, discounted at 8%:

$ .....   BATON ROUGE, La., June 13, 1889.

Three months after date I promise to pay to the order of G. T. Webster, "Cashier,"

at the First National Bank of Baton Rouge,

....Dollars for value received, with interest at the rate of 8 per cent per annum from maturity until paid.

....

STOCKS AND INVESTMENTS.

364. A **corporation** is a company authorized by law to transact business as a single individual, having the same legal rights and obligations.

365. **Stock** is the capital of a corporation or company; and a **share** is one of the equal parts into which the stock is divided, and is transferable like other property.

The value of a share is usually \$100, and, in this book, will be so regarded, unless otherwise stated.

Stock is at par when it may be sold in open market for its face value; when it sells for less, it is below par, or at a discount; when it sells for more it is above par, or at a premium.

The market quotations are given in percentage. Thus, stock at 100 is at par; at 105, 5% above par; at 97, 3% below par.

A dividend is a sum paid to stockholders out of the earnings of the company, and is always reckoned on the par value of the stock.

A stock broker is one who buys and sells stocks for a commission, called brokerage. Brokerage is usually $\frac{1}{4}$% or $\frac{1}{8}$% of the par value of the stock.

A bond is an instrument in writing under seal, given to secure the payment of a sum of money at a specified time. Bonds are issued for the purpose of borrowing money.

The principal bonds bought and sold as stock are government, state, city, and railroad bonds; and these are quoted according to the rate of interest which they bear. Thus, the U. S. 4's are bonds issued by the United States, bearing 4% interest.

366. ***To find the cost of any number of shares of stock.***

(1) Find the cost of 250 shares of Louisiana railroad stock, the market value being $102\frac{1}{4}$, and brokerage $\frac{1}{4}$%.

Solution: The cost of one share, including brokerage, will be $\$102\frac{1}{2}$; hence, 250 shares will cost $250 \times \$102\frac{1}{2} = \$25625$.

EXERCISE LXXXVI.

Find the cost, brokerage being $\frac{1}{8}$%:

2. Of 500 shares of stock at $102\frac{1}{2}$.
3. Of 36 shares of stock at a premium of $3\frac{3}{8}$%.

4. Of 180 shares of stock at 92.

5. Of 72 shares of stock at a discount of $3\frac{1}{4}\%$.

6. A broker bought 90 shares of stock at $1\frac{1}{4}\%$ discount, and sold them at $1\frac{1}{2}\%$ premium; find the gain.

7. A bought 50 shares of city stock at $98\frac{1}{8}$ and sold it at $99\frac{1}{2}$, paying $\frac{1}{4}\%$ brokerage in each transaction; how much did he make?

367. ***To find the number of shares that can be bought for a given amount.***

(8) When the brokerage is $\frac{1}{8}\%$, how many shares of stock at $107\frac{3}{4}$ can be purchased for \$4315.

Solution: Since one share with the brokerage will cost $\$107\frac{7}{8}$, \$4315 will buy as many shares as $\$107\frac{7}{8}$ is contained times in \$4315, or 40 shares.

9. How many shares of telegraph stock at $102\frac{5}{8}$ can be bought for \$5150, brokerage $\frac{3}{8}\%$?

10. I wish to invest \$7515 in railroad stock, at a premium of 25%; how many shares can I buy, brokerage being $\frac{1}{4}\%$?

11. Find the number of shares of mining stock at $88\frac{1}{2}$, that can be bought for \$10635, brokerage $\frac{1}{8}\%$.

12. A invested \$7620 in city stock at 5% discount, and sold it at $2\frac{1}{4}\%$ premium, paying $\frac{1}{4}\%$ brokerage each time; find the gain.

13. A man sold a house, from which he received a yearly rent of \$425, for \$9717, and invested the proceeds in 4% bonds at $78\frac{7}{8}$, paying $\frac{1}{8}\%$ brokerage; by how much was his yearly income increased?

368. ***To find the amount of an investment when the income and rate are given.***

(14) How much will it cost me to buy enough of 6% bank stock at $102\frac{7}{8}$, to secure an income of \$900 a year, brokerage being $\frac{1}{8}\%$?

Solution: The income of 1 share is $6; hence, the number of shares is $900 ÷ $6, or 150. Now, 150 shares at $103 each = $15450, Ans.

15. My income from railroad stock bought at $107\frac{1}{2}$, brokerage $\frac{1}{4}$%, is $8400; what did it cost me if it pays an annual dividend of 6%?

16. How much must be invested in $3\frac{1}{4}$% city bonds, bought at $96\frac{3}{4}$, brokerage $\frac{1}{8}$%, to yield an annual income of $260?

17. What sum must be invested in U. S. 4's, at 105, to yield $3000 annually?

18. The Illinois Central railroad stock pays a quarterly dividend of $3\frac{1}{2}$%, and can be bought at $109\frac{5}{8}$; what sum must be invested in it to endow a professorship in a college with an annual income of $2240, brokerage being $\frac{1}{4}$%?

19. A man bought 5% stock at $94\frac{1}{2}$, brokerage $\frac{1}{8}$%, which pays him an annual income of $1680; how much larger or smaller would his income have been had he invested in 6% bonds at $107\frac{25}{28}$, brokerage $\frac{1}{4}$%?

369. ***To find the rate per cent of income from money invested in bonds.***

(20) I bought bonds paying 5%, at 106; what per cent do I realize on the money invested, brokerage $\frac{1}{4}$%?

Solution: The cost of one bond is $$106\frac{1}{4}$ and it yields me $5 annual income. Hence, the rate required = $5 ÷ $$106\frac{1}{4}$ = $4\frac{12}{17}$%.

21. What per cent income is yielded by 7% State bonds bought at $124\frac{3}{4}$, brokerage $\frac{1}{4}$%?

22. A lady bought railroad stock paying 6% at $79\frac{7}{8}$; what per cent does she get of the money invested, brokerage $\frac{1}{8}$%?

23. Which is the better investment, to buy 8% stock at $20\frac{1}{8}$% discount, or 5% stock at $37\frac{5}{8}$% discount, brokerage in each being $\frac{1}{8}$%? What is the difference in the rate of income?

24. What per cent of his money will a man gain by investing in Pacific Railroad 6's at $104\frac{3}{4}$, brokerage $\frac{1}{4}\%$?

25. What is the difference in the rate of income between 5's bought at 80 and 6's bought at 90?

370. ***To find the price at which stock should be bought to yield a certain rate of income.***

(26) At what price should 4% bonds be bought to yield an income of 5%, brokerage being $\frac{1}{4}\%$?

Solution: The annual income of 1 share is \$4, which is to be 5% of the cost; hence, the cost is $\$4 \div 5\% = \$80$. This includes the price and brokerage; hence, the price is $80 - \frac{1}{4} = 79\frac{3}{4}$, or $20\frac{1}{4}\%$ discount.

At what per cent premium or discount must:

27. 4% bonds be bought to yield an income of 3%?

28. 5% bonds be bought to yield an income of 8%?

29. 6% stock be bought to yield an income of 10%, brokerage being $\frac{1}{4}\%$?

30. 8% stock be bought to yield an income of 5%, brokerage being $\frac{1}{8}$ of 1%?

31. A bought 6% stock at 20% premium; at what per cent discount must B buy $4\frac{1}{2}\%$ stock to realize the same rate of income?

EXCHANGE.

371. **Exchange** is the process of paying debts in distant places by the remittance of drafts instead of money.

A **draft** or a **bill of exchange** is an order made by one party on another to pay a certain sum to a third party.

The person who makes the order is the **drawer**, the person to whom it is addressed, the **drawee**, and the person to whom the money is payable, the **payer**.

Exchange is of two kinds, viz.: domestic or inland, and foreign.

Domestic exchange is exchange between different parts of the same country.

For *foreign exchange,* see Appendix, Art. 482.

A sight draft is one payable at sight, and a time draft is one payable at some future time.

When a draft can be bought for its face, it is said to be *at par*; when the cost is less than the face, it is *below par*, or at a *discount*; and when the cost is more than the face, it is *above par*, or at a *premium*. The rate per cent which the cost of a draft is more or less than its face, is called the *Rate of Exchange*.

NOTE.—The rate of exchange between two places depends chiefly on their relative trade. If New Orleans owes New York, drafts on New York are at a premium in New Orleans; if New York owes New Orleans, drafts on New York are at a discount; if the trade of the two cities with each other is equal, exchange is at par.

The following is the usual form of a sight draft:

$500. SHREVEPORT, La., May 2, 1889.

At sight, pay to the order of H. Skolfield, five hundred dollars, value received, and charge to the account of

JOHN S. YOUNG.

To R. M. Walmsley, New Orleans La.

In time drafts, in the place of the words "at sight," insert "—— days after sight," or "—— days after date." "After sight" means after acceptance.

Discount is allowed on time drafts, and is computed on the amount or face of the draft, according to the rules of bank discount.

372. ***To find the cost of a draft.***

(1) What is the cost of a sight draft on Mobile for $3650, at $\frac{3}{4}$% premium?

Solution: Since the rate of premium is $\frac{3}{4}$%, $1 of exchange will cost 1.00\frac{3}{4}$; hence, the cost of $3650 will be 3650 times 1.00\frac{3}{4}$, or $3677.375, Ans.

(2) Find the cost of a draft for $8320, payable in 60 days, at a premium of 1$\frac{1}{4}$%, money being worth 6%.

Solution: Since the rate of premium is 1$\frac{1}{4}$, $1 of exchange is equal to $1.0125; but the bank discount on $1 for 63 da. at 6% is $.0105; hence, $1 of exchange will cost $1.0125 — $.0105, or $1.002. Therefore $8320 will cost 8320 times $1.002, or $8336.64, Ans.

Formula.—**Cost of Draft = Number of $ in the Face × by Cost of $1 Exchange.**

EXERCISE LXXXVII.

How much will it cost me to pay a bill:

3. Of \$650 in New York, exchange being at 2% premium?

4. Of 3240 in New Orleans, exchange being at $\frac{1}{2}$% disc.?

5. Of \$7268.40 in Memphis, exchange being at $1\frac{3}{8}$% premium?

6. Of \$4375.50 in Mobile, exchange being at $1\frac{1}{2}$% disc.?

7. The face of a sight draft is \$395.75, discount $\frac{4}{5}$%; what is the cost?

8. Find the cost of a draft for \$650, payable in 60 days, at a premium of $\frac{1}{2}$%, money being worth 8%.

9. When exchange is at $\frac{3}{8}$% discount, and interest 6%, what must I pay for a draft for \$7216.85, payable in 30 days?

10. What cost a draft on Cincinnati for \$12620, payable in 90 days, premium being $1\frac{3}{4}$%, and interest 6%?

11. I owe a debt of \$320 in Galveston, payable in 45 days; find the cost of a draft sufficient to discharge the debt, exchange being at $\frac{3}{4}$% discount, and interest 7%.

373. ***To find the face of a draft.***

(12) What is the face of a sight draft which cost \$2020, exchange being at 1% premium?

Solution: Since the premium is 1%, a \$1 draft will cost \$1.01; therefore \$2020 will buy a draft of as many dollars as \$1.01 is contained times in \$2020, or \$2000, Ans.

(13) How large a draft on Savannah can be bought in Atlanta, due 90 days after sight, for \$3000, exchange being $2\frac{1}{2}$% premium, and interest 6%?

Solution: The cost of \$1 sight draft would be \$1.025. But the bank discount on \$1 for 93 days is \$.0155; hence, the cost of \$1 draft is \$1.025 — \$.0155 = \$1.0095, and \$3000 will therefore buy as many dollars of exchange as \$1.0095 is contained times in \$3000, or \$2971.77.

Formula.—Number of \$ in Face = Cost of Draft ÷ Cost of \$1 Exchange.

14. Find the face of a sight draft costing $756.40, when exchange was $2\frac{1}{8}\%$ premium.

15. What is the face of a sight draft that can be bought for $2540, when exchange is at $1\frac{3}{5}\%$ discount?

16. What was the face of a sight draft for which $1256.25 was paid, exchange being at $\frac{1}{2}\%$ premium?

17. If a draft payable 60 days after sight costs $798.80, when exchange is at $1\frac{1}{4}\%$ premium and interest 8%, what is its face?

18. A merchant in Baton Rouge paid $4265 for a draft to settle a debt in New York at 30 days' sight, exchange at $3\frac{1}{4}\%$ premium, interest 8%; what was the amount of the debt?

19. A father has a son at college, and sent him as large a draft as could be bought for $395.40, due in 21 days after sight, exchange being $\frac{3}{4}\%$ discount, and interest 6%; how much money did the son receive?

QUESTIONS.

What is interest? Principal? Rate? Amount? What is the rule for computing interest by years? By months? By aliquot parts? 6% method? Banker's method? Exact interest? What is annual interest? How computed?

What is a promissory note? The face? Maker? Indorser? Partial payments? Indorsements? United States rule? Mercantile rule? Compound interest? How computed?

What is true discount? How computed? What is a bank? Bank of issue? Of discount? Savings bank? Bank discount? How computed? How find the face of a note?

What is a corporation? Stock? Share? When is stock at par? At a premium? At a discount? What is a stock broker? Brokerage? A bond?

What is exchange? A draft? The drawer? Drawee? Payer? Domestic exchange? Sight draft? Time draft? How find the cost? How find the face?

CHAPTER V.

PROPORTION AND AVERAGES.

RATIO.

374. Ratio is the relation of one number to another of the same kind, expressed by their quotient.

(1) The ratio of 6 ft. to 2 ft. is 6 ft. ÷ 2 ft., or 3.

375. The **sign** of ratio is the colon (:), which is equivalent to ÷.

(1) 8 : 4 is read, the ratio of 8 to 4, and is equivalent to 8 ÷ 4, or 2. 12 : 3 = 4 is read, the ratio of 12 to 3 is equal to 4, or 12 contains 3, 4 times.

The **terms** are the two numbers compared; the **antecedent**, the first term or dividend; the **consequent**, the second term or divisor; and a **couplet**, both terms together.

376. A **simple ratio** is the ratio of two numbers; and a **compound ratio** is the product of two or more simple ratios.

The compound ratio obtained by multiplying the two simple ratios, 2 : 3 and 5 : 7, is usually written thus, $\left\{\begin{matrix}2:3\\5:7\end{matrix}\right\}$. Hence,

$$\left\{\begin{matrix}3:4\\5:7\end{matrix}\right\}=\frac{3\times 5}{4\times 7};\ \left\{\begin{matrix}2:4\\3:7\\5:9\end{matrix}\right\}=\frac{2\times 3\times 5}{4\times 7\times 9};\ \text{etc., etc.}$$

The **reciprocal** of a ratio, or inverse ratio, is the result of interchanging the places of its terms.

(1) The reciprocal or inverse of 2:3 is 3:2, or $\frac{3}{2}$.

377. Since the *antecedent* corresponds to the *numerator*, and the *consequent* to the *denominator*, *changes* on the terms

of a ratio have the same effect upon its value as like changes have upon the terms of a fraction. Hence, the

PRINCIPLES:

1. *Multiplying the antecedent, or Dividing the consequent.*	*Multiplies the ratio.*
2. *Dividing the antecedent, or Multiplying the consequent.*	*Divides the ratio.*
3. *Multiplying or dividing both terms by the same quantity.*	*Does not alter the value of the ratio.*

378. The ratio, antecedent, and consequent are so related to each other, that if any two of them are given the other may be found. Thus:

Formulas.—
1. **The Ratio = Antecedent ÷ Consequent.**
2. **The Consequent = Antecedent ÷ Ratio.**
3. **The Antecedent = Consequent × Ratio.**

EXERCISE LXXXVIII.

What is the ratio:

1. Of 20 to 5?
2. Of 180 ft. to 63 ft?
3. Of 3 to 12?
4. Of 84 in. to 132 in?
5. Of 1.5 to 6.5?
6. Of 25.6 to 1.08?
7. Of 6. to .75?
8. Of 3.375 to 18.9?

When the terms are compound numbers, they must be reduced to the same denomination.

Find the value:

9. Of 3 gal.:2 qt.
10. Of 10 yards:9 meters.
11. Of 8 in.:2 ft.
12. Of 21 quarts:20 liters.
13. Of 3 s.:4 d.
14. Of 77 grains:5 grams.
15. Of 22 yd.:8 rd.
16. Of £3 6d.:15s. 3 far.
17. Of 15 s. to 4 s. 6 d.
18. Of 3 pk. 1 pt.:6 bu. 4 qt.

When the terms are fractions they may be changed into integers by multiplying both by the L. C. M. of their denominators, which *does not* change the ratio.

What is the value of:

19. $\frac{1}{2} : \frac{1}{3}$? **20.** $\frac{2}{3} : \frac{3}{4}$? **21.** $2\frac{1}{2} : 3\frac{1}{4}$?

22. $\frac{3}{15} : \frac{7}{25}$? **23.** $\frac{11}{49} : \frac{5}{63}$? **24.** $13\frac{1}{8} : 17\frac{1}{2}$?

25. How much more is 6 : 2 than 7 : 3?

26. How much more is 7 : 8 than 6 : 7?

Find the value of:

27. $(3 : 4) + (5 : 6)$. **28.** $(3, : 5) - (2\frac{1}{2} : 9)$.

29. $(2 : 6) \times (5 : 20)$. **30.** $(5 : 3) \div (1.2 : \frac{3}{4})$.

31. $(2.4 : 6.6) \times (3.5 : 8.40)$. **32.** $(\frac{24}{5} : \frac{36}{4}) \times (\frac{21}{8} : \frac{35}{6})$.

Reduce to simple ratios:

33. $\left\{\begin{array}{c} 3 : 4 \\ 2 : 9 \end{array}\right\}$ **34.** $\left\{\begin{array}{c} 2\frac{2}{3} : 2\frac{1}{4} \\ 1\frac{4}{5} : 1\frac{1}{3} \end{array}\right\}$. **35.** $\left\{\begin{array}{r} 6 : 8 \\ 20 : 21 \\ 14 : 25 \end{array}\right\}$

36. Antecedent $= 5\frac{1}{3}$, ratio $= \frac{4}{9}$, consequent $=$?

37. Consequent $= 3\frac{1}{2}$, ratio $= \frac{5}{8}$, antecedent $=$?

38. Consequent $= 3\frac{1}{3}$, antecedent $= \frac{3}{4}$, ratio $=$?

39. How much less is 3 : 5 than $(3 + 1) : (5 + 1)$?

40. How much more is 7 : 5 than $(7 + 3) : (5 + 3)$?

41. If 5 lb. of tea are worth as much as 11 lb. of coffee, and 7 lb. of coffee as much as 15 lb. of sugar, find the ratio of the value of tea to that of sugar.

PROPORTION.

379. Proportion is an equality of ratios.

(1) 8:4 = 6:3, and 12:2 = 30:5, are proportions.

380. The **sign** of proportion is the double colon (::), which is equivalent to =.

(1) 8:4 :: 6:3 is read, 8 is to 4 as 6 is to 3, and is equivalent to 8:4 = 6:3, which is read, the ratio of 8 to 4 is equal to the ratio of 6 to 3, or 8 contains 4 as many times as 6 contains 3.

The **terms** or **proportionals** are the four numbers compared; the **antecedents,** the first and third terms; the **consequents,** the second and fourth terms; the **extremes,** the first and fourth terms; and the **means,** the second and *third* terms.

(1) In 5:10 :: 3:6, 5, 10, 3 and 6 are the terms or proportionals; 5 and 3 the antecedents; 10 and 6 the consequents; 5 and 6 the extremes; and 10 and 3 the means.

381. When the second term is equal to the third it is said to be a **mean proportional** between the extremes.

(1) In 4:6 :: 6:9, 6 is a mean proportional between 4 and 9.

382.—PRINCIPLE. *In any proportion, the product of the extremes is equal to the product of the means.*

For, take any proportion, as .... $3 : 2 :: 9 : 6$,
which means .... $\frac{3}{2} = \frac{9}{6}$.
Multiplying both sides by 2×6........ $3 \times 6 = 2 \times 9$.

Hence, the relation of the four terms of a proportion to each other is such, that if any three of them are given, the other or missing term may be found thus:

To find either extreme, *Divide the product of the means by the other extreme.*

To find either mean, *Divide the product of the extremes by the other mean.*

Proportion may be simple or compound.

SIMPLE PROPORTION.

383. A **simple proportion** is an equality of two simple ratios.

384. ***To find the missing term of a simple proportion.***

(1) Find the value of x in $6 : 14 :: 9 : x$.

Explanation.—Since one of the extremes is missing I divide the product of the means by the given extreme, and obtain 21.

Operation.

$$x = \frac{14 \times 9}{6} = 21, \text{ Ans.}$$

(2) Find the value of x in $\frac{3}{5} : x :: 55 : 49\frac{1}{2}$.

Explanation. — Since one of the means is missing I divide the product of the extremes by the given mean, and obtain $\frac{3}{5}$.

Operation.

$$x = \frac{\frac{3}{5} \times 49\frac{1}{2}}{55} = \frac{3}{5}, \text{ Ans.}$$

EXERCISE LXXXIX.

Find the value of x in the following:

. $5 : 15 :: 3 : x$. 4. $15 : 100 :: 12 : x$.

. $12 : 3 :: x : 2$. 6. $85 : 120 :: x : 144$.

. $20 : x :: 15 : 3$. 8. $\$9.20 : x :: 161 : 217$.

. $x : 5 :: 2 : 7$. 10. $x : 3.2$ yd. $:: 5.04 : 62.5$.

. $\frac{1}{8} : \frac{2}{5} :: \frac{5}{16} : x$. 12. $\frac{3}{4} : \frac{5}{8} :: \$1\frac{1}{12} : x$.

3. $\frac{3}{5} : x :: 2 : \frac{10}{21}$. 14. $178\frac{1}{2}$ yd. : $29\frac{3}{4}$ yd $:: x : \$4\frac{1}{4}$.

5. $\frac{2}{3} : 5\frac{1}{4} :: 1\frac{1}{7} : x$. 16. $27\frac{6}{7} : x :: 22\frac{2}{7} : 28.8$.

Proportion is applied to the solution of problems which volve quantities that increase or decrease in the same ratio.

(17) If 28 yd. of cloth cost \$3.50, how much will 46 yd. ›st?

Statement: 28 yd. : 46 yd. :: \$3.50 : \$$x$.

Operation: $x = \frac{46 \times 3.50}{28} = 5.75$.

The cost of the cloth increases in the same ratio with the number yards; hence, the ratio of 28 yd. to 46 yd. is equal to the ratio of 3.50, the cost of 28 yd., to \$$x$, which represents the cost of 46 yd.

RULE.—I. *Make that term which has the same unit as the nswer the third term.*

II. *If the question requires the answer to be greater than e third term, make the greater of the two remaining numrs the second term; but if the answer ought to be less than e third term, make the least of the two numbers the second rm.*

III. *Divide the product of the second and third terms by e first, and the quotient will be the fourth term.*

18. If 3 yd. of cloth cost \$4.50, what will 5 yd. cost?

19. If 160 caps cost \$450, what will 840 caps cost?

20. If 4 hats cost \$12, how many hats can be bought for 27.

21. How many yards of linen may be bought for \$28.50, hen 6 yards cost \$4.50?

22. If 6 cloaks can be made from 27 yards of cloth, how many yards will be required for 20 cloaks?

23. If 2.75 yards of cloth are worth \$23.10, what is the value of 16.875 yards at the same rate?

24. If 4 boys can build a fence in 15 hours, how long will it take 6 boys to build it?

25. If 18 men can harvest a field of wheat in 21 days, how many men can harvest it in 14 days?

26. What number multiplied by 15 will give a product equal to 21 multiplied by 25?

27. There are 51 rows in an orchard and 60 trees in each row; how many trees would there be in each row if they were arranged in 68 rows?

28. How many yards of cambric, 30 inches wide, will be required to line 15 yards of silk, 20 inches wide?

29. How long will provisions sufficient for 475 men 56 days last 133 men?

30. If 6 men can do as much work as 10 boys, how many boys can do as much work as 15 men?

31. If $16\frac{1}{2}$ lb. of butter cost as much as $46\frac{1}{4}$ lb. of sugar, how many pounds of sugar will cost as much as $19\frac{7}{8}$ lb. of butter?

32. If I go a journey in $15\frac{5}{8}$ days, at 40 miles a day, how long would it take me at $33\frac{1}{3}$ miles a day?

33. An upright pole 6 feet high casts a shadow 9 feet long; what is the length of the shadow of a tree 81 feet high, at the same time?

34. The length of a shadow of a tree is 111 ft. 4 in., and the length of the shadow of 4 ft. 6 in. of the tree is 6 ft.; what is the height of the tree?

35. A grocer has a false quart measure, containing 1 pt. 3 gi.; what is the real worth of the liquor that he sells for \$16?

36. If he uses $14\frac{5}{8}$ oz. for a pound, how much does he cheat by selling coffee for \$25.60?

37. If $\frac{3}{4}$ of a load of potatoes is worth \$7.80, how much is $\frac{10}{13}$ of the load worth?

38. If $\frac{3}{8}$ of $\frac{5}{7}$ of a lot of land cost $\frac{11}{14}$ of $\$157\frac{1}{2}$, what will $\frac{1}{3}$ of $\frac{5}{22}$ of the lot cost?

39. The ages of three boys are proportional to $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$; if the youngest is 12 yr. old, how old are the other two?

40. The property of A is assessed at \$6430; of B, \$5250; and of C, \$2040.50. If A's taxes are \$22.505, what are the taxes of B and C?

41. If $1\frac{1}{4}$ lb. of beef and $1\frac{3}{8}$ lb. of flour are allowed for a ration, how much will 560 rations cost if the price of beef is $11\frac{3}{8}$ cts. and of flour $3\frac{1}{4}$ cts. a pound?

42. A's land is $\frac{1}{6}$ more in quantity than B's, but B's is $\frac{1}{5}$ better in quality; what is B's land worth if A's is worth \$1750?

COMPOUND PROPORTION.

385. **A compound proportion** is an equality of two ratios, one or both of which are compound.

(1) $\left\{\begin{matrix} 3 : 4 \\ 5 : 6 \end{matrix}\right\} :: 10 : 16$, is a compound proportion, of which 3 and 5 are the factors of the first term, and 4 and 6 the factors of the second term.

386. ***To find the missing term of a compound proportion.***

Find the missing term, denoted by x, in the following:

(1) $\left\{\begin{matrix} 2 : 5 \\ 3 : 4 \end{matrix}\right\} :: 9 : x. \qquad x = \frac{5 \times 4 \times 9}{2 \times 3} = 30$, Ans.

(2) $\left\{\begin{matrix} 3 : 5 \\ 2 : x \end{matrix}\right\} :: \left\{\begin{matrix} 4 : 21 \\ 7 : 20 \end{matrix}\right\}. \qquad x = \frac{3 \times 2 \times 21 \times 20}{5 \times 4 \times 7} = 18$, Ans.

EXERCISE XC.

Find the missing term in the following;

3. $\left\{\begin{matrix} 2 : 3 \\ 4 : 5 \end{matrix}\right\} :: 24 : x.$ 4. $\left\{\begin{matrix} 3 : 4 \\ 5 : 6 \end{matrix}\right\} :: x : 48.$

5. $\left\{\begin{matrix} 2 : 5 \\ 3 : x \end{matrix}\right\} :: \left\{\begin{matrix} 3 : 14 \\ 7 : 15 \end{matrix}\right\}.$ 6. $\left\{\begin{matrix} 5 : 8 \\ x : 10 \end{matrix}\right\} :: \left\{\begin{matrix} 14 : 6 \\ 9 : 4 \end{matrix}\right\}.$

7. $\left\{\begin{matrix} 9 : 15 \\ 4 : x \\ 10 : 12 \end{matrix}\right\} :: 8 : 12.$ 8. $\left\{\begin{matrix} 1\frac{3}{4} : 5\frac{1}{4} \\ 12 : 5\frac{1}{3} \end{matrix}\right\} :: \left\{\begin{matrix} 6 : 8 \\ 10\frac{2}{3} : x \\ 3\frac{3}{4} : 10 \end{matrix}\right\}.$

(9) If 16 horses eat 96 bu. of oats in 42 da., in how many days will 7 horses eat 66 bu.?

NOTE.—All the terms of every problem in compound proportion appear in couplets, except one, and this always has the same unit or denomination as the required term. Thus, in the preceding example, denoting the required term by x, the couplets are 16 horses and 7 horses, 96 bu. and 66 bu., and 42 da. and x da.

Explanation.—Since the unit of the required term is days, I make the third term 42 da.

Statement.

$$\left\{\begin{matrix} 7 \text{ horses} : 16 \text{ horses} \\ 96 \text{ bu} : 66 \text{ bu.} \end{matrix}\right\} :: 42 \text{ da.} : x \text{ da.}$$

$$x = 66.$$

In forming the first and second terms I consider each couplet separately, just as if the answer depended on that couplet. Thus:

1°. Since 7 horses require more time to eat the oats than 16 horses the first ratio is 7 horses : 16 horses, the greater number being in the second term.

2°. Since it takes the horses 42 da. to eat 96 bu. it will take them less time to eat 66 bu.; hence, the second ratio is 96 bu. : 66 bu., the less number being in the second term.

RULE I.—*Make the number, like the answer, the third term.*

II. *With this third term and each remaining couplet form a proportion as if the answer depended solely on that proportion.*

III. *Multiply the third term by the product of all the*

second terms, and divide the product by the product of all the first terms.

10. If 10 men earn $90 in 4 da., how much will 16 men earn in 5 da.?

11. If 35 men earn $462 in 6 da., in how many days can 55 men earn $484?

12. If 18 men earn $273 in 7 da., how many men can earn $260 in 12 da.?

13. If 36 horses in 5 da. eat $75 worth of oats, how much will it cost to feed 32 horses on oats 9 da.?

14. If it cost $21.70 to carry 1240 lb. of freight 125 mi., how much should be paid for carrying 1750 lb. 97 mi.?

15. If 9 men can cut 36 acres of grass in 14 da., how many acres will 19 men cut in 11 da.?

16. How many days will 1500 lb. of beef serve 120 men, if 300 lb. serve 40 men 15 da.

17. If 32 men build a wall 36 feet long, 8 feet high and 4 feet wide, in 4 days, in what time will 48 men build a wall 864 feet long, 6 feet high, and 3 feet wide?

18. If 12 men mow 25 acres of grass in 2 da. of $10\frac{1}{2}$ hr., how many men will it require to mow 80 acres in 6 da. of $9\frac{3}{5}$ hr.?

19. If 5 lb. of yarn will make $16\frac{2}{3}$ yd. of cloth, $1\frac{1}{2}$ yd. wide, how many pounds will be required to make a piece 100 yd. long and $1\frac{1}{4}$ yd. wide?

20. How many days of 7 hr. each will it take a man to travel 390 mi., if he goes 130 mi. in 3 da. of 14 hr. each?

21. If 9 men can dig a drain $26\frac{2}{3}$ rd. long, 6 ft. deep, and 4 ft. wide, in 16 da. of $10\frac{4}{5}$ hr. each, how many men will be required to dig a drain 400 rd. long, $6\frac{2}{5}$ ft. deep and 5 ft. wide, in 108 da. of 8 hr. each?

22. If 15 men can perform a piece of work in 24 da., how many men can perform another piece of work 9 times as great in $\frac{1}{5}$ of the time?

23. In how many days, of 10 hr. each, will 75 masons build a wall 500 ft. long, 3 ft. thick, $21\frac{1}{3}$ ft. high, if 20 masons build one 45 ft. long, 2 ft. thick, and $15\frac{5}{8}$ ft. high in 15 da., of 7 hr. each?

CAUSE AND EFFECT.

387. In many problems the quantities may be classified into **causes** and **effects**, and treated according to the following

PRINCIPLES.—I. *Like causes produce like effects.*

II. *The ratio of the effects is equal to the ratio of the causes which produce them.*

388. ***To find the unknown cause or effect.***

(1) If 5 men earn $155 in a week, how much will 9 men earn in the same time?

First Cause.		Second Cause.		First Effect.		Second Effect.
5	:	9	::	$155	:	$x

$$\begin{array}{r|l} 5 & 9 \\ & \\ x & 155 \end{array} \qquad x = 279.$$

Explanation.—5 men working a certain time is a cause that produces the effect of earning $155. These are the first cause and first effect.

Again, the work of 9 men is a second cause, which produces an unknown effect, denoted by x. The solution of the problem consists in finding what this effect will be, on the principle that effects are in the same ratio as their causes.

(2) If 6 men, working 20 da. of 8 hr. each, can saw 340 cords of wood, how long will it take 16 men working 9 hr. a day to saw 204 cords?

$$\begin{array}{cccc} \text{First Cause.} & \text{Second Cause.} & \text{First Effect.} & \text{Second Effect.} \\ \left\{\begin{array}{ccc} 6 & : & 16 \\ 20 & : & x \\ 8 & : & 9 \end{array}\right\} & & :: \ 340 & : \ 204 \end{array}$$

Explanation.—The first cause is 6 men laboring 20 da. of 8 hr. each, and the first effect is the sawing of *340 cords* of wood.

The second cause is 16 men laboring an unknown number of days (denoted by x) of 9 hr. each, and the second effect is the sawing of 204 cords of wood.

$$\begin{array}{r|l} 6 & 16 \\ 20 & x \\ 8 & 9 \\ 204 & 340 \end{array} \qquad x = 4.$$

Arranging these terms as in the margin, and canceling, gives 4 days as the missing element of the second cause.

RULE.—*Denote the required number by x; draw a vertical line; on the left place the first cause and second effect; on the right, the second cause and first effect, and proceed according to Art.* 170.

EXERCISE XCI.

3. If 5 lb. beef cost 42 cts. what will $3\frac{1}{3}$ lb. cost?

4. If 5 men can build a wall in 21 days, how many men will be required to build it in 7 days?

First cause, 5 men, 21 days; second cause, x men, 7 days; first effect, 1 wall; second effect, 1 wall.

5. How long will it take 15 men to perform a work which 21 men can do in 35 days?

6. How many yards of paper, $2\frac{1}{2}$ feet wide, will cover a wall 12 feet long and 9 feet high?

7. How many planks 10 feet long and 9 inches wide will floor a room which requires 125 planks 12 feet long and 6 inches wide?

8. How much would $9\frac{3}{5}$ yards of flannel cost, if 41 yards cost \$16.40?

9. What cost $50\frac{1}{4}$ yards of cloth at the rate of $5\frac{1}{2}$ yards for $\$27\frac{1}{2}$?

10. What cost a silver pitcher weighing 9 oz. 13 pwt. 8 gr., at the rate of 7 s. 6 d. per ounce?

11. If it cost \$91 to shingle a roof 39 feet long and 21 feet wide, what will it cost to shingle a roof 35 feet long and 18 feet wide?

12. If it cost \$120 to build a wall 40 feet long, $1\frac{1}{2}$ feet thick and 14 feet high, what will be the thickness of a wall 180 feet long and 21 feet high, that can be built for \$675?

13. A man borrows \$1200, and keeps it 2 yr. 5 mo. 5 da.; how long should he lend \$1750 to compensate for the favor?

14. If \$100 gain \$6 in 12 mo., how much will \$340.20 gain in 2 yr. 8 mo. 12 da.?

15. If \$100 gain \$20 in 12 mo., how long will it take \$615.38 to gain \$607.17?

16. How much will \$100 gain in 12 mo. if \$540 gain \$27.09 in 8 mo. 18 da.?

17. If \$100 gain \$8 in 12 mo., what sum will gain \$106.40 in 2 yr. 4 mo. 15 da.

18. If \$240 gain \$23.04 from Feb. 15, 1878, to April 27, 1879, how much will \$252 gain from August 2, 1877, to March 9, 1878?

SPECIFIC GRAVITY.

389. The **specific gravity** of a solid or liquid substance is the ratio of the weight of the substance to that of an equal volume of water.

(1) A cu. ft. of water weighs 62.5 lb., or 1000 oz., and a cu. ft. of limestone weighs 2950 oz.; hence, the specific gravity of limestone is 2950 ÷ 1000, or 2.95.

Illustration of Principles.—I weigh a stone suspended by a thread; it weighs 3.125 lb. I then slip a pail of water under it, and raise the pail until the stone is covered with water; it now weighs $1.171\frac{7}{8}$ lb., and has lost $1.953\frac{1}{8}$ lb. From this I learn:

(1) A bulk of water equal to that of the stone weighs $1.953\frac{1}{8}$ lb. For, when a substance is under water the water buoys it up just the amount of the weight of the water displaced by it.

(2) The specific gravity of the stone is 3.125 lb. ÷ $1.953\frac{1}{8}$ lb. or 1.6.

(3) Since a cu. ft. of water weighs 1000 oz., 1 oz. of water occupies 1.728 cu. in., and 1 lb., 27.648 cu. in.; hence, $1.953\frac{1}{8}$ lb. occupy $1.953\frac{1}{8} \times 27.648$ cu. in., or 54 cu. in. That is, *the volume of the stone is 54 cu. in.*

NOTES.—1. For a table of the specific gravities of some well known substances, including those mentioned in subsequent problems, see Appendix, Art. 489.

2. The specific gravity of a substance multiplied by 1000 expresses the weight of a cu. ft. of the substance in ounces.

(1) What is the weight of a cubic block of cast iron, each edge being 2 ft.

Solution.—$2 \times 2 \times 2 = 8$, number cu. ft. in the block.
$8 \times 7207 = 57656$, number oz. in the block.
57656 oz., $\div$ 16, $= 3603\frac{1}{2}$ ℔., Ans.

(2) Find the solid contents of a piece of gold which weighs 1200 ℔.

Solution.—1200 ℔. = 19200 oz.; weight of one cu. foot of gold = 19258 oz.
$19200 \div 19258 = .99 +$ cu. ft., Ans.

(3) A piece of wood, weighing 12 oz. in air, is fastened to a piece of metal which weighs 15 oz. in air and $13\frac{3}{4}$ oz. in water; and both are found to weigh, under water, $9\frac{3}{4}$ oz.; find the specific gravity of the wood.

Solution.—Both lose, in water, $27 - 9\frac{3}{4} = 17\frac{1}{4}$ (oz.)
The lead alone loses $1\frac{1}{4}$ "
Weight of water displaced by the wood, 16 "
Hence, $12 \div 16 = .75$, the specific gravity of the wood.

EXERCISE XCII.

What is the specific gravity of a substance:

4. Which weighs 15 ℔. in air and 12 ℔. in water?

5. Which weighs 12 ℔. in air and 3 ℔. in water?

6. Which weighs $6\frac{1}{2}$ oz. in air and $5\frac{5}{8}$ oz. in water?

Find the volume of a substance:

7. Which loses 8 ℔. when weighed in water?

8. Which loses 4.25 ℔. when weighed in water?

9. Which loses 15 oz. when weighed in water?

What is the weight of a rectangular:

10. Block *of slate 5 in.* by 8 in. by 12 in.?

11. Block of maple 8 ft. by 1 ft. by 6 in.?

12. Block of marble 5 ft. by 3 ft. by 2 ft.?

13. Find the weight of a gallon of pure milk.

14. A poplar rail is floating on the water; what part of it is above the water?

15. A piece of walnut is floating on the water; what part of it is beneath the water?

16. A body weighs, in air, 600 grains; when attached to a piece of copper the combination weighs 2647 grains in air, and loses, when weighed in water, 834 grains. The copper alone loses 230 grains; required the specific gravity of the body.

17. Wishing to ascertain the exact number of cubic inches in a very irregular fragment of stone, I ascertained its loss of weight in water to be 5.346 oz.; required its size.

18. A dry cypress log containing 40 cu. ft. is floating on the water; find the largest number of boys, weighing 67 pounds each, it will support without sinking.

PRORATA DIVISION.

390. Prorata division is dividing a number into parts having a given ratio to each other.

391. ***To perform prorata division.***

(1) Divide 45 into 3 parts proportional to 2, 3 and 4.

Explanation.—Since multiplying two or more numbers by the same number does not change their ratio, I multiply the given proportionals by 5, because 9, the sum of the proportionals, is contained 5 times in 45, the given number, and obtain 10, 15, 20, Ans.

Operation.

$$\begin{array}{r} 2 \times 5 = 10 \\ 3 \times 5 = 15 \\ \underline{4} \times 5 = \underline{20} \\ 9 \times 5 = 45 \end{array}$$

The parts may also be found by the proportions:

$$\left\{\begin{array}{l} 9 : 2 :: 45 : 10, \text{ 1st part.} \\ 9 : 3 :: 45 : 15, \text{ 2d } " \\ 9 : 4 :: 45 : 20, \text{ 3d } " \end{array}\right.$$

RULE.—I. *Make the number to be divided the third term; each proportional part successively the second term; and their sum the first.*

II. *The product of the second and third terms of each proportion, divided by the first, will be the corresponding part required.*

NOTE.—If the given proportionals are fractions, convert them into integers by multiplying each of them by the L. C. M. of all the denominators, then proceed with the integers as above.

EXERCISE XCIII.

2. Divide 35 into 2 parts proportional to 3 and 4.

3. Divide 228 into 2 parts proportional to 8 and 11.

4. Divide 32 into 2 parts proportional to $\frac{1}{3}$ and $\frac{1}{5}$.

5. Divide 151.7 into 2 parts proportional to $\frac{3}{4}$ and $\frac{5}{7}$.

6. A basket contains 60 apples, of which there are 2 bad to every 3 good ones; how many of each are there?

7. In counting 825 balls I observed that for every 7 white balls, there were 11 black and 15 red balls; how many of each were there?

8. The ages of three boys are to each other as $\frac{1}{2}$. $\frac{1}{3}$, $\frac{1}{4}$; if $65 be divided among them in proportion to their ages, how much will each receive?

9. A farmer raised 945 bushels of grain, composed of oats, wheat and corn in the proportion of $3\frac{1}{4}$, $4\frac{2}{3}$ and $7\frac{5}{6}$; how many bushels were there of each kind?

10. Divide 54 cts. into 2 parts such that the greater shall be 5 times the less.

11. A, B and C own 108 horses, of which B owns 2 times as many as A, and C 3 times as many as B; how many does each own?

12. A, B and C build a bridge for $300; how much should each pay if A is worth $3240; B, $4160, and C, $3600?

PARTNERSHIP.

392. **Partnership** is the association of two or more persons for the transaction of business.

The association is called a firm, house or company; and the persons associated, partners.

The capital, joint stock, or stock in trade is the money or property furnished by the partners; the assets are the various kinds of property which the firm possesses; the liabilities are the debts of the firm; and the net capital is the excess of the property above the liabilities.

Partnership may be simple or compound.

Simple Partnership.

393. Partnership is **simple** when the capital of the partners is invested for the same time.

394. ***To prorate or apportion the profit or loss when the partnership is simple.***

(1) A and B formed a partnership; A furnished $500 and B $400; they gained $252. What was each man's share of the profits?

By Analysis.—$\$500 + \$400 = \$900$, the whole capital. Hence, A had $\frac{500}{900}$, or $\frac{5}{9}$ of the capital, and should have $\frac{5}{9}$ of the profits; $\frac{5}{9}$ of $\$252 = \$140$, A's share of the profits.

B had $\frac{400}{900}$, or $\frac{4}{9}$ of the capital, and should have $\frac{4}{9}$ of the profits: $\frac{4}{9}$ of $\$252 = \$112$, B's share of the profits.

By Percentage.—The gain, $252, is $\frac{252}{900} = \frac{28}{100} = 28\%$ of the whole capital. Therefore, $\$500 \times .28 = \$140$, A's share; and $\$400 \times .28 = \$112$, B's share.

By Proportion.—$\$500 + \$400 = \$900$.
$\$900 : \$252 :: \$500 :$ A's gain, $= \$140$.
$\$900 : \$252 :: \$400 :$ B's gain, $= \$112$.

RULE.—*Take such a part of the gain or loss as each partner's stock is of the whole capital.* Or,

Find the per cent which the profit or loss is of the whole capital, and multiply each man's capital by it.

EXERCISE XCIV.

2. A and B formed a partnership; A put in \$300 and B \$500; they gained \$160; what was each man's share of the gain?

3. A, B and C are partners; A furnishes \$1560; B, \$1730, and C \$1810; they lose \$255; what is each man's share of the loss?

4. The capital of two partners is proportional to 5 and 7, and their profits are \$540; what is the share of each?

5. The capital of four partners are proportional to 1.75, 2.24, 2.51 and 3.2, and their profits are \$58.20; find each partner's share?

6. A and B owned two bales of cotton, which they sold at a gain of \$1.80 each; A owned $\frac{2}{3}$ of one bale, and B owned $\frac{5}{8}$ of the other; find each man's share of the entire profits.

7. A and B traded in partnership two years, making an annual profit of \$3240; during the first year A owned $42\frac{6}{7}\%$ of the stock, and during the second year B owned $55\frac{5}{9}\%$ of it; what is each partner's share of the total profits?

Compound Partnership.

395. Partnership is **compound** when the capital of the partners is invested for unequal times.

396. ***To prorate the profit or loss when the partnership is compound.***

(1) A and B enter into partnership; A furnishes \$300 for 8 months, and B \$400 for 9 months; they gain \$85; what is each one's share of the profit?

Solution.

$\$300 \times 8 = \$2400.$ $\frac{2400}{6000} = \frac{2}{5}$; $\frac{2}{5}$ of \$85 = \$34, A's share.

$\$400 \times 9 = \underline{\$3600.}$ $\frac{3600}{6000} = \frac{3}{5}$; $\frac{3}{5}$ of \$85 = \$51, B's share.

\$6000.

Explanation.—The use of \$300 for 8 mo. = the use of 8 × \$300, or \$2400 for 1 mo., and the use of \$400 for 9 mo. = the use of 9 × \$400, or \$3600 for 1 mo. The respective capitals, then, are equivalent to \$2400 and \$3600, employed for the same time. Hence, I divide the profit proportional to these sums as in simple partnership.

RULE.—*Multiply each partner's capital by the number of time units it is employed; consider these products as their respective capitals, and proceed as in simple partnership.*

EXERCISE XCV.

2. Two merchants traded in company; A put in \$215 for 6 months, and B \$390 for 9 months, but by misfortune they lost \$200; what will each lose?

3. Three men, A, B and C, trade in company on the following terms: A puts in \$300 for 5 months; B \$400 for 8 months, and C \$500 for 3 months; and they gained \$100; what is each man's share of the gain?

4. J. W. Jones commenced business January 1, with a capital of \$3200; and May 1, he took Wm. Smith into partnership, with a capital of \$4200; and at the end of the year they gained \$240; what was each man's share of the gain?

5. The amounts invested by two partners are proportional to 9 and 12; the capital of the first is in trade $5\frac{3}{4}$ months, and that of the latter $6\frac{3}{4}$ months; if they gain \$356.40, what will be the share of each?

6. Two men, A and B, hire a pasture for \$91.05; A puts in 450 sheep for 7 weeks, and B 365 sheep for 8 weeks; what should each pay?

7. F, G and H are partners for a year. F puts in \$7300 for 3 months, and then adds \$2700; G puts in \$9200 for 5 months, and then withdraws \$4200; H puts in \$8500 for 8 months, and then withdraws \$3500; apportion a gain of \$5618.

8. Two partners begin business, each with a capital of $3000. A adds $600 at the end of 3 months, and $800 more at the end of 7 months; B adds $1000 at the end of 5 months; what is the share of each, at the year's end, of a profit of $4420?

9. A and B are partners; A's stock is 15% more than B's, but B's is 20% longer in the business; they gain $1410; find the share of each.

10. A begins business with $6000; at the end of 6 months he takes in B, and 6 months after, their gain is $3300, of which A's share was $1800; find B's capital.

11. A and B are partners, and their capital is in the ratio of 4 to 5; after 3 months A withdraws $\frac{2}{3}$ of his and B a *part* of his, and at the year's end A receives $800 gain, and B $875; what part of his capital did B withdraw?

12. A and B join capitals, which are as $\frac{1}{2}$ and $\frac{1}{3}$; after 4 months A sells B a *part* of his capital, and at the end of the year each receives the same gain; what part of A's capital did B buy?

13. A, B and C entered into business as partners, A and B each putting in $5000, and C a certain sum; at the end of two years A took out $1000, B $2000, and C $3000, and at the end of the fourth year their loss was $3600, of which C's part was $1050; find A's and B's share of the loss, and C's original capital.

AVERAGES.

397. An **average** is a medial value or quality made out of unequal quantities having unequal values or qualities. See Art. 92.

AVERAGE OF MIXTURES.

398. ***To find the average value of a mixture when the quantity and price of each article are given.***

(1) If a mixture be made of 9 lb. of candy worth 12 cts. a pound, 7 lb. at 16 cts., and 4 lb. at 20 cts., what will 1 lb. of it be worth?

Solution:	9 lb. of candy	@ 12 cts.	= \$1.08
	7 lb. " "	@ 16 cts.	= 1.12
	4 lb. " "	@ 20 cts.	= .80
Hence,	20 lb. of the mixture		= \$3.00
and	1 lb. " " "		= $\frac{1}{20}$ of \$3.00 = 15 cts.

RULE.—*Find the value of each article, and divide the total value by the number of articles.*

NOTES.—1. If an article cost nothing, as water, its value is 0; but the quantity must be added to the other articles.

2. This process is often called Alligation Medial.

EXERCISE XCVI.

2. A farmer mixed 24 bu. oats, worth 35 cts. a bushel, with 17 bu. corn @ 55 cts., and 9 bu. of rye @ 45 cts.; what was the mixture worth per bushel?

3. A man bought 20 acres of land @ \$40, 30 acres @ \$60, 40 acres @ \$32$\frac{1}{2}$, and 60 acres @ \$38; what was the average price per acre?

4. If 7 gal. alcohol, worth \$1.20 per gal., and 5 gal. @ \$1.40, are mixed with 4 gal. water, what is a gallon of the mixture worth?

5. A man rode 5 hr. at the rate of 4$\frac{1}{2}$ mi. per hr., 4 hr. at the rate of 4$\frac{1}{8}$ mi., and 3 hr. at 5 mi. per hour; what was his average rate per hour?

6. If 9 gal. of a mixture containing 80% of whisky, and 6 gal. of a mixture containing 95% of whisky are mixed with 10 gal. of water, what per cent of whisky will the mixture contain?

Alligation Alternate.

399. ***To find the proportional parts of a mixture, the average price and the price of each ingredient being given.***

(1) A grocer desires to mix 4 kinds of sugars, worth 5, 7, 10 and 12 cents a pound, so that the mixture should be worth 9 cents a pound; how much of each must be taken?

Operation.

	Col.	1	2	3
9	5	$\frac{1}{4}$		3
	7		$\frac{1}{2}$	1
	10		1	2
	12	$\frac{1}{3}$		4

Explanation.—In selling the sugars which are worth less than 9 cts. for 9 cts., there will be a gain; and in selling those which are worth more than 9 cts. for 9 cts., there will be a loss. Now, I desire to take such amounts of each that the *gain* will balance the *loss*. On 1 lb. of the 5-ct. sugar there will be a gain of 4 cts.; hence, on $\frac{1}{4}$ of a lb. there will be a gain of 1 ct. On 1 lb. of the 12-ct. sugar there will be a loss of 3 cts., hence, on $\frac{1}{3}$ of a lb. there will be a loss of 1 ct. That is, if I take $\frac{1}{4}$ of a lb. of the 5-ct. sugar as many times as I take $\frac{1}{3}$ of a lb. of the 12-ct. sugar, the gain and loss will be exactly equal. Hence, I write $\frac{1}{4}$ and $\frac{1}{3}$ in the first column for the proportional amounts to be taken of the sugars whose prices they stand opposite.

In the same manner I find the proportional amounts $\frac{1}{2}$ and 1 for the 7-ct. and 10-ct. sugars, which I write in the second column.

Now, since I may take $\frac{1}{3}$ any number of times, provided I also take $\frac{1}{4}$ the same number of times, I may multiply them by 12, the L. C. M. of their denominators, and thus reduce them to 4 and 3, which I write in the third column. Similarly multiplying $\frac{1}{2}$ and 1 by 2, I obtain 1 and 2, which I write in the third column. Hence, the required proportional amounts are 3 lb. of the 5-ct. sugar, 1 lb. of the 7-ct., 2 lb. of the 10-ct., and 4 lb. of the 12-ct.

It is evident that problems of this kind may have an unlimited number of sets of answers; for any multiple of the proportional numbers found also expresses the proportional quantities to be taken of each.

RULE.—I. *Write the several prices or qualities in a column, and the average price or quality at the left.*

II. *Form couplets by comparing any price or quality less, with one that is greater than the average rate, and place the reciprocal of the difference between the price and average, opposite the price.*

III. *Multiply the fractional terms of each of the couplets by the L. C. M. of their denominators, and the products will be the proportional numbers required.*

EXERCISE XCVII.

2. Dried fruits worth 9, 12, 15, and 18 cts. a pound are combined in a mixture worth 14 cts. a pound; find the proportional weights of the different kinds.

3. In what proportions must syrups, worth 40, 45, 50, and 64 cts. per gallon, be mixed that the mixture may have an average value of 48 cts. per gallon?

4. A mixture of sugars worth 6, 7, $7\frac{1}{2}$, 9, $9\frac{1}{3}$, and 11 cts. a pound is sold at 8 cts. a pound; what proportion of the several kinds in the mixture?

5. 5 times the sum of four numbers is equal to 2 times the first plus 3 times the second plus 6 times the third plus 9 times the fourth; find the numbers.

6. A farmer has hogs worth $10, $6, and $4 a head; what number of each should he sell to obtain an average price of $8 a head?

As *one* of the values is above the average, and *two* below, I repeat the former so as to compare it with each of the latter.

8	10	$\frac{1}{2}$		2		2	
	10		$\frac{1}{2}$		1	1	3
	6		$\frac{1}{2}$		1	1	1
	4	$\frac{1}{4}$		1		1	1

7. A grocer has soda worth 12, 15, and 20 cts. a pound; how much of each must he take to form a mixture worth 18 cts. a pound?

8. In what proportional parts must gold respectively 14, 18, and 24 carats fine be combined to make a metal 22 carats fine?

9. A merchant sold coffees at \$.18, \$.21, \$.27, and \$.35 a pound; what proportional amounts did he sell if the average price received was \$.30 a pound?

10. Find four numbers such that 15 times their sum is equal to the sum of 7 times the first, 11 times the second, 13 times the third, and 18 times the fourth.

11. How much water and how much whisky worth \$.90, \$.95, and \$1.15 a gallon are required to form a mixture worth \$.50 a gallon?

(12) A grocer has sugars worth 5, 6, 9, and 12 cts. a pound: how much of each must he take to form a mixture of 60 pounds, worth 8 cts. a pound?

		1	2	3	4	
8	5	$\frac{1}{3}$		4		$4 \times 6 = 24$
	6		$\frac{1}{2}$		1	$1 \times 6 = 6$
	9		1		2	$2 \times 6 = 12$
	12	$\frac{1}{4}$		3		$3 \times 6 = 18$
						10)60(6

I find the proportionals, 4, 1, 2, 3, as before. Adding these, I find they would form a mixture of 10 ℔. But since the required mixture is 60 ℔., or 6 times 10 ℔., I multiply each of the proportionals by 6, and obtain 24 ℔. at 5 cts., 6 ℔. at 6 cts., 12 ℔. at 9 cts., and 18 ℔. at 12 cts.

If the quantity of the whole compound is not exactly divisible by the sum of the proportionals, it may often be made divisible by taking certain multiples of the numbers in columns 3, 4, etc., as in the next example.

(13) A paid \$100 for 100 head of hogs, pigs and chickens. The hogs cost \$9 each, the pigs \$4 each, and the chickens \$$\frac{1}{2}$ each; how many of each did he buy?

Solution: The average price is \$100 ÷ 100, or \$1.

		1	2	3	4		5	6	8	
1	9	$\frac{1}{8}$		1		1	3		3	3 hogs
	4		$\frac{1}{3}$		1	1		7	7	7 pigs
	$\frac{1}{2}$		2		6	6		42	42	90 chickens.
	$\frac{1}{2}$	2		16		16	48		48	
						24			100	

I obtain the proportionals 1, 1, 6, 16 as before; but their sum, 24, is not an exact divisor of 100. Hence, I change the proportionals by multiplying the numbers in the third column by 3, and those in the fourth column by 7, writing the results in columns 5 and 6 respectively; this gives the proportionals in column 8, whose sum is an exact divisor of 100.

14. A grocer mixed 4 kinds of tea, worth 5 s., 8 s., 11 s., and 12 s. a pound, taking 40 pounds in all, worth 9 s. a pound; how much of each kind did he take?

15. How much candy at 9, 11 and 14 cts. a pound, will form a mixture of 56 pounds, worth 12 cts. a pound?

16. How much ginger at 15, 18 and 25 cts. a pound, must be taken to form a mixture of 155 ℔., worth 21 cts. a pound.

17. A goldsmith would mix gold of 14, 15, 18 and 24 carats fine so as to produce a ring weighing 4 pwt., 20 carats fine; how much of each should he take?

18. How much brandy, whisky, rum and water, at $1.10, $.80, $.30 and $0 per gallon, respectively, must be taken to form a mixture of 100 gallons, worth $100?

19. A has $1000 at interest at 8%, which he wishes to divide into 3 parts and place at interest at 5, 7, and 10 per cent, respectively, so that the interest shall remain the same; find the three parts.

20. The use of $2200 for $4\frac{1}{4}$ months is equivalent to the use of three sums for $2\frac{1}{2}$, $3\frac{1}{3}$, and $5\frac{1}{5}$ months, respectively; find the three sums.

EQUATION OF PAYMENTS.

400. **Equation of payments** is the process of finding the time when two or more debts, due at different times, may be paid without loss to either debtor or creditor.

The time of payment is called the **equated time**; the **term of credit** is the time to elapse before a debt is due; and the **average term of credit** is the time to elapse before several debts, due at different times, may be equitably paid together.

401. ***To find the average term of credit and the equated time of several debts.***

(1) A owes B \$400 due in 5 months, and \$500 due in 7 months; find the average term of credit.

Explanation.	Operation.
Explanation.—The use of \$400 for 5 months = the use of \$2000 for 1 month; and the use of \$500 for 7 months = the use of \$3500 for 1 month. Hence, the entire debt = the use of \$5500 for 1 month, which = the use of \$900	$400 \times 5 = 2000$ $500 \times 7 = 3500$ $900 \quad)5500$ $6\frac{1}{9}$

for as many months as 900 is contained times in 5500, or $6\frac{1}{9}$ months, Ans.

(2) Find the equated time for paying \$400 due June 7, \$500 due July 17, and \$600 due Oct. 5.

Solution.

\$ 400 × 0 =	0,	(0 da. from June 7 to June 7.)	
500 × 40 =	20000	(40 da. " " 7 to July 17.)	
600 × 120 =	72000	(120 da. " " 7 to Oct. 5.)	
\$1500	)92000	61 da. " " 7, is Aug. 7, Ans.	
	$61\frac{1}{3}$		

Explanation.—I find the time that each debt has to run, counting from the earliest date when a debt is due, and multiply each debt by its term of credit thus found. I next divide the sum of the products by the sum of the debts, and the quotient is the average term of credit. Finally, I find the equated time by adding the average term of credit to the earliest date when a debt is due.

NOTES.—1. The same result is obtained by finding the interest at any convenient rate on each debt for its term of credit as found by the rule, and dividing the sum of the interest by the interest on the sum of the debts for one day, if the terms of credit were expressed in days, or for one month, if in months.

2. When the equated time contains a fraction of a day, it is called 1 day, if equal to ½ a day or more; if less, it is disregarded.

3. The equated time may be found by using the latest date instead of the earliest, and counting the time backward instead of forward.

EXERCISE XCVIII.

3. What is the average term of credit of \$500 due in 2 mo., \$600 due in 5 mo., and \$900 due in 7 mo.?

4. A person owes \$3600, of which $\frac{1}{3}$ is due in 4 mo., $\frac{1}{4}$ in 6 mo., and the remainder in 8 mo.; what is the average term of credit?

5. A grocer owes \$4500, to be paid $\frac{1}{3}$ in 5 mo., $\frac{1}{6}$ in 10 mo., $\frac{1}{9}$ in 18 mo., and the remainder in 20 mo.; what is the average term of credit?

6. Bought a bill of goods Jan. 15, 1888, amounting to \$1500, of which $\frac{1}{5}$ is due in 30 da., $\frac{1}{4}$ in 60 da., $\frac{3}{10}$ in 90 da., and the remainder in 120 da.; find the average term of credit and the equated time.

7. A owes \$600, \$900, and \$1500, due in 30, 60 and 90 days respectively, from June 15, 1885; find the equated time of settlement.

8. Bought a bill of goods Jan. 1, 1886, of which \$3000 are due in 9 mo., \$5000 in 12 mo., and \$8000 in 15 mo.; what is the equated time of payment?

(9) A owed \$1000 due in 8 mo., of which he paid \$300 at the end of 3 mo., \$500 at the end of 6 mo., and then gave his note for the remaining \$200; for what time was the note drawn?

Explanation.—At the end of 8 months A had evidently lost the use of \$300 for five months, and the use of \$500 for 2 months, or the use of \$2500 for 1 month; hence, to balance this loss, he should have the use of \$200 for as many months as 200 is contained times in 2500, or $12\frac{1}{2}$ months. Therefore, the note was drawn for 2 mo. + $12\frac{1}{2}$ mo. = $14\frac{1}{2}$ mo.

Operation.

$$300 \times 5 = 1500$$
$$500 \times 2 = 1000$$
$$2500 \div 200 = 12\tfrac{1}{2}$$
$$12\tfrac{1}{2} + 2 = 14\tfrac{1}{2}.$$

10. I owe \$912 and \$500; if I pay the first 15 days before due, how many days after due should I pay the last?

11. A owes \$840, due Oct. 3; he pays \$400, July 1; \$200 Aug. 1; when will the balance be due?

12. On a bill of goods bought March 1, 1889, amounting to \$1200, on 60 days' credit, the following payments were made: March 21, \$400; April 5, \$150; April 20, \$400; what is the equated time for the payment of the balance?

13. Jan. 15, 1889, A contracted a debt of \$1100, of which \$350 were due in 3 mo., and \$750 in 6 mo.; but at the end of 2 mo. he paid \$200, and 3 mo. afterward \$500; when is the balance due?

(14) Bought goods as follows: Aug. 15, 1888, on 3 mo. credit, \$600; Sept. 10, on 4 mo., \$750; Nov. 5, on 6 mo., \$900. Find the equated time of payment.

Solution.

\$600 is due Nov. 15, 1888.	600	×	0	=	00
\$750 " " Jan. 10, 1889.	750	×	56	=	42000
\$900 " " May 5, 1889.	900	×	171	=	153900
	2250				)195900
					87

Average term of credit is 87 da., and date of payment is 87 da. from Nov. 15, 1888, or Feb. 10, 1889, Ans.

NOTE.—When the terms of credit begin at different dates, as in the preceding example, first find the date when each item matures, and then proceed as in the solution of example 2.

15. Each of the following bills were bought on a credit of 4 mo.: March 10, 1879, \$200; April 15, \$160; May 1, \$440; find the equated time.

16. What is the equated time of the following bills: Jan. 14, 1886, \$875 on 30 da.; Jan. 22, \$930 on 60 da.; March 22, \$1800 on 60 da.; April 20, \$3600 on 90 da.?

17. Bought goods as follows:

Jan. 6, 1889, \$300, on a credit of 3 mo.
Feb. 12, 1889, \$400, " " " 4 mo.
Mar. 18, 1889, \$250, " " " 3 mo.

Find the equated time.

18. Find the equated time of the following bill:

J. P. Hargrove,

To *Jones & Skinner*, Dr.

Jan. 1	To mdse. on 4 mo. credit.	$600.
Feb. 7......	" 5 "	370
March 15....	" 4 "	560
April 20	" 6 "	420

Averaging Accounts.

401. Averaging accounts is finding the average or equated time for paying the balance, or calculating the cash balance at any particular time.

402. ***To find the equated time for paying the balance of an account.***

(1) Find the equated time for the payment of the balance of the following account:

Dr. J. C. Moon. *Cr.*

1880.			1880.		
Feb. 15	To mdse @ 3 mo.	$400	June 4	By cash . .	$400
Mar. 3	" " @ 4 mo.	500	" 10	" " . .	200
" 24	" " @ 5 mo.	900	July 5	" " . .	600
Apr. 15	" " @ 3 mo.	1000			

1° Solution—By Products.

May 15.	400 ×	0 =	0	June 4.	400 ×	20 =	8000
July 3.	500 ×	49 =	24500	June 10.	200 ×	26 =	5200
July 15.	1000 ×	61 =	61000	July 5.	600 ×	51 =	30600
Aug. 24.	900 ×	101 =	90900		1200		43800
	2800		176400				
	1200		43800				
	1600		)132600 = 83				

Hence, the equated time is 83 da. *after* May 15, or *Aug.* 6.

Explanation.—I first find the time when each item becomes due, viz.: May 15, July 3, etc., and take the earliest of these dates (May 15) as the date of reference.

I next multiply each item on both sides by the number of days from the date of reference to the maturity of each item, and divide the difference between the sums of the products (132600) by the difference between the sums of the items (1600). The quotient is the average term of credit, 83 days.

To find the equated term I add 83 days to the date of reference, and obtain Aug. 6.

Analysis.—On the day of reference the purchaser has lost, in the aggregate, the use of \$176400 for 1 day; and the seller, the use of \$43800 for 1 day; hence, to balance the loss, the former is entitled to the use of \$132600 for 1 day, or to the use of \$1600 (the difference between the sums of debits and credits) for as many days as 1600 is contained times in 132600, or 83 days.

Note.—The cash balance of an account is the sum which, paid on any specified day, will exactly balance the account. In the preceding example the cash balance on May 15 is the true present worth of \$1600 payable Aug. 6, or in 83 days. Reckoning at 6%, it is \$1600 $\div$ 1.013$\frac{5}{6}$ = \$1578.16.

2° Solution—By Interest.

The interest at 12%, of

\$ 400	for 101 da. =	\$13.46$\frac{2}{3}$		\$ 400	for 81 da. =	\$10.80
500	" 52 da. =	8.66$\frac{2}{3}$		200	" 75 da. =	5.00
1000	" 40 da. =	13.33$\frac{1}{3}$		600	" 50 da. =	10.00
900	" 0 da. =	0		\$1200		\$25.80
\$2800		\$35.46$\frac{2}{3}$				

\$2800 — \$1200 = \$1600; \$35.46$\frac{2}{3}$ — \$25.80 = \$9.66$\frac{2}{3}$.

Int. for 1 da. of \$1600 = \$.53$\frac{1}{3}$; \$9.66$\frac{2}{3}$ $\div$ \$.53$\frac{1}{3}$ = 17$\frac{1}{2}$.

Hence, the equated time is 18 da. before Aug. 24, or Aug. 6.

Explanation.—Aug. 24 is selected as the date of reference, that being the latest date at which any debt becomes due. I then compute the interest on each debt and payment from the time of its date to Aug. 24. The rest of the process is obvious.

Analysis.—On the day of reference the purchaser would gain \$35.46$\frac{2}{3}$ interest, and the seller \$25.80 interest; that is, the former gains \$9.66$\frac{2}{3}$ more than the latter, which is equivalent to the interest of the balance (\$1600) for about 18 days. Hence, the equated time is 18 days *before Aug. 24*, or Aug. 6.

RULES.—I. *Write the date at which each item on both sides matures, and assume the date of the earliest item on either side as the date of reference. Find the number of days from this standard to the maturity of the respective items.*

II. *Multiply each item by its number of days, and divide the difference between the sums of products by the difference between the sums of items; the quotient will be the average time.*

III. *If the greater sum of items and the greater sum of products are both on the same side, add the average time to the assumed date; if on opposite sides, subtract it; and the result will be the date when the balance of the account is equitably due.*

NOTE.—If, at any assumed date, the balance of interest (or discount) is in favor of the person who owes the balance of the account, the time must be reckoned *forward;* if the reverse, *backward.*

EXERCISE XCIX.

Find the equated time of the following:

2.

Dr. J. F. TAYLOR. *Cr.*

1888.			1888.		
Feb. 5	To mdse @ 3 mo.	$400	Feb. 10	By cash.	$350
Mar. 20	" " @ 4 mo.	500	Mar. 23	" "	400
Apr. 1	" " @ 5 mo.	800	May 7	" "	600

3. Also find the cash balance May 10, at 6% interest.

4.

Dr. J. R. HOLMES. *Cr.*

1889.			1889.		
June 2	To mdse @ 30 da.	$150	June 15	By cash.	$ 80
June 17	" " @ 60 da.	300	July 9	" "	200
July 25	" " @ 90 da.	250	Aug. 7	" "	360

5. Also find the cash balance Aug. 15, at 8% interest.

6.

Dr. W. P. THEUS. *Cr.*

1888.			1888.		
Nov. 15	To mdse @ 2 mo.	$600	Nov. 30	By cash.	$500
Dec. 10	" " @ 3 mo.	900	Dec. 25	" "	700
Dec. 20	" " @ 3 mo.	1000	1889.		
1889.			Jan. 10	" "	1100
Jan. 5	" " @ 4 mo.	1200	Jan. 30	" "	1000

7. Also find the cash balance Feb. 1, 1889, at 6% interest.

QUESTIONS.

What is ratio? State the three principles of ratio.

What is proportion? The terms? Antecedents? Consequents? Extremes? Means? Principle? How find the missing term of a proportion?

What kind of problems may be solved by the principles of proportion? Give the rule for simple proportion. Compound proportion.

State the principles of cause and effect. How find the unknown cause or effect?

What is specific gravity? What three principles are illustrated?

What is prorata division? How performed?

What is partnership? Simple partnership? Compound partnership? How apportion a profit or loss in simple partnership? In compound partnership?

What is an average? How perform alligation medial? Alligation alternate?

What is equation of payments? Equated time? Term of credit? Average term of credit?

What is averaging accounts? How find the equated time for paying the balance of an account?

CHAPTER VI.

HIGHER OPERATIONS.

INVOLUTION.

403. A **power** is the product of equal factors.

(1) $3 \times 3 \times 3 \times 3 = 81$ is a power of 3.

Powers are named according to the number of times the factor is taken to produce the power.

The first power is the number itself; the second power, also called a square, is the product of two equal factors; the third power, also called a cube, is the product of three equal factors; etc., etc.

404. An **exponent** is a small figure placed above a number on the right to denote the power.

(1) $3^1 = 3$, the first power of 3.
(2) $3^2 = 3 \times 3 = 9$, the second power or square of 3.
(3) $3^3 = 3 \times 3 \times 3 = 27$, the third power or cube of 3.
(4) $3^4 = 3 \times 3 \times 3 \times 3 = 81$, the fourth power of 3.

405. Involution is finding a power of a number.

406. ***To find any power of a number.***

(1) Find the value of 12^3, or the third power of 12.

Solution: $12 \times 12 = 144, \times 12 = 1728$, Ans.

RULE.—*Multiply the number by itself, and continue the multiplication till that number has been used as factor as many times as are indicated by the exponent.*

NOTES.—1. The number of multiplications will be ***one less*** than the exponent, because the root is used *twice* in the first multiplication, once as multiplicand and once as multiplier.

2. When the power to be obtained is of a high degree multiply by some of the powers instead of by the root continually; thus, to ob-

tain the 9th power of 2, multiply its 6th power (64) by its 3d power (8); or, its 5th power (32) by its 4th power (16); the rule being, that *the product of any two or more powers of a number is that power whose degree is equal to the sum of their degrees.*

(3) Any power of a fraction is equal to that power of the numerator divided by that power of the denominator.

EXERCISE C.

2. Copy, complete and learn the following table:

Numbers	1	2	3	4	5	6	7	8	9
Squares									
Cubes									
Numbers	.1	.2	.3	.4	.5	.6	.7	.8	.9
Squares									
Cubes									

3. Find the square or second power of 25; 35; 46; 4.5; .08; .055; $\frac{3}{4}$; $\frac{5}{8}$; $7\frac{1}{2}$; $3\frac{1}{3}$; 2.45.

4. Find the cube or third power of 15; 45; 72; 4.8; .5; .07; .003; $\frac{2}{3}$; $\frac{5}{8}$; $2\frac{1}{4}$; $1\frac{5}{8}$.

Complete the following:

5. $75^2 = (\quad)$.
6. $125^2 = (\quad)$.
7. $3.1^4 = (\quad)$.
8. $2.05^3 = (\quad)$.
9. $.3^5 = (\quad)$.
10. $3^7 = (\quad)$.
11. $(2\frac{1}{2})^4 = (\quad)$.
12. $(3\frac{3}{4})^3 = (\quad)$.
13. $2^{10} = (\quad)$.
14. $5^8 = (\quad)$.

EVOLUTION.

407. A **root** of a number is one of the equal factors that will, if multiplied together, produce the number.

(1) $5 \times 5 \times 5 = 125$; hence, 5 is a root of 125.

408. The **sign of evolution** is $\sqrt{\ }$, which is called the **radical sign.**

The **index** of the root is the figure placed over the radical sign to denote what root is to be taken. When no index is written, 2 is understood.

(1) $\sqrt{16}$, or $\sqrt[2]{16} = 4$; or the square root of 16 is 4.

(2) $\sqrt[3]{27} = 3$; or the cube root of 27 is 3.

(3) $\sqrt[4]{625} = 5$; or the fourth root of 625 is 5.

409. Evolution is finding a root of a number.

A perfect power is a number whose exact root can be found; and an imperfect power is a number whose exact root can not be found.

EXTRACTION OF THE SQUARE ROOT.

410. Extraction of the square root is the process of finding one of the two equal factors that produce a number.

A **perfect square** is a number which can be resolved into two equal factors. The perfect squares between 1 and 100 are $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$, $6^2 = 36$, $7^2 = 49$, $8^2 = 64$, $9^2 = 81$. When a number is not a perfect square, it contains a perfect square and something over. Thus, the greatest perfect square contained in 20 is 16; in 34, 25; in 60, 49; in 90, 81.

411. The extraction of the square root of a number depends on the formation of the number with respect to the tens and units of its root.

412. ***To analyze the square of a number.***

Find the composition of the square of 34.

Parallel Operations.

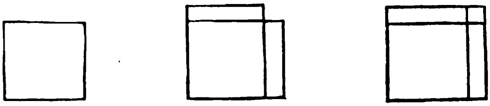
$$\begin{array}{rlll}
34 = & & 30 + 4 = 3 \text{ tens} + 4 \text{ units.} \\
34 = & & 30 + 4 \\
\hline
136 = & & 30 \times 4 + 4^2 \\
102 = & 30^2 + & 30 \times 4 \\
\hline
1156 = & 30^2 & + 2(30 \times 4) + 4^2 \\
= & 900 & + 240 + 16.
\end{array}$$

Hence, representing the tens by t, and the units by u, we have the

Formula: $(t + u)^2 = t^2 + 2tu + u^2$.

That is, *the square of any number composed of tens and units is equal to the square of the tens, + twice the product of the tens by the units, + the square of the units.*

Geometrical Illustration.



A B C

If each side of figure (A) is 30, and the width of the additions in the other figures is 4, then

$A = 30^2$,

$B = 30^2 + 2(30 \times 4)$,

$C = 30^2 + 2(30 \times 4) + 4^2 = 34^2$.

Complete the following and verify the results:

1. $32^2 = 30^2 + 2(\quad) + (\quad)$.
2. $67^2 = (\quad) + 2(\quad) + 7^2$.
3. $58^2 = (\quad) + 2(\quad) + (\quad)$.
4. $7.3^2 = 49 + 2(\quad) + (\quad)$.
5. In a similar manner, find the squares of 54; 38; 65; 92; 81; 35; 4.8.

413. The preceding examples illustrate the following principles:

If a number, which is a perfect square, be separated into periods of two figures each, beginning at the right:

1°. *There will be as many figures in the square root of the number as there are periods.*

For, the square of the unit's figure of the root will lie wholly in the first period; the square of the ten's figure of the root will lie wholly in the second period; the square of the hundred's figure of the root will lie wholly in the third period, etc., etc.

2°. *The square root of the greatest square contained in the left-hand period is the left-hand figure of the root.*

3°. *If a number, whose square root is composed of tens and units, be diminished by the square of the tens of the root, the remainder will be equal to* $2tu + u^2$, which is the

same as the sum of twice the tens and units, multiplied by the units.

Thus, take 625, whose square root is 25. The square of the tens of the root is 20^2, or 400; now, $625 - 400 = 225$, which is equal to the sum of 2×20 and 5, multiplied by 5.

414. ***To find the square root of a number.***

(1) Find the square root of 1156.

Operation.

$$\begin{array}{lr|l}
 & & 11{,}56 \mid \underline{34} \\
 & & 9 \\
\text{Trial divisor, } 2 \times 30 = & 60 & 256 \\
 & \underline{4} & \\
\text{Complete divisor,} & 64 & \underline{256}
\end{array}$$

Explanation.—Separating the number into periods of two figures each, I find that the root will be expressed by two figures.

I write 9, the greatest square in the left-hand period, under the period, and 3, the square root of 9, for the tens of the root.

Subtracting 9 from 11, and to the remainder annexing the next period, I have 256. Now, since $256 = (2t + u) \times u$, or $2 \times 30 +$ *units*, multiplied by the *units*, the units $= 256 \div (2 \times 30 + \text{units})$.

The last term of this divisor, being small in comparison with the the first, may be temporarily omitted; which gives 2×30 for a *trial divisor*, and 4 for a *trial quotient*, or unit figure of the root. With this unit figure I now complete the divisor, by adding the omitted term (units $= 4$) to the trial divisor. Multiplying this complete divisor (64) by 4, I obtain 256, which is equal to the dividend. Hence, 34 is the required root.

In obtaining any figure of the root except the first, the figure or figures of the root already found are regarded as tens, and the figure sought as units; that is, each succeeding figure of the root is found in the same manner as the second figure of a root expressed by two figures.

(2) Extract the square root of 5227.29.

Full Process.

$$\begin{array}{r|l}
 & 52{,}27.29 \mid \underline{72.3} \\
 & 49 \\
2 \times 70 = 140 & 3\ 27 \\
140 + \ 2 = 142 & \underline{2\ 84} \\
2 \times 72.0 = 144.0 & 43.29 \\
144.0 + .3 = 144.3 & \underline{43.29}
\end{array}$$

Abbreviated Method.

$$\begin{array}{r|l}
 & 52{,}27.29 \mid \underline{72.3} \\
 & 49 \\
142 & 3\ 27 \\
 & \underline{2\ 84} \\
144.3 & 43.29 \\
 & \underline{43.29}
\end{array}$$

RULE.—I. *Separate the number into 2-figure periods, beginning at the decimal point.*

II. *Find the greatest square in the left hand period, and place its root on the right for the first root-figure. Subtract its square from the left-hand period, and to the remainder annex the next period for a dividend.*

III. *Double the part of the root already found, and place it on the left for a trial divisor; divide the dividend, exclusive of its right-hand figure, by the trial divisor, and take the quotient for the next figure of the root, and also annex it to the trial divisor, to form the complete divisor.*

IV. *Multiply the complete divisor by the second figure of the root; subtract the product from the dividend, and to the remainder annex the next period for a dividend.*

V. *Proceed with the second, and with each succeeding dividend, in the same manner as with the first.*

NOTES.—1. If any dividend is less than the divisor write a 0 in the root, and also on the right of the divisor, and annex the next period to the dividend for a new dividend.

2. If there is a remainder after all the periods have been used, annex periods of decimal 0's, and extend the work to any desired degree of exactness, writing + after the root.

3. To extract the square root of a mixed number first reduce it to a mixed decimal number, or to an improper fraction.

4. To find the square root of a fraction whose terms are perfect squares, extract the square root of the numerator and of the denominator for the terms of the required root; when the terms are not perfect squares reduce the fraction to a decimal, and then extract the square root.

EXERCISE CI.

Find the square root of:

3. 225.	4. 1225.	5. 1849.
6. 3025.	7. 1764.	8. 3136.
9. 9216.	10. 5625.	11. 2704.

12. 15625.	**13.** 50625.	**14.** 94864.
15. 11881.	**16.** 280900.	**17.** 9980.01.
18. 2.3409.	**19.** .060025.	**20.** .004096.
21. $\frac{144}{289}$.	**22.** $\frac{1089}{4225}$.	**23.** $\frac{2304}{58081}$.
24. $11\frac{1}{9}$.	**25.** $42\frac{1}{4}$.	**26.** $52\frac{9}{16}$.

Find the square root of the following to four places of decimals:

27. 2.	**28.** 3.	**29.** 5.	**30.** 7.
31. $\frac{3}{8}$.	**32.** $6\frac{2}{5}$.	**33.** $5\frac{3}{7}$.	**34.** $3\frac{3}{20}$.

Find the value of:

35. $\sqrt{49}+\sqrt{121}-\sqrt{16}$. **36.** $\sqrt{2704}+\sqrt{1225}-\sqrt{784}$.

37. $\sqrt{64}-\sqrt{.09}-\sqrt{.81}$. **38.** $\sqrt{10.24}-\sqrt{.1296}-\sqrt{.20\frac{1}{4}}$.

39. A square field contains 21025 sq. yd.; what is the length of one side?

40. Find the side of a square field equal in area to a rectangular field 676 yd. long and 256 yd. wide.

41. A garden contains 31212 sq. ft., and its length is 3 times its breadth; find its dimensions.

42. A rectangular field, whose length is 4 times its breadth, contains 90 acres; how much more will it cost to fence it at $3 per rod than an equivalent square field?

43. Find the edge of a square which contains 3 times the area of a square whose edge is 25 inches.

44. The area of the bottom of a rectangular box is 400 sq. in.; of one side, 256 sq. in.; and of one end, 144 sq. in.; find the dimensions of the box.

45. The distance passed over by a body falling under the influence of gravity is $16\frac{1}{12}$ ft. multiplied by the square of the number of seconds; in how many seconds will a ball fall from a state of rest through a distance of 400 yards?

46. If each side of a square was 3 yards longer the area would be 381 sq. yd. more; find the length of the side.

EXTRACTION OF THE CUBE ROOT.

415. Extraction of the cube root is the process of finding one of the three equal factors of a number.

A perfect cube is a number which can be resolved exactly into three equal factors. The integral perfect cubes between 1 and 1000 are 2^3 or 8, 3^3 or 27, 4^3 or 64, 5^3 or 125, 6^3 or 216, 7^3 or 343, 8^3 or 512, 9^3 or 729.

416. The extraction of the cube root of a number depends on the formation of a number with respect to the tens and units of its root.

417. ***To analyze the cube of a number.***

Find the composition of the cube of 34.

Parallel Operations.

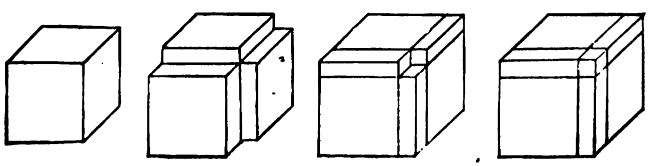
$$
\begin{array}{rrl}
34^2 = & 1156 = & 30^2 + 2(30 \times 4) + 4^2 \\
& 34 = & 30 + 4 \\
\hline
& 4624 = & (30^2 \times 4) + 2(30 \times 4^2) + 4^3 \\
& 3468 = & 30^3 + 2(30^2 \times 4) + (30 \times 4^2) \\
\hline
34^3 = & 39304 = & 30^3 + 3(30^2 \times 4) + 3(30 \times 4^2) + 4^3 \\
& = & 27000 + 10800 + 1440 + 64
\end{array}
$$

Hence, representing the tens by t and the units by u, we have the

Formula: $(t + u)^3 = t^3 + 3t^2u + 3tu^2 + u^3$.

That is, *the cube of any number composed of tens and units is equal to the cube of the tens, + three times the square of the tens by the units, + three times the tens by the square of the units, + the cube of the units.*

Geometrical Illustration.



A B C D

If each edge of figure A is 30, and the thickness of the additions in the other figures is 4, then

$A = 30^3$.

$B = 30^3 + 3(30^2 \times 4)$.

$C = 30^3 + 3(30^2 \times 4) + 3(30 \times 4^2)$.

$D = 30^3 + 3(30^2 \times 4) + 3(30 \times 4^2) + 4^3 = (34)^3$.

Complete the following, and verify the results:

1. $35^3 = 30^3 + 3(\ \ ) + 3(\ \ ) + (\ \ )$.
2. $62^3 = (\ \ ) + 3(\ \ ) + 3(\ \ ) + 8$.
3. $75^3 = (\ \ ) + 3(\ \ ) + 3(\ \ ) + (\ \ )$.
4. $(4.3)^3 = 64 + 3(\ \ ) + 3(\ \ ) + .027$.
5. In a similar manner, find the cubes of 31, 25, 74, 92, 68, 1.8, 3.6.

418. The preceding examples illustrate the following principles:

If a number, which is a perfect cube, be separated into periods of three figures each, beginning at the right:

1°. *There will be as many figures in the cube root of the number as there are periods.*

For the cube of the units of the root will lie wholly in the first period; the cube of the ten's figure of the root will lie wholly in the second period; the cube of the hundred's figure of the root will lie wholly in the third period; etc., etc.

2°. *The cube root of the greatest cube contained in the left-hand period is the left-hand figure of the root.*

3°. *If a number, whose cube root is composed of tens and units, be diminished by the cube of the tens of the root, the remainder will be equal to* $3t^2u + 3tu^2 + u^3$, *which is the same as the sum of* 3 *times the square of the tens*, 3 *times the tens by the units, and the square of the units, multiplied by the units.*

Thus, take 15625, whose cube root is 25. The cube of the tens of the root is 20^3, or 8000. Now, $15625 - 8000 = 7625$, which is equal to the sum of 3×20^2, $3 \times 20 \times 5$, and 5^2, multiplied by 5

418. *To find the cube root of a number.*

(1) Find the cube root of 389017.

Operation.

		389,017	73
		343	
Trial divisor, 3×70^2	$=$	14700	46,017
$3 \times 70 \times 3$	$=$	630	
3^2	$=$	9	
Complete divisor, ...		15339	46,017

Explanation.—Separating the number into periods of three figures each, I find the root will be expressed by two figures.

I write 343, the greatest cube in the left-hand period, under the period, and 7, the cube root of 343, for the tens of the root.

Subtracting 343 from 389, and to the remainder annexing the next period, I have 46017. Now since $46017 = (3t^2 + 3tu + u^2) \times u$, or 3×70^2 plus $3 \times 70 \times$ units plus units ^{2}, multiplied by the units, the units $= 46017 \div (3 \times 70^2$ plus $3 \times 70 \times$ units $+$ units$^2)$.

The last two terms of this divisor, being small in comparison with the first, may be temporarily omitted, which gives 3×70^2, or 14700, for a trial divisor, and 3 for the units of the root. With this unit figure I now complete the divisor by adding the omitted terms, $3 \times 70 \times$ units $+$ units$^2 = 3 \times 70 \times 3 + 3^2$, to the trial divisor. Multiplying this complete divisor, 15339, by 3, I obtain 46017, which is equal to the dividend.

Remark: Three times the square of the tens is the convenient trial divisor. This is in most instances a greater part of the complete divisor; for example, the least number of tens above *one* ten is 2, and the greatest figure in unit's place can not exceed 9: the cube of 29 is 24389, the first complete divisor is 1821, the first trial divisor being 1200, a greater part of it.

In extracting the cube root, only two periods of figures are considered in connection. Hence,

Any figure of the root after the second, is found in the same manner as the second figure of a root expressed by two figures.

This principle is fully illustrated in the solution of the following example.

(2) Find the cube root of 244140.625.

Full Process.

		244,140.625	62.5
		216	
1° Trial divisor,	3×60^2 =	10800	28140
	$3 \times 60 \times 2$ =	360	
	2^2 =	4	
1° Complete divisor		11164	22328
2° Trial divisor,	3×620^2 =	1153200	5812625
	$3 \times 620 \times 5$ =	9300	
	5^2 =	25	
2° Complete divisor		1162525	5812625

Abbreviated Method.

	244,140.625	62.5
	216	
1° Trial divisor . .	10800	28140
1° Complete divisor.	11164	22328
2° Trial divisor . .	1153200	5812625
2° Complete divisor.	1162525	5812625

RULE.—I. *Separate the number into 3-figure periods, beginning at the decimal point.*

II. *Find the greatest cube in the left-hand period, and place its root on the right for the first root-figure. Subtract its cube from the left-hand period, and to the remainder annex the next period for a dividend.*

III. *Square the part of the root already found, multiply the result by 3, annex two ciphers, and place it on the left as a* TRIAL DIVISOR: *divide the dividend by the trial divisor, and take the quotient as the second figure of the root. To obtain the* TRUE DIVISOR, *add to the trial divisor 3 times the product of the last root-figure by the root previously found, annexing one cipher, and also add the square of the last root-figure.*

IV. *Multiply the true divisor by the last root-figure, subtract the product from the dividend, and to the remainder annex the next period for a new dividend.*

V. *Proceed with the second, and each succeeding dividend, in the same manner as with the first, until all the periods are used.*

NOTE.—The notes under Art. 414 are applicable to cube root with the change of "square" to "cube".

EXERCISE CII.

Find the cube roots of the following:

3. 32768.	**4.** 19683.
5. 74088.	**6.** 421.875.
7. 157464.	**8.** .005088448.
9. 7.301384.	**10.** .000042875.
11. 48228544.	**12.** .000523606616.
13. $\frac{12167}{15625}$.	**14.** $\frac{1953125}{13312053}$.

15. $81\frac{10}{27}$. **16.** $5\frac{104}{125}$. **17.** $405\frac{28}{125}$.

Find the value of the following to three places of decimals:

18. $\sqrt[3]{2}$. **19.** $\sqrt[3]{3}$. **20.** $\sqrt[3]{5}$.

21. $\sqrt[3]{\frac{2}{3}}$. **22.** $\sqrt[3]{.03}$. **23.** $\sqrt[3]{38\frac{22}{31}}$.

24. A cubical block of granite contains 110592 cu. in.; what is its edge?

25. What is the height of a cubical box of the same capacity as a box 54 in. long, 49 in. wide, and 28 in. deep?

26. A cubical cistern having a capacity of 2197 cu. ft., is lined on the sides and bottom with copper, at 60 cts. per sq. foot; what is the cost of the lining?

27. The dimensions of a room are in the ratio of 3, 2, 1, and its capacity is 16464 cu. ft.; find the dimensions.

28. Find the edge of a cube whose volume is 3 times that of a cube whose edge is 2 ft.

29. I wish to construct a cubical bin that will contain 500 bushels of wheat; required one of its edges.

30. A cubical block of marble dropped into a rectangular reservoir 6 ft. long and $4\frac{1}{2}$ ft. wide, raises the water $1\frac{1}{2}$ in.; what is the edge of the cube?

31. How many sq. in. of leather will it take to cover a cubical block of metal which weighs $94\frac{39}{128}$ lb. in water, and 100 lb. out of water?

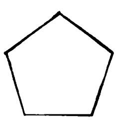
MENSURATION.

419. Mensuration is the process of measuring lines, surfaces, solids, and angles.

In mensuration, the **length** of lines, the **area** of surfaces, the **volume, capacity** or **solid contents** of solids, and the **size** of angles are determined by means of the relations they sustain to certain given quantities. In general, the discovery of these relations is due to Geometry, Trigonometry, and Calculus. Hence, the **rules** of mensuration are the **translations** of **formulas** taken from the Higher Mathematics.

POLYGONS.

420. A **polygon** is a plane figure bounded by straight lines.



The **perimeter** of a polygon is the distance around it, or the sum of all its sides.

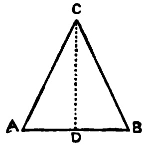
A polygon is **regular** when it has all its sides equal and all its angles equal.

The **base** of a polygon is the side on which it is supposed to stand; and the **diagonal** of a polygon is a straight line joining two angles not adjacent.

A polygon of three sides is a **triangle**; one of four sides, a **quadrilateral**; of five sides, a **pentagon**; of six sides, a **hexagon**; etc.

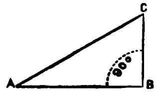
Triangles.

421. A triangle is **scalene** when it has no equal sides; **isosceles**, when it has two equal sides, and **equilateral**, when its three sides are equal.



The **vertical angle** of a triangle is the angle opposite the base, as the angle C; and the **altitude** is the perpendicular distance from the vertical angle to the line of the base, as the line CD.

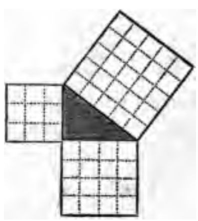
422. A **right triangle** is a triangle that has one right angle.



The side AC, opposite the right angle, is the **hypothenuse**; the side AB, the **base**; and BC, the **perpendicular**.

423. ***Any two sides of a right triangle being given, to find the third side.***

THEOREM.—The square described on the hypothenuse of a right triangle is equal to the sum of the squares described on the other two sides.



A

Hence, *the square of either side about the right angle is equal to the square of the hypothenuse diminished by the square of the other side.*

From this theorem, which is demonstrated by geometry, and illustrated by figure A, the following rules are derived:

I. ***To find the hypothenuse:***

RULE.—*Extract the square root of the* SUM *of the square of the base and the square of the perpendicular.*

II. ***To find the base or perpendicular:***

RULE.—*Extract the square root of the* DIFFERENCE *between the square of the hypothenuse and square of the given side.*

NOTE.—The square of the Difference may be found by the following simple rule: *Multiply the sum of the given sides by their difference.*

EXERCISE CIII.

(1) The sides forming the right angle of a triangle are 12 and 5 ft.; find the hypothenuse.

Solution: $\sqrt{12^2+5^2}=\sqrt{169}=13$ (ft.), Ans.

2. What is the length of a ladder which will reach a window 28 ft. high, when its foot is placed 21 ft. from the house?

3. The hypothenuse of a right triangle is 85 and the perpendicular 68; what is the base?

4. A rectangular garden is 87 by 116 yd.; how far is it from one corner to the diagonally opposite corner?

5. The width of a house is 48 ft., and the height of the gable 10 ft.; find the length of the rafters.

6. The diagonal of a croquet ground is 51 ft., and its length 45 ft.; what is its width?

7. A pole and its shadow form a right angle; the shadow is 50 ft. long, and the distance from the top of the pole to the end of the shadow is 75 ft; find the height of the pole?

8. How far is it from a point 70 mi. north of this place to a point 45 mi. east of this point?

9. A room is 20 ft. by 16 ft. by 12 ft.; how long a line will extend from one of the lower corners to the upper corner farthest from it?

424. ***To find the area of a triangle, its base and altitude being given.***

(10) The base of a triangle is 16 inches, and its altitude 9 inches; what is the area of the triangle?

Solution: $\frac{16\times 9}{2}=72$, the area in sq. in.

RULE.—*Multiply the base by the altitude, and take half the product.*

What is the area of a triangle:

11. Whose base is 36 rd., and altitude 20 rd.?

12. Whose base is 123 yd., and altitude 14 rd.?

How many acres in a triangular field:

13. Whose base is 15 ch., and altitude 8 ch.?

14. Whose base is 363 yd., and altitude 160 yd.?

15. Find the value, at $6 per acre, of a triangular piece of land whose base is 80 rd., and altitude 60 rd.

16. Find the number of sq. ft. in the gable end of a building 40 ft. wide, and 12½ feet from the beam to the ridgepole.

17. The area of a triangle, whose base is 12 in., is 20 sq. in.; find its altitude.

18. The base of a triangular field is 227 rd., and its area 46 A. 17½ sq. rd.; what is its altitude?

425. ***To find the area of a triangle when the three sides are given.***

(19) What is the area of a triangle whose sides are 5, 12, and 13 inches?.

Solution: $5 + 12 + 13 = 30$; $30 \div 2 = 15$.
$15 - 5 = 10$, $15 - 12 = 3$, $15 - 13 = 2$.
$15 \times 10 \times 3 \times 2 = 900$; $\sqrt{900} = 30$, Ans. in sq. in.

RULE.—*From half the sum of the three sides subtract each side; then multiply the half sum and the three remainders together, and extract the square root of the product.*

20. Find the area of a triangle whose sides are 12, 16, and 20 feet.

21. Find the area of an equilateral triangle whose sides are each 12 feet.

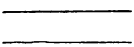
22. What is the area of an isosceles triangle whose base is 48 ft., and sides 32 ft.?

23. The three sides of a triangular field are 25, 26, and 17 ch.; what is it worth at $25 per acre?

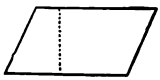
24. The base of a triangle is 75 ft., and the other two sides 65 and 20 ft.; find the altitude.

Suggestion: Divide twice the area by the base.

Quadrilaterals.



426. Parallel lines are lines which have the same direction.



427. A **parallelogram** is a quadrilateral whose opposite sides are parallel. The opposite sides are also equal.

When a parallelogram is right-angled, it is a **rectangle**; and when the four sides of a rectangle are equal, it is a **square**.

The **altitude** of a parallelogram is the perpendicular distance between the base and the opposite side.

428. ***To find the area of a parallelogram.***

(1) What is the area of a parallelogram whose base is 12 in. and altitude 9 in.?

Solution: $12 \times 9 = 108$, the area in sq. in.

RULE.—*Multiply the base by the altitude.*

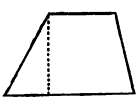
EXERCISE CIV.

2. Find the area of a garden, in the form of a parallelogram, whose base is 12 rd., and altitude 8 rd.

3. How many acres in a field, in the form of a parallelogram, whose base is 87.5 chains, and altitude 37.5 chains?

4. The area of a parallelogram is 20 sq. yd., and its base is 12 ft.; what is its altitude?

5. The diagonal of a rectangular garden is $9\frac{1}{8}$ ch., and one of its sides $5\frac{1}{2}$ ch.; what is it worth, at $65 per acre, exclusive of the surface occupied by a house 66 ft. by 44 ft.?



429. A **trapezoid** is a quadrilateral having two of its sides parallel.

The altitude of a trapezoid is the perpendicular distance between its parallel sides. The parallel sides are called **bases**.

430. *To find the area of a trapezoid.*

(6) What is the area of a trapezoid whose bases are 16 ft. and 12 ft., and altitude 10 ft.?

Solution: $\frac{16+12}{2} \times 10 = 140$; hence, the area is 140 sq. ft.

RULE.—*Multiply half the sum of the parallel sides by the altitude.*

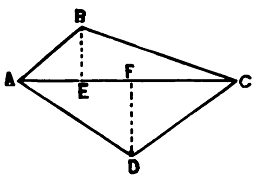
7. The parallel sides of a trapezoid are 11 ft. and 7 ft., and its altitude 10 ft.; how many sq. yd. in its area?

8. Required the area of a trapezoid whose bases are 30.5 and 45.5 ch., and its altitude 18.5 ch.

9. Required the area of a board 5 ft. long, 8 in. wide at one end, and 4 in. wide at the other.

10. A section of land is cut into two trapezoids by running a line from the N. W. corner of the N. E. $\frac{1}{4}$ to the S. W. corner of the S. E. $\frac{1}{4}$ of the S. W. $\frac{1}{4}$; find the area of the two trapezoids.

431. A **trapezium** is a quadrilateral of which no two sides are parallel.



432. *To find the area of a trapezium.*

(11) What is the area of the trapezium ABCD, whose diagonal AC is 20 ft., the perpendicular BE, 5 ft., and the perpendicular FD, 9 ft.?

Solution: The area

Of the triangle ABC, $\frac{1}{2}$ of 20×5, = 50 sq. ft.
Of the triangle ACD, $\frac{1}{2}$ of 20×9, = 90 sq. ft.
Of the trapezium ABCD, = 140 sq. ft., Ans.

(12) Find the area of the trapezium ABCD, whose diagonal AC is 65 ft., and whose sides AB, BC, CD and DA are 25, 60, 39, and 52 ft. respectively.

Solution: The area

Of the triangle ABC, see Art. 425,	=	750 sq. ft.
Of the triangle ACD, " " 425,	=	1114 sq. ft.
Of the trapezium ABCD,	=	1864 sq. ft.

RULE.—*Find the areas of the two triangles formed by drawing a diagonal, and their sum will be the area of the trapezium.*

NOTE.—The area of any polygon may be found by cutting it into triangles, and taking the sum of their areas.

13. Find the area of a trapezium divided by a diagonal of 80 rods into triangles, whose altitudes are 35 and 55 rods.

14. In a trapezium ABCD, AB is 20 ch., BC 21 ch., CD 29 ch., DA 29 ch., and the diagonal AC 29 ch.; find its area in acres.

15. Find the area of a polygon made up of 3 triangles, whose bases are 20, 25 and 12 rods, and whose altitudes are 7, 12 and 17 rods.

16. What is the area of a trapezium ABCD, of which AB, BC, CD, DA and AC are respectively 10, 12, 14, 16, and 18 rods?

17. A's land lies in a certain section, and is bounded by straight lines, as follows: beginning at the N. E. corner of S. E. $\frac{1}{4}$ of S. E. $\frac{1}{4}$, thence to the centre of the N. W. $\frac{1}{4}$ of N. E. $\frac{1}{4}$, thence to the centre of the S. W. $\frac{1}{4}$ of N. W. $\frac{1}{4}$, thence to the centre of the west boundary line of the S. E. $\frac{1}{4}$ of S. W. $\frac{1}{4}$, and thence to the point of starting; what is A's land worth at \$12$\frac{1}{2}$ per acre?

CIRCLES.*

433. ***To find the circumference of a circle.***

(1) What is the circumference of a circle whose diameter is 10 ft.?

Solution: 3.1416 × 10 ft. = 31.416 ft., Ans.

RULE.—*Multiply the diameter by* 3.1416.

Conversely, to find the diameter, *divide the circumference by* 3.1416.

EXERCISE CV.

Find the circumference of a circle:

2. Whose diameter is 180 feet.

3. Whose radius is 275 inches.

4. What is the diameter of a circle whose circumference is 65.5 inches?

5. What is the radius of a circle whose circumference is 500 feet?

6. A buggy wheel is $4\frac{1}{2}$ ft. in diameter; find the length of the tire.

7. What is the diameter of a mile circular race track?

434. ***To find the area of a circle.***

(8) Find the area of a circle whose radius is 10 inches.

Solution: $10^2 = 100$, × 3.1416 = 314.16, the area in sq. in.

RULE.—*Multiply the square of the radius by* 3.1416, ***or the square of the diameter by*** .7854.

Conversely, to find the radius, ***divide the area by*** 3.1416, ***and extract the square root of the quotient.***

9. Find the area of a circular garden whose radius is 80 yards.

10. What is the diameter of a circle whose area is 706.86 sq. ft.?

* *For definition of terms, see Art. 258.*

11. What is the value of a circular field 120 rd. in diameter, at $20 per acre?

12. A horse is fastened by a rope long enough to allow him to graze on $1\frac{1}{4}$ acres; how long is the rope?

13. How far is it around a circular garden which contains 16 acres?

14. The circumference of a circular garden is 160 rd., and the perimeter of a square garden is also 160 rd.; how much larger is the area of the former than that of the latter?

SOLID BODIES.

Prisms and Cylinders.

435. A **Prism** is a solid, two faces of which are equal and parallel polygons, and the other faces parallelograms.

The **bases** of a prism are the equal and parallel polygons; the **lateral faces** are all the faces except the bases; the **lateral surface** is the sum of the lateral faces; the **lateral edges** are the straight lines in which the lateral faces meet; and the **altitude** is the perpendicular distance between the bases.

A prism is triangular, quadrangular, pentagonal, hexagonal, etc., according as its bases are triangles, quadrilaterals, pentagons, etc.

436. A **right prism** is a prism whose lateral faces are rectangles, or whose lateral edges are perpendicular to the bases.

437. A **cylinder** is the solid that is formed by turning a rectangle around one of its sides, as an axis.

The **bases** are the two circles forming the ends; the **altitude** is the perpendicular distance between the bases, and the **lateral surface** is the entire surface except the bases.

438. ***To find the lateral or entire surface of a right prism or cylinder.***

(1) Find the entire surface of a right prism whose altitude is 9 ft., and the base of which is an equilateral triangle 10 ft. on a side.

Solution: Perimeter of base,	3×10,	$=$	30 ft.
Lateral surface,	9×30,	$=$	270 sq ft.
Area of base, Art. 425,		$=$	43.3 "
Area both bases,	2×43.3,	$=$	86.6 "
Entire surface,	$270 + 86.6$,	$=$	356.6 "

(2) Find the entire surface of a cylinder whose altitude is 5 ft., and the diameter of the base 10 ft.

Solution: Circumference of base,	3.1416×10,	$=$	31.416 ft.
Lateral surface,	31.416×5,	$=$	157.08 sq. ft.
Area of both bases,	$2 \times .7854 \times 10^2$,	$=$	157.08 "
Entire surface,			314.16 sq. ft.

RULE.—I. *Multiply the perimeter or circumference of the base by the altitude, and the product will be the convex surface.*

II *Add the area of the upper and lower bases to the convex surface, and the sum will be the entire surface.*

EXERCISE CVI.

3. A quadrangular prism is 12 ft. long, and its base is a rectangle 5 ft. by 3 ft.; find its (1) lateral surface; (2) entire surface.

4. A triangular prism is 15 in. long, and the sides of its bases are 3, 4, 5 in.; find its (1) lateral surface; (2) entire surface.

5. Find the convex surface of a cylinder whose length is 30 in., and circumference of base 15 in.

6. What is the entire surface of a cylinder whose altitude is 35 in., and the radius of its base $6\frac{1}{2}$ in.?

7. The entire surface of a square shaft of marble is 1088 sq. ft., and each side of its base is 8 ft.; what is its height?

8. The diameter of the base of a cylinder is 1 ft. 3 in. and its entire surface is 49.5783 sq. ft.; find its height.

439. ***To find the volume of a right prism or cylinder.***

(9) Find the volume of a right prism, the altitude being 12 in., and the base a square 5 in. on a side.

Solution: Area of base, $5 \times 5, = 25$ sq. in.
Volume, $12 \times 25, = 300$ cu. in.

(10) Find the volume of a cylinder 8 ft. long, the diameter of base being 18 in.

Solution: 18 in $= 1\frac{1}{2}$ ft.
Area of base, $(1\frac{1}{2})^2 \times .7854, = 1.76715$ sq. ft.
Volume, $8 \times 1.76715, = 14.13720$ cu. ft.

RULE.—*Multiply the area of the base by the altitude.*

11. Required the volume of a triangular prism 4 ft. long, each side of its base being 30 in.

12. Required the volume of a cylinder whose length is 15 ft., and its diameter 4 ft.

13. A cylindrical tank is 12 ft. high and 10 ft. in diameter; how many gallons of water will it hold, allowing $7\frac{1}{2}$ gal. to the cu. ft.?

14. A piece of iron in the form of a right triangular prism is 15 ft. long, and the sides of its bases 1, 2.4, 2.6 inches; what is its weight, allowing 7200 oz. to the cu. ft.?

15. A bushel measure is a cylinder $18\frac{1}{2}$ in. in diameter and 8 in. deep; what is its capacity in cu. in.?

16. A railroad embankment is in the form of a right quadrangular prism 100 ft. long, its base being a trapezoid, whose parallel sides are 10 and 26 feet, and its altitude 12 ft.; how many cu. yd. of dirt does it contain?

17. If an ordinary brick was weighed in water and out of water, what would be the difference in the weights?

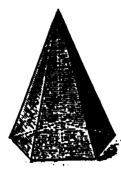
18. A cylindrical vessel 6 in. in diameter and 7 in. deep is full of water; if the water was poured into another cylindrical vessel, 8 in. in diameter, what would be the depth of the water?

19. A cylindrical bar of brass is 10 ft. long and $2\frac{1}{2}$ in. in diameter; how much less will it weigh in water than out of water?

20. The lengths of three cylinders are 50, 40 and 30 in.; the diameters of the bases of the first and second are 20 and 15 in., respectively, and the volume of the first is equal to the volumes of the other two; find the diameter of the third.

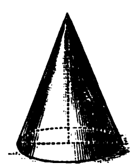
Pyramids and Cones.

440. A **pyramid** is a solid, one face of which is a polygon, and the other faces are triangles, which meet at a common point, called the vertex.



The **altitude** of a pyramid is the perpendicular distance from the vertex to the base.

441. A **cone** is the solid that is formed by turning a right triangle around one of the sides that includes the right angle.



The **altitude** is the side used as an axis, and the base is the circle generated by the other side.

The **slant height** of a regular pyramid is the straight line drawn from its vertex perpendicularly to one side of the base.

The **slant height** of a cone is a straight line from the vertex to the circumference of the base.

The **lateral surface** of a pyramid or cone is all the surface except the base.

442. ***To find the lateral or entire surface of a pyramid or cone.***

NOTE.—In this book a pyramid is supposed to have all its faces equal isosceles triangles.

(1) The base of a pyramid is a square 10 in. on a side, and the slant height 30 in.; what is the entire surface?

Solution:	Perimeter of base,	$4 \times 10,$ =	40 in.
	Lateral surface,	$\frac{1}{2}(40 \times 30),$ =	600 sq. in.
	Area of base,	$10 \times 10,$ =	100 "
	Entire surface,		700 sq. in.

(2) Find the entire surface of a cone, the diameter of whose base is 5 in., and the slant height 20 in.

Solution:	Circumference of base,	$5 \times 3.1416,$ =	15.7080 in.
	Lateral surface,	$\frac{1}{2}(15.7080 \times 20),$ =	157.080 sq. in
	Area of base,	$5^2 \times .7854,$ =	19.635 "
	Entire surface,	=	177.715 sq. in.

RULE.—I. *Multiply the perimeter or circumference of the base by half the slant height, and the product will be the lateral surface.*

II. *To the lateral surface add the area of the base, and the sum will be the entire surface.*

NOTE.—The slant height of cones may be found by adding the square of the altitude to the square of the radius of the base, and extracting the square root of the sum.

EXERCISE CVII.

3. Find the convex surface of a cone, the slant height being 23 in., and the circumference of the base 14 in.

4. Find the convex surface of a pentagonal pyramid whose slant height is 15 ft., and each side of the base 3 ft.

5. The base of a pyramid is a triangle 12 yd. on a side, and the slant height is 20 yd.; (1) find the lateral surface; (2) find the entire surface.

6. What is the (1) lateral surface and (2) the entire surface of a cone formed by turning a right triangle around its perpendicular as an axis, the perpendicular of the triangle being 8 in., and its base 6 inches?

7. What is the difference between the entire surface of a cone whose altitude is 12 ft., and the circumference of its base 40 ft., and that of a regular pyramid, whose altitude is 12 ft., and its base a square, whose perimeter is 40 ft.?

443. ***To find the volume of a pyramid or cone.***

(8) What is the volume of a pyramid whose altitude is 12 ft., and base a square 5 ft. on a side?

Solution: Area of base, $5 \times 5, = 25$ sq. ft.
Volume, $25 \times 4, = 100$ cu. ft.

(9) Find the volume of a cone whose altitude is 15 in., and radius of base 10 in.

Solution: Area of base, $10^2 \times 3.1416, = 314.16$ sq. in.
Volume, $314.16 \times 5, = 1570.80$ cu. in.

RULE.—*Multiply the area of the base by one-third of the altitude.*

10. Find the volume of the cone whose dimensions are given in example 6.

11. Find the volume of a pyramid whose altitude is 30 in., and the base a triangle 14 in. on each side.

12. Find the weight of a conical piece of silver whose altitude is 12 in., and radius of its base 5 in.

13. A pyramidical piece of common stone is 45 in. high, and its base is a hexagon whose area is 360 sq. in.; how much less would it weigh in water than in the air?

14. Find the difference between the volumes of the cone and pyramid whose dimensions are given in example 7.

The Sphere.

444. A sphere is a solid bounded by a curved surface, all points of which are equally distant from a point within called the centre.

The diameter of a sphere is a straight line drawn through the centre, and terminated both ways by the surface. The radius is a straight line drawn from the centre to the surface.

445. ***To find the surface of a sphere.***

(1) Find the surface of a sphere whose diameter is 10 in.

Solution: Surface, $10^2 \times 3.1416, = 314.16$ sq. in.

RULE.—*Multiply the square of the diameter by* 3.1416.

NOTE.—The surface of a sphere is 4 times the area of a circle having the same diameter.

EXERCISE CVIII.

2. What is the surface of a sphere whose diameter is 3 ft?

3. Find the surface of a globe $2\frac{1}{2}$ ft. in diameter.

4. The earth is nearly a sphere, having a diameter of 7912 miles; what is its surface?

5. What will it cost, at \$.50 a square foot, to gild a spherical ball whose radius is $12\frac{1}{2}$ inches?

446. ***To find the volume of a sphere.***

(6) Find the volume of a sphere whose diameter is $3\frac{1}{3}$ ft.

Solution: Volume, $(3\frac{1}{3})^3 \times .5236, = 19.39 +$ cu. ft.

RULE.—*Multiply the cube of the diameter by* .5236.

NOTES.—1. The volume of a sphere may be found by multiplying the surface of the sphere by one-third of the radius.

2. The decimal .5236 is $\frac{1}{6}$ of 3.1416.

7. What is the volume of a sphere whose diameter is 4 ft?

8. Find the weight of a ball of iron 5 in. in diameter.

9. A ball of gold is 6 in. in diameter; how much less would it weigh in water than in air?

10. What length of wire $\frac{1}{50}$ of an inch in diameter can be drawn from a ball of copper 10 in. in diameter, allowing *10% for* waste?

SIMILAR QUANTITIES.

447. Similar surfaces are surfaces having the same form and corresponding dimensions proportional.

All circles are similar figures; so also are any two equiangular triangles. Any two polygons are similar when the angles of the first are successively equal to the angles of the second taken in the same order, and the ratios of the dimensions of the first to the corresponding dimensions of the second are equal.

The corresponding dimensions are any lines similarly situated, as radii, diameters, sides, diagonals, etc.

PRINCIPLES.—1°. *The areas of similar surfaces are to each other as the squares of their corresponding dimensions.*

2°. *The corresponding dimensions of similar surfaces are to each other as the square roots of their areas.*

NOTE.—Articles whose values are determined by their quantity of surface vary in value as their areas vary.

(1) The area of a triangular field, whose altitude is 6 chains, is 4.8 acres; find the area of a similar field whose altitude is 9 chains.

Solution: $6^2 : 9^2 :: 4.8$ A. : xA. Hence, $x = 10.8$.

(2) If one side of a field containing 42.3 acres is 18 rd., what is the length of the corresponding side of a similar field, which contains 75.2 acres?

Solution: 42.3 A. : 75.2 A. :: 18^2 rd. : x^2rd.

Hence, $x^2 = (18^2 \times 75.2) \div 42.3 = 576$, and $x = \sqrt{576} = 24$.

EXERCISE CIX.

3. A rug 4 ft. long weighs 10 lb.; find the weight of a similar rug 6 ft. long.

4. If it cost \$.50 a ft. to dig a well 3 ft. in diameter, what is the diameter of a well which cost \$1 a ft.?

5. If a 2-inch pipe will empty a cistern in 20 hours, in what time will a 4-inch pipe empty it?

6. If a gate 6 inches square will empty a mill-pond in 21 hours, how large must a similar gate be to empty it in $2\frac{1}{3}$ hours?

7. A sphere 10 in. in diameter was gilded for \$5; at that rate what would it cost to gild a sphere 12 in. in diameter?

8. If a copper wire $\frac{1}{20}$ of an inch in diameter will sustain a weight of 150 ℔., how large a wire may sustain a weight of 6000 ℔.?

9. The altitude of a triangular field is 13 chains; how far from the vertical angle must a line be run parallel to the base to divide the field into two equal parts?

10. Find the diameter of a circle whose area is equal to the sum of the areas of two circles, whose diameters are 12 and 5 in.

448. Similar solids are those which have the same form, and their corresponding dimensions proportional.

The corresponding dimensions of *spheres* are their *diameters*, *radii*, and *circumferences*; those of cubes are *their* sides.

The corresponding dimensions of *cylinders* and *cones* are their *altitudes*, and the *diameters* or the *circumferences* of their bases.

Pyramids are similar when their *bases* are similar polygons, and their *altitudes* proportional.

PRINCIPLES.—1°. *The volumes of similar solids are proportional to the cubes of their corresponding dimensions.*

2°. *The dimensions of similar solids are proportional to the cube roots of their volumes.*

NOTE.—Similar solids generally vary in weight, value, and many other respects, in the same proportion as they vary in volume.

(11) If a ball 4 in. in diameter weighs 20 ℔., what is the weight of a ball 6 in. in diameter?

Solution: $4^3 : 6^3 :: 20\text{lb} : x\text{lb}$.
Hence, $x = (6^3 \times 20) \div 4^3 = 67\frac{1}{2}$.

(12) If a cubic block of marble, whose edge is 2 ft., weighs 10 tons, what is the edge of a cube of marble which weighs $33\frac{3}{4}$ tons?

Solution: $10 \text{ T} : 33\frac{3}{4} \text{ T} :: 2^3 \text{ ft.} : x^3 \text{ ft.}$

Hence, $x^3 = (33\frac{3}{4} \times 2^3) \div 10 = 27$, and $x = \sqrt[3]{27} = 3$.

13. If a log of wood 20 ft. long is worth \$4, what is the value of a similar log 25 ft. long?

14. If a cheese 10 in. in diameter is worth \$4.50, what is the value of another cheese 12 in. in diameter, and thick in proportion?

15. If a conical stack of hay whose height is 12 ft. contains 5 tons, what must be the height of a similar stack to contain $23\frac{4}{27}$ tons?

16. If a cistern 6 ft. in diameter will contain 30 hhd. of water, what is the diameter of a similar cistern that will contain $1111\frac{1}{9}$ hhd.?

17. If a hog whose girth is 5 ft. 6 in. weighs 160 lb., what is the weight of a hog of similar build whose girth is 6 ft.?

18. If it is worth \$10 to dig a cellar whose dimensions are each 6 ft., what is it worth to dig one whose dimensions are each 10 ft.?

19. I have a conical stack of hay whose height is 12 ft., of which I desire to sell one-half, taken from the top; how much of the height must be taken off?

20. The values of two balls of copper are \$18.90 and \$87.50; if the diameter of the smaller is $10\frac{1}{2}$ in., what is the diameter of the second?

21. Find the diameter of a ball whose volume is equal to the volumes of three balls, whose diameters are 3, 4, and 5 inches.

22. A spherical shell 6 in. in diameter weighs half as *much as if it were solid*; find the diameter of its cavity.

23. A silver dollar is about $1\frac{1}{2}$ in. in diameter and $\frac{1}{16}$ of an in. thick; what would be the dimensions of a dime if it were similarly made?

24. Four women own equally a ball of yarn 6 in. in diameter; find the diameters of the resulting balls after each woman has in turn taken off her share.

QUESTIONS.

What is a power? An exponent? Involution? How find any power of a number?

What is a root? Evolution? Sign of evolution? Extraction of square root? On what does it depend? Repeat the formula. Give the rule.

What is extraction of the cube root? On what does it depend? Repeat the formula. Give the rule.

What is mensuration? What is a polygon? A regular polygon? The perimeter of a polygon? Diagonal of a polygon? Give names of particular polygons. What is a right triangle? Give the rule for finding any side of a right triangle. What is the altitude of a triangle? Give rule for the area of a triangle when the base and altitude are given. When the three sides are given.

Define the various quadrilaterals. What is the ru'e for finding the area of a parallelogram? Of a trapezoid? Of a trapezium whose sides and a diagonal are given?

How is the circumference of a circle found? The area?

What is a prism? Mention particular kinds. Give the rule for the convex and the entire surface of a prism. For the volume. Define a cylinder. Give the rule for finding the convex surface of a prism. For finding the volume.

What is a pyramid? The slant height? The altitude? What is the rule for finding the convex surface? The volume.

Define a cone. Give rule for finding the convex surface. The volume.

What is a sphere? How is its surface found? Its volume?

Define similar surfaces. Give the principles. What are similar solids? Give the principles.

APPENDIX.

449. Table of prime numbers from 1 to 1009.

1	59	139	233	337	439	557	653	769	883
2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	613	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	859	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	1009

SHORT METHODS.

450. ***To multiply any integer by 11.***

(1) Multiply 83469 by 11.

Explanation.—For the first figure of the product I write the unit-figure of the multiplicand; then I add the units to tens, tens to hundreds, etc., carrying, and writing the results in the product.

Operation.
83469
918159, Ans.

EXERCISE CX.

Find the following products:

2. 45×11. **3.** 352×11. **4.** 4327×11.

5. 74×11. **6.** 576×11. **7.** 8509×11.

451. ***To multiply by any number of two figures ending with 1.***

(8) Multiply 375 by 31.

Explanation.—I place, or conceive to be placed, a 0 on the right of the multiplicand. I then multiply by the ten's figure of the multiplier, and in addition	**Operation.** 3750 11625, Ans.

to carrying as usual, I also carry the figure on the left of the one multiplied.

Thus, 3 times 0 are 0, and 5 is 5; 3 times 5 are 15 and 7 are 22, write 2 and carry 2; 3 times 7 are 21 and 2 and 3 are 26, write 6 and carry 2; 3 times 3 are 9 and 2 are 11, which write.

Find the following products:

9. 83 × 41. **10.** 238 × 91. **11.** 4378 × 51.
12. 75 × 31. **13.** 507 × 61. **14.** 8564 × 71.

NOTE.—This process may, with an evident modification, be employed in multiplying by such multipliers as 601, 7001, etc.

452. ***To multiply two integers, each of which is a little less than 100 or 1000, etc.***

NOTE.—The complement of a number is the difference between it and a unit of the next higher order. The complement of 97 is 100 — 97, or 3; of 995, 1000 — 995, or 5.

(15) Multiply 96 by 89.

Explanation.—From the less of the two numbers (89), I subtract the complement of the other (4), and to the remainder (85), regarded as units of the next higher	**Operation.** 89 4 8544, Ans.

order (hundreds), add or annex the product of the complements (44).

Find the following products:

16. 99 × 91. **17.** 93 × 88. **18.** 998 × 985.
19. 92 × 95. **20.** 79 × 92. **21.** 992 × 983.
22. 99 × 81. **23.** 88 × 94. **24.** 995 × 997.

Find the square:

25. Of 97. **26.** Of 93. **27.** Of 99.

453. ***To multiply two integers, each of which is a little larger than 100, or 1000, etc.***

(28) Multiply 104 by 112.

Explanation.—To the larger of the two numbers (112) I add the excess above 100 of the other (4), and to the sum (116), regarded as units of the next higher order (hundreds), add or annex the product of the two excesses (48).

Operation.

112
4
11648, Ans.

Multiply:

29. 110 by 104. **30.** 121 by 105. **31.** 1024 by 1004.

32. 108 by 107. **33.** 144 by 102. **34.** 1032 by 1008.

Find the square:

35. Of 102. **36.** Of 105. **37.** Of 109.

454. ***To multiply two integers whose unit figures are the same.***

(38) Multiply 76 by 46.

Explanation.—I say 6 times 6 are 36, write 6 and carry 3; 6 times 11, (the sum of 4 and 7) are 66 and 3 are 69; write 9 and carry 6; 4 times 7 are 28 and 6 are 34.

Operation.

76
46
3496, Ans.

Multiply:

39. 43 by 63. **40.** 65 by 85. **41.** 103 by 123.

42. 74 by 54. **43.** 92 by 42. **44.** 252 by 202.

455. ***To square any integer of two figures.***

(45) What is the square of 48?

Explanation.—I say 8 times 8 are 64, write 4 and carry 6; 8 times 8 (twice the tens) are 64 and 6 are 70, write 0 and carry 7; 4 times 4 are 16 and 7 are 23.

Operation.

48
2304, Ans.

Find the square of:

46. 34. **47.** 83. **48.** 76.

49. 43. **50.** 94. **51.** 67.

52. 76. **53.** 57. **54.** 29.

456. ***To multiply two integers of two figures each when the tens are the same, and the sum of the units is 10.***

(55) Multiply 83 by 87.

Explanation.—I multiply one of the tens (8) by the other plus 1 (9), and to the product (72), considered as hundreds, add the product of the units (21).

Operation.

$$\begin{array}{r} 83 \\ \underline{87} \\ 7221 \end{array}$$

Multiply:

56. 24 by 26.	**57.** 81 by 89.	**58.** 102 by 108.
59 69 by 61.	**60.** 62 by 68.	**61.** 114 by 116.
62. 73 by 77.	**63.** 46 by 44.	**64.** 127 by 123.

In a similar manner find the square of:

65. 35.	**66.** 85.	**67.** 45.	**68.** 105.
69. 65.	**70.** 25.	**71.** 95.	**72.** 115.

457. ***To multiply two numbers whose average is a multiple of 5 or 10.***

(73) Multiply 78 by 72.

Solution: $\frac{1}{2}$ of $(78 + 72) = 75$; $\frac{1}{2}$ of $(78 - 72) = 3$;
$75^2 = 5625$; $3^2 = 9$; $5625 - 9 = 5616$, Ans.

Find the following products:

74. 24×36.	**75.** 37×43.	**76.** 94×106.
77. 41×49.	**78.** 78×82.	**79.** 195×205.

458. ***To multiply a mixed number by a mixed number.***

(80) Multiply $9\frac{1}{4}$ by $11\frac{2}{3}$.

Explanation.—I first multiply the integers; next I multiply one of the integers by the fractional part of the other factor; next I multiply that factor by the fractional part of the first factor, then add the three results.

Operation.

$$\begin{array}{rcl} 9 \times 11 &=& 99 \\ 9 \times \frac{2}{3} &=& 6 \\ 11\frac{2}{3} \times \frac{1}{4} &=& \underline{2\frac{11}{12}} \\ && 107\frac{11}{12}, \text{ Ans.} \end{array}$$

Find the following products:

81. $8\frac{1}{3} \times 9\frac{1}{2}$.	**82.** $9\frac{3}{4} \times 8\frac{1}{2}$.	**83.** $12\frac{1}{7} \times 10\frac{2}{3}$.
84. $12\frac{2}{5} \times 10\frac{1}{6}$.	**85.** $15\frac{1}{3} \times 7\frac{2}{5}$.	**86.** $20\frac{5}{6} \times 18\frac{3}{5}$.

NOTE.—When the sum of the fractions is equal to 1, and the integers the same, we may proceed thus: *multiply one of the integers by the other plus* 1, *and to the product annex the product of the fractions.*

Thus, $8\frac{1}{4} \times 8\frac{3}{4} = (8 \times 9) + (\frac{1}{4} \times \frac{3}{4}) = 72\frac{3}{16}$.

Multiply:

87. $5\frac{1}{3}$ by $5\frac{2}{3}$. **88.** $9\frac{2}{5}$ by $9\frac{3}{5}$. **89.** $11\frac{3}{7}$ by $11\frac{4}{7}$.

459. ***To divide one fraction by another.***

(90) Divide $16\frac{1}{2}$ by $3\frac{1}{3}$.

Explanation.—I multiply both dividend and divisor by the L. C. M. of the denominators (6), then divide as in integers.

Operation.

$$\begin{array}{r|l} 3\frac{1}{3} & 16\frac{1}{2} \\ \hline 20 & 99 \\ \hline & 4\frac{19}{20}, \text{ Ans.} \end{array}$$

Divide:

91. $12\frac{3}{4}$ by $3\frac{1}{2}$. **92.** $3\frac{1}{3}$ by $\frac{5}{6}$. **93.** $\frac{11}{12}$ by $3\frac{1}{4}$.

94. $10\frac{1}{2}$ by $2\frac{1}{3}$. **95.** $9\frac{4}{5}$ by $4\frac{2}{3}$. **96.** $17\frac{1}{5}$ by $7\frac{1}{6}$.

CIRCULATING DECIMALS.

460. A **circulating** or **repeating decimal** is a decimal in which a figure or set of figures is constantly repeated in the same order.

Thus, the decimal value:

Of $\frac{2}{3}$ is .666666 + etc., extended indefinitely;

Of $\frac{3}{11}$ is .272727 + etc., extended indefinitely;

both of which are circulating decimals.

461. A **repetend** is the figure or set of figures repeated. In the preceding examples 6 and 27 are the repetends. A repetend is written but once, and is indicated by placing a dot above the first and the last figure.

Thus, .2525 + etc., $= .\dot{2}\dot{5}$, and .666 + etc., $= .\dot{6}$.

Hence, $\frac{2}{3} = .\dot{6}$ and $\frac{3}{11} = .\dot{2}\dot{7}$.

462. A **pure circulating decimal** is a decimal that consists only of a repetend.

Thus, $.\dot{4}\dot{3}$, $.\dot{2}1\dot{5}$, are pure circulating decimals.

463. A **mixed circulating decimal** is a decimal that consists of an unrepeating or *finite* part and a repetend.

Thus, $.2\dot{7}\dot{3}$, $.54\dot{1}7\dot{3}$, are mixed circulating decimals.

464. **Similar repetends** are repetends that begin at the same decimal place.

Thus, $.\dot{5}$ and $.\dot{1}\dot{7}$; $.8\dot{2}\dot{3}$ and $.5\dot{6}5\dot{3}$, are similar repetends.

465. **Conterminous repetends** are repetends that end at the same decimal place.

Thus, $.5\dot{1}\dot{7}$ and $.\dot{3}1\dot{3}$, $.3\dot{2}8\dot{7}$ and $.51\dot{6}\dot{7}$, are conterminous repetends.

466. ***To reduce a pure circulating decimal to a common fraction.***

(1) Reduce $.\dot{3}\dot{6}$ to a common fraction.

$$\begin{array}{lrl} \text{Solution:} & 1 \text{ time } .\dot{3}\dot{6} = & .363636 + \text{etc.} \\ & 100 \text{ times } .\dot{3}\dot{6} = & 36.363636 + \text{etc.} \\ \text{Subtracting,} & 99 \text{ times } .\dot{3}\dot{6} = & 36. \\ \text{Hence,} & .\dot{3}\dot{6} = & \frac{36}{99} = \frac{4}{11}. \end{array}$$

RULE.—*Write the repetend, with the dots and decimal point omitted, as the numerator of a fraction whose denominator consists of as many* 9's *as there are figures in the repetend. Reduce the result to simplest form.*

EXERCISE CXI.

2. Express as common fractions $.\dot{7}$; $.\dot{1}\dot{3}$; $.\dot{3}5\dot{2}$.

3. Reduce to common fractions in their simplest form. $.\dot{1}\dot{5}$; $.\dot{1}4\dot{4}$; $.\dot{3}7\dot{8}$; $.\dot{1}72\dot{8}$.

4. What common fractions are equivalent to $.\dot{4}\dot{5}$, $.\dot{5}3\dot{1}$, $.\dot{3}28\dot{5}$, and $.\dot{4}2857\dot{1}$?

467. ***To reduce a mixed circulating decimal to a common fraction.***

(5) Reduce $.3\dot{5}\dot{4}$ to a common fraction.

$$\begin{array}{lrl} \text{Solution:} & 10 \text{ times } .3\dot{5}\dot{4} = & 3.5454 + \text{etc.} \\ & 1000 \text{ times } .3\dot{5}\dot{4} = & 354.5454 + \text{etc.} \\ \text{Subtracting,} & 990 \text{ times } .3\dot{5}\dot{4} = & 351 \\ \text{Hence,} & .3\dot{5}\dot{4} = & \frac{351}{990} = \frac{39}{110}. \end{array}$$

RULE.—*Subtract the finite part from the entire decimal; write the remainder as the numerator of a fraction whose denominator consists of as many 9's as there are figures in the repetend, with as many ciphers annexed as there are figures in the finite part. Reduce the resulting fraction to its simplest form.*

6. Reduce to common fractions in their simplest form: $.5\dot{1}\dot{8}$; $.16\dot{2}\dot{7}$; $.25\dot{3}\dot{6}$; $.231\dot{4}2\dot{9}$.

7. Reduce to improper fractions: $2.2\dot{7}$; $4.2\dot{3}$; $1.8\dot{1}$; $1.2\dot{3}\dot{5}$.

8. Express as common fractions: $2.3\dot{3}\dot{9}$; $5.7\dot{4}$; $8.12\dot{6}\dot{3}$; $.32\dot{6}\dot{3}$; $.18\dot{9}$.

468. ***To make two or more repetends similar and conterminous.***

(9) Required to make $.\dot{6}$, $.3\dot{2}\dot{5}$ and $.15\dot{1}2\dot{4}$ similar and conterminous.

Solution.

		Similar.		Similar and Conterminous.
$.\dot{6}$	=	$.66\dot{6}$	=	$.6\dot{6}66666\dot{6}$.
$.3\dot{2}\dot{5}$	=	$.32\dot{5}\dot{2}$	=	$.32\dot{5}2525\dot{2}$.
$.15\dot{1}2\dot{4}$	=	$.15\dot{1}2\dot{4}$	=	$.15\dot{1}24124\dot{4}$.

Explanation.—$.\dot{6}$ extended indefinitely gives the same result as $.6\dot{6}66666\dot{6}$ extended indefinitely, and $.3\dot{2}\dot{5}$ extended indefinitely gives the same result as $.32\dot{5}2525\dot{2}$ extended indefinitely; etc.

RULE.—*Extend each repetend until all begin at the same decimal place and all end at the same decimal place.*

Make the following similar and conterminous:

10. $.\dot{4}6\dot{3}$, $.5\dot{8}\dot{1}$ and $.63\dot{2}\dot{4}$.

11. $.\dot{3}2\dot{7}$, $.6\dot{3}\dot{5}$ and $.176\dot{3}$.

12. $.3\dot{2}1\dot{4}$, $.75\dot{4}\dot{3}$ and $.\dot{5}23\dot{9}$.

13. $16.\dot{1}\dot{5}$, $37.1\dot{2}4\dot{8}$ and $.003\dot{1}\dot{7}$.

469. ***To add circulating decimals.***

RULE.—*Make the given repetends, when dissimilar, similar and conterminous. Add as in addition of finite decimals, observing to increase the repetend of the amount by the number, if any, to be carried from the left-hand column of the repetends.*

EXAMPLES.

14. Add $3.\dot{5}$, $7.65\dot{1}$, $1.\dot{7}6\dot{5}$, $6.\dot{1}7\dot{3}$, $51.\dot{7}$, 3.7, $27.\dot{6}3\dot{1}$, and $1.\dot{0}0\dot{3}$ together.

15. Add $5.0\dot{7}7\dot{0}$, $.\dot{2}\dot{4}$, and $7.\dot{1}2494\dot{3}$.

16. Add $3.0\dot{4}$, $6.\dot{4}5\dot{6}$, $23.\dot{3}\dot{8}$, $.2\dot{4}\dot{8}$.

17. Required the value of $.\dot{3} + .45 + .\dot{4}\dot{5} + .\dot{3}5\dot{1} + .646\dot{8} + .64\dot{6}\dot{8} + .6\dot{4}6\dot{8}$, and $.\dot{6}46\dot{8}$.

18. Find the value of $1.25 + 3.\dot{4} + .6\dot{3}\dot{7} + 7.8\dot{8}\dot{5} + 7.875 + 7.8\dot{7}\dot{5} + 11.\dot{1}$.

470. ***To subtract circulating decimals.***

RULE.—*Make the repetends, when dissimilar, similar and conterminous. Subtract as in subtraction of finite decimals; observing to regard the repetend of the subtrahend as increased by* 1, *when it exceeds that of the minuend.*

EXAMPLES.

19. From $7.\dot{1}$ take $5.0\dot{2}$.

20. From $16.13\dot{4}\dot{7}$ take $11.08\dot{8}\dot{4}$.

21. From $3.\dot{1}2\dot{3}$ take $0.\dot{7}\dot{1}$.

22. From $10.0\dot{5}6\dot{3}$ take $8.\dot{2}\dot{7}$.

23. From $104.\dot{1}\dot{4}$ take $13.\dot{6}3\dot{7}$.

24. Find the value of $5.5\dot{1} + 7.7\dot{3}\dot{1} - 1.\dot{8}\dot{3} - .\dot{1}7\dot{3}$.

471. Circulating decimals may be multiplied and divided by first reducing them to common fractions, performing the operations required, and then reducing the result to decimal form.

Find the value of:

25. $16.\dot{2}0\dot{4} \times 32.\dot{7}\dot{5}$.

26. $13.\dot{4}0\dot{1} \times 6.7\dot{5}\dot{4}$.

27. $3.973\dot{4}\dot{8} \div .208\dot{3}$.

28. $.06\dot{6}66\dot{5} \div .9\dot{4}3\dot{2}$.

472. Value, composition and weight of U. S. coins.

COIN.	VALUE.	COMPOSITION.	WEIGHT.
BRONZE.			
One cent.	1 cent.	95 parts copper, 5 pts. tin & zinc.	48 gr. Troy.
NICKEL.			
3-cent piece.	3 cents.	75 parts copper, 25 parts nickel.	30 gr. Troy.
5-cent piece.	5 cents.	75 " " 25 " "	77.16 gr. Troy.
SILVER.			
Dime.	10 cents.	90 parts silver, 10 parts copper.	2.5 grams.
Quarter dollar.	25 cents.	90 " " 10 " "	6.25 "
Half dollar.	50 cents.	90 " " 10 " "	12.5 "
Dollar.	100 cents.	90 " " 10 " "	412.5 gr. Troy.
GOLD.			
Dollar.	100 cents.	90 parts gold, 10 parts copper.	25.8 gr. Troy.
Quarter eagle.	2½ dollars.	90 " " 10 " "	64.5 " "
Three dollar.	3 dollars.	90 " " 10 " "	77.4 " "
Half eagle.	5 dollars.	90 " " 10 " "	129 " "
Eagle.	10 dollars.	90 " " 10 " "	258 " "
Double eagle.	20 dollars	90 " " 10 " "	516 " "

473. The names and values of the English coins are as follows:

The gold coins are the sovereign (= £1), half sovereign (= 10 s.), guinea (= 21 s.), and half-guinea (= 10 s. 6 d.) The *silver* coins are the crown (= 5 s.), the half-crown (= 2 s. 6 d.), the florin (= 2 s.), the shilling, and the six-penny, four-penny, and three-penny pieces. The penny, half-penny, and farthing are the *copper* coins.

474. MISCELLANEOUS TABLES.

The following linear units are often used:

$1\frac{1}{6}$ statute miles	= 1 geographic or nautical mile.
60 geographic, or $69\frac{1}{2}$ statute mi., nearly,	= 1 degree on the equator.
360 degrees	= 1 circumference of the earth.

A *knot*, used for measuring distances at sea, is equivalent to a nautical mile.

4 inches	=	1 hand, for measuring the height of horses.
9 inches	=	1 span.
18 inches	=	1 cubit.
6 feet	=	1 fathom, for measuring depths at sea.
120 fathoms	=	1 cable's length.
3.3 feet	=	1 pace, for measuring approximate distances.
5 paces	=	1 rod. " " "

BOOK MEASURE.

A sheet folded into	The book is	A sheet of paper makes
2 leaves	a folio,	4 pp. (pages).
4 "	a quarto or 4to,	8 "
8 "	an octavo or 8vo,	16 "
12 "	a duodecimo or 12mo,	24 "
16 "	a 16mo,	32 "
18 "	an 18mo,	36 "

The weight of a bushel of certain articles is as follows:

Barley . .	48 lb.	Corn in ear .	68 lb.	Peas . . .	60 lb.
Beans . .	60 "	Clover seed .	60 "	Potatoes . .	60 "
Buckwheat .	42 "	Flax seed . .	56 "	Rye . . .	56 "
Bran . . .	20 "	Hemp seed .	44 "	Salt . . .	56 "
Corn . . .	56 "	Oats . . .	32 "	Timothy seed	45 "
Corn meal .	50 "	Onions . .	60 "	Wheat . .	60 "

NOTE.—In Louisiana the weight of a bushel of oats, barley and rye is each 32 pounds.

The following are in common use:

100 lb. of	grain or flour	=	1 cental.
100 "	dry fish	=	1 quintal.
100 "	nails	=	1 keg.
196 "	flour	=	1 barrel.
200 "	beef or pork	=	1 barrel.
240 "	lime	=	1 cask.
280 "	salt at N. Y. Salt Works	=	1 barrel.

The old French linear and land measure is still partly used in Louisiana, and in other French settlements of the United States.

TABLE.

12 lines	= 1 inch.	6 feet	= 1 toise.
12 inches	= 1 foot.	32 toises	= 1 arpent.

1024 sq. toises = 1 sq. arpent.

The French foot equals 12.79 English inches.

The arpent is the old French name for acre, and is equal to about $\frac{5}{6}$ of an English acre.

DIFFERENCE BETWEEN DATES.

475. There are four methods of finding the time between two dates. To illustrate them let it be required to find the time from June 25, 1886, to Nov. 3, 1889.

First Method.

Explanation.—The process is the same as in denominate numbers, giving 3 yr. 4 mo. 8 da.

Operation.

1889yr.	11mo.	3da.
1886	6	25
3	4	8

Second Method.

Explanation.—I first find the number of whole years, 3 yr. I next find the number of whole months, 4mo. Finally, I find the number of days from Oct. 25 to Nov. 3, 9 da.

Operation.

June 25, '86 to June 25, '89	= 3 yr.
June 25 to Oct. 25,	= 4 mo.
Oct. 25 to Nov. 3,	= 9 da.

Third Method.

Operation.—From June 25, '86 to June 25, '89 = 3 yr.
From June 25 to Nov. 3 = 131 da.

Explanation.—I first find the number of whole years, aud then the number of days, giving 3 yr. 131 da.

Fourth Method.

Solution:	From June 25, '86 to June 25, '87	= 365 da.
	" " " '87 " " '88	= 366 "
	" " " '88 " " '89	= 365 "
	" June 25 to Nov. 3	= 131 "
Hence,	From June 25, '86 to Nov. 3, '89	= 1227 "

EXERCISE CXII

Find by each method the time:

1. From Jan. 29, 1882, to Aug. 15, 1884.
2. From June 18, 1863, to March 1, 1868.

NOTES.—1. Bankers and other accountants generally use "Time Tables," which give the exact number of days between any two dates less than a year apart, and "Interest Tables" which give the interest of $1 at different rates for years, months, and days. These tables are designed for use in business, and not for pupils in the schools.

2. It is well to observe that the method of expressing time is not in accordance with the ordinary principles of Arithmetical Notation. In general, where no sign is written, *plus* is understood. Thus, 375 means 300 + 70 + 5; 4 gal. 3 qt. 1 pt. means 4 gal. + 3 qt. + 1 pt. But 1889 yr. 6 mo. 5 da. 7 hr. 13 min. does not mean 1889 yr. + 6 mo. + etc. It means 7 hr. + 13 min. of the 5th da. of the 6th mo. of the 1889th yr., or 1888 yr. + 5 mo. + 4 da. + 7 hr. + 13 min.

476. TABLE OF LONGITUDES.

	°	′	″	
Austin, Tex	97	44	12	W.
Baltimore, Md	76	36	59	W.
Baton Rouge, La	91	11	13	W.
Berlin, Ger	13	23	0	E.
Bombay, India	72	48	0	E.
Boston, Mass	71	3	30	W.
Charleston, S. C	79	55	49	W.
Chicago, Ill	87	37	45	W.
Cincinnati, O	84	29	45	W.
Constantinople, Tur	28	59	0	E.
Denver, Col	104	59	33	W.
Galveston, Tex	94	47	30	W.
Greenwich, Eng	0	0	0	
London, Eng		5	38	W.
Mobile, Ala	88	2	28	W.
Nashville, Tenn	86	49	0	W.
New Orleans, La	90	3	49	W.
New York City	74	0	24	W.
Paris, France	2	20	22	E.
Pekin, China	116	26	0	E.
Philadelphia, Pa	75	0	10	W.
Portland, Me	70	15	18	W.
Richmond, Va	77	26	4	W.
Rio Janeiro, Brazil	43	20	0	W.
Rome, Italy	12	27	0	E.
Salt Lake City, U	111	53	47	W.
San Francisco, Cal	122	26	45	W.
Savannah, Ga	81	5	26	W.
Sitka, Alaska	135	19	42	W.
St. Louis, Mo	90	12	14	W.
St. Petersburg, Rus.	30	16	0	E.
Sydney, Australia	151	11	0	E.
Vienna, Austria	16	20	0	E.
Washington, D. C.	77	0	36	W.

477. The following table shows the legal and allowable rates of interest in the several States:

NAME OF STATE.	RATE PER CENT.		NAME OF STATE.	RATE PER CENT.	
Alabama	8	8	Missouri	6	10
Arizona	10	Any.	Montana	10	Any.
Arkansas	6	10	Nebraska	10	12
California	10	Any.	Nevada	10	Any.
Canada	6	6	New Hampshire	6	6
Colorado	10	Any.	New Jersey	6	6
Connecticut	6	6	New Mexico	6	12
Dakota	7	12	New York	6	6
Delaware	6	6	North Carolina	6	8
District Columbia	6	10	Ohio	6	8
Florida	8	Any.	Oregon	10	12
Georgia	7	Any.	Pennsylvania	6	6
Idaho	10	24	Rhode Island	6	Any.
Illinois	6	8	South Carolina	7	Any.
Indiana	6	8	Tennessee	6	6
Iowa	6	10	Texas	8	12
Kansas	7	12	United States	6	6
Kentucky	6	10	Utah	10	Any.
Louisiana	5	8	Vermont	6	6
Maine	6	Any.	Virginia	6	8
Maryland	6	6	Washington Territory	10	Any.
Massachusetts	6	6	West Virginia	6	6
Michigan	7	10	Wisconsin	7	10
Minnesota	7	10	Wyoming	12	Any.
Mississippi	6	10			

NOTE.—When the per cent of interest is not mentioned in the note or contract, the first column gives the per cent that may be collected by law. If stipulated in the note, a per cent of interest as high as that in the second column may be collected.

478. ***To compute interest by cancellation.***

Find the interest of $142 for 2 yr. 3 mo. 18 da., at 8%.

Explanation.—I draw a vertical line; on the left I place the principal, time expressed in months and decimal of a month and the rate; on the right I place 12, and proceeding according to the rule of cancellation, obtain $26.528.

Operation.

$$\begin{array}{r|l} 142 & \\ 9.2\quad \cancel{27.6} & \cancel{12}\ \cancel{3} \\ .02\quad \cancel{.08} & \end{array}$$

$$142 \times 9.2 \times .02 = 26.528.$$

For exercises, let the student solve the examples on page 222.

479. COMPOUND INTEREST TABLE.

Showing the amt. of $1, *at various rates, compound int. from* 1 *to* 20 *years*

Yrs	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.
1	1.025000	1.030000	1.035000	1.040000	1.050000	1.060000
2	1.050625	1.060900	1.071225	1.081600	1.102500	1.123600
3	1.076891	1.092727	1.108718	1.124864	1.157625	1.191016
4	1.103813	1.125509	1.147523	1.169859	1.215506	1.262477
5	1.131408	1.159274	1.187686	1.216653	1.276282	1.338226
6	1.159693	1.194052	1.229255	1.265319	1.340096	1.418519
7	1.188686	1.229874	1.272279	1.315932	1.407100	1.503630
8	1.218403	1.266770	1.316809	1.368569	1.477455	1.593848
9	1.248863	1.304773	1.362897	1.423312	1.551328	1.689479
10	1.280085	1.343916	1.410599	1.480244	1.628895	1.790848
11	1.312087	1.384234	1.459970	1.539454	1.710339	1.898299
12	1.344889	1.425761	1.511069	1.601032	1.795856	2.012197
13	1.378511	1.468534	1.563956	1.665074	1.885649	2.132928
14	1.412974	1.512590	1.618695	1.731676	1.979932	2.260904
15	1.448298	1.557967	1.675349	1.800944	2.078928	2.396558
16	1.484506	1.604706	1.733986	1.872981	2.182875	2.540352
17	1.521618	1.652848	1.794676	1.947901	2.292018	2.692773
18	1.559659	1.702433	1.857489	2.025817	2.406619	2.854339
19	1.598650	1.753506	1.922501	2.106849	2.526950	3.025600
20	1.638616	1.806111	1.989789	2.191123	2.653298	3.207136

Yrs	7 per cent.	8 per cent.	9 per cent.	10 per cent.	11 per cent.	12 per cent.
1	1.070000	1.080000	1.090000	1.100000	1.110000	1.120000
2	1.144900	1.166400	1.188100	1.210000	1.232100	1.254400
3	1.225043	1.259712	1.295029	1.331000	1.367631	1.404908
4	1.310796	1.360489	1.411582	1.464100	1.518070	1.573519
5	1.402552	1.469328	1.538624	1.610510	1.685058	1.762342
6	1.500730	1.586874	1.677100	1.771561	1.870414	1.973822
7	1.605781	1.713824	1.828039	1.948717	2.076160	2.210681
8	1.718186	1.850930	1.992563	2.143589	2.304537	2.475963
9	1.838459	1.999005	2.171893	2.357948	2.558036	2.773078
10	1.967151	2.158925	2.367364	2.593742	2.839420	3.105848
11	2.104852	2.331639	2.580426	2.853117	3.151757	3.478549
12	2.252192	2.518170	2.812665	3.138428	3.498450	3.895975
13	2.409845	2.719624	3.065805	3.452271	3.883279	4.363492
14	2.578534	2.937194	3.341727	3.797498	4.310440	4.887111
15	2.759031	3.172169	3.642482	4.177248	4.784588	5.473565
16	2.952164	3.425943	3.970306	4.594973	5.310893	6.130392
17	3.158815	3.700018	4.327633	5.054470	5.895091	6.866040
18	3.379932	3.996019	4.717120	5.559917	6.543551	7.689964
19	3.616527	4.315701	5.141661	6.115909	7.263342	8.612760
20	3.869684	4.660957	5.604411	6.727500	8.062309	9.646291

480. ***Computations by the table.***

(1) Find the compound interest of \$340 for 2 years, at 6%, compounded semi-annually.

Solution: The compound amount of \$1 for 4 yr. at 3%, as shown by the table, is \$1.125509; hence, the compound amount of \$340 for the same time and rate is 340 × \$1.125509, or \$382.67. Therefore, the compound interest = \$382.67 — \$340 = \$42.67.

NOTE.—When the interest is compounded semi-annually, the tabular amount at half the annual rate is taken for twice the number of years. When it is compounded quarterly, the tabular amount at $\frac{1}{4}$ the rate is taken for four times the number of years; etc.

EXERCISE CXIII.

2. Find the compound interest of \$1200 for 11 years, at 7%.

3. What is the amount of \$400 for 12 years, at 4% compound interest?

4. Find the compound amount of \$450 for 15 yr. 6 mo., at $3\frac{1}{2}$%.

5. A man deposits for his son \$200 in a savings bank, at $3\frac{1}{2}$% compound interest. If the deposit is made when the son is 1 year old, how much will it be worth when he is 21, interest compounded annually?

481. ***To find the principal, when the compound interest, time and rate are given.***

(6) What principal, at 7% compound interest, will amount to \$52.432 in 4 years?

Solution: I find from the table that the compound amount of \$1 for the given time and rate is \$1.310796; hence, the required principal is as many dollars as \$1.310796 is contained times in \$52.432, or \$40.

7. What principal, at 6% compound interest, will produce \$2370 in 10 years?

8. What sum will amount to \$640.405 in 6 yr., at 8%, compounded *semi-annually*?

9. What principal must be invested, at 6% compound interest, to yield $1026.54 interest in 3 yr. 2 mo. and 12 da.?

NOTES.—1. When the principal, amount and time are given, the rate may be found exactly or approximately thus:

Divide the amount by the principal and the quotient will be the amount of $1 *for the given time and rate. In the table, opposite the given number of years, find this quotient or the number nearest it in value, and over it may be found the rate required.*

2. When the principal, amount and rate are given, the time may be found thus:

1°. *Divide the amount by the principal.*

2°. *If the quotient be found in the table under the given rate, the years opposite will be the required number of intervals; but if not found exactly, in the table, take the number next less, noting its deficiency, its number of years, and its gain during a full interval.*

3°. *Divide the deficiency by the interval gain, and annex the quotient to the number of full intervals; the result will be the required time.*

10. Find the time in which $5000 will amount to $9430.46, at 8% compound interest, payable semi-annually.

Solution: $9430.46 ÷ 5000 = $1.886092, the amount of $1 at 4%. The number next lower in the 4% column is $1.872981, the amount for 16 intervals, and is less than $1.886092 by $.013111.

According to the table, the gain of an entire interval is $1.947901 — $1.872981, or $.074920. Now $.013111 ÷ $.074920 = $\frac{7}{40}$; hence the time is $16\frac{7}{40}$ intervals of 6 mo. each, or 8 yr. 1 mo. $1\frac{1}{2}$ da.

11. In what time will $500 amount to $703.55, at 5% compound interest?

12. At what rate, by compound interest, will $1000 amount to $1593.85 in 8 years?

13. In what time will $300 amount to $557.401, at 6% compound interest?

14. At what rate compound interest will any sum double itself in 10 years?

15. In what time will any sum double itself, at 6% compound interest?

FOREIGN EXCHANGE.

482. Foreign exchange is the method of making payments between *different* countries.

483. A **set of exchange** consists of three bills of the *same date* and *tenor*, *First*, *Second*, and *Third* of exchange. They are sent by different mails in order to save time in case of miscarriage. When one is *paid*, the others are *void*.

484. The following is the common form of a foreign bill of exchange:

Exchange for £500. CHARLESTON, *May* 2, 1885.

At sight of this First of Exchange, Second and Third of the same tenor and date unpaid, pay to the order of HENRY P. DAVIS, *Five hundred pounds sterling, value received, and charge the same to the account of*

TO TRESVANT & CO., London. WILLIAM D. LEONARD.

The foregoing is the form of the *first* bill; in the second and third bills, the only change required is the substitution of the word *second* or *third* for *first*.

Foreign exchange is affected by the course of trade, and also by the comparative value of the currencies of the two countries concerned, both of which are generally included in the quotations of exchange.

485. Exchange with Europe is chiefly done through large commercial centres, as London, Paris, Geneva, Amsterdam, Antwerp, Hamburg, Frankfort, and Berlin.

The unit of exchange on London is the *pound* sterling; on Paris and Antwerp it is the *franc*; on Berlin, Hamburg and Frankfort it is *four marks*.

486. In quoting exchange on a foreign country, it is customary to quote the *value* of the *money unit* of that country in U. S. *money*.

These values are published annually by the Secretary of the Treasury. Those given on the 1st day of January, 1889, are as follows:

COUNTRY.	MONETARY UNIT.	STANDARD.	VALUE IN U. S. MONEY.
Austria	Florin	Silver	$ .38, 6
Belgium	Franc	Gold and silver	.19, 3
Bolivia	Boliviano	Silver	.68
Brazil	Milreis	Gold	.54, 6
Brit. Pos. in N. A.	Dollar	Gold	1.00
Chili	Peso	Gold and silver	.91, 2
Cuba	Peso	Gold and silver	.92, 6
Denmark	Crown	Gold	.26, 8
Ecuador	Sucre	Silver	.68
Egypt	Pound	Gold	4.94, 3
France	Franc	Gold and silver	.19, 3
German Empire	Mark	Gold	.23, 8
Great Britain	Pound	Gold	4.86, 6½
Greece	Drachma	Gold and silver	.19, 3
India	Rupee	Silver	.32, 8
Italy	Lira	Gold and silver	.19, 3
Japan	Yen	Silver	.73, 4
Liberia	Dollar	Gold	1.00
Mexico	Dollar	Silver	.73, 9
Netherlands	Florin	Gold and silver	.40, 2
Norway	Crown	Gold	.26, 8
Peru	Sol	Silver	.68
Portugal	Milreis	Gold	1.08
Russia	Rouble	Silver	.54, 4
Sandwich Islands	Dollar	Gold	1.00
Spain	Peseta	Gold and silver	.19, 3
Sweden	Crown	Gold	.26, 8
Switzerland	Franc	Gold and silver	.19, 3
Tripoli	Mahbub	Silver	.61, 4
Turkey	Piaster	Gold	.04, 4
U. S. of Colombia	Peso	Silver	.68

487. ***To find the cost of a bill of exchange.***

(1) What is the cost of a bill of exchange in London for £120 5 s. 6d., exchange being quoted at $4.86?

Solution : £120 5 s. 6 d. = £120.275. Since £1 = $4.86, £120.275 = *120.275* × $4.86 = $584.536½, Ans.

(2) What will it cost in U. S. money to settle a debt of 18270 francs in Paris, exchange being 5.22 francs to a dollar?

Solution: Since 5.22 francs = \$1, 18270 francs are equal to as many dollars as 5.22 is contained times in 18270, or \$3500, Ans.

(3) What cost a bill on Berlin for 540 marks, at \$.94½, brokerage being ½%?

Solution: Since 4 marks are worth \$.945, the worth of 540 marks is 540 times ¼ of \$.945, or \$127.58 = cost less brokerage. \$127.58 + ½% of \$127.58 = \$128.21 +, Ans.

488. The method of finding the *face* of a foreign bill of exchange is essentially the same as that of domestic bills.

(4) What is the face of a bill of exchange on London, bought for \$4500, at \$4.87½ in gold?

Solution: Since \$4.875 will buy a bill of £1, \$4500 will buy as many pounds as \$4.875 is contained times in \$4500, and \$4500 ÷ 4.875 = £923.076, or £923. 1 s. 6¼ d., Ans.

EXERCISE CXIV.

5. What is the face of a bill on Dublin, for which \$6500 was paid in gold, at \$4.86?

6. Find the face of a bill on Geneva, which cost \$1500 gold, exchange 5.16.

7. Find the cost of a bill for 1000 francs on Antwerp, at 5.17½ fr. to a dollar, gold at 1% premium?

8. What would be the cost of a sight bill on Berlin for 1680 marks, when exchange is quoted at .96½?

9. What would be the cost in St. Louis of a sight draft on Hamburg for 3200 marks, when exchange is quoted at .96⅜?

10. How large a sight draft on London can be bought in New York for \$1174.20, when sterling exchange is quoted at 4.89¼?

11. What will a bill on London for £200 12 s., payable in 60 days, cost in New York, when sterling exchange is quoted at $4.85\frac{3}{8}$?

12. What will a sight draft on London for £300 8 s. cost, when sterling exchange is quoted at 4.88?

13. Wishing to pay a bill of £860 15 s. in Liverpool, I buy a bill of exchange at 60 days' sight on London; what does it cost, exchange being at 4.87 and brokerage $\frac{1}{2}$%?

14. Find the cost of a bill of exchange on Geneva, Switzerland, for 25600 francs, exchange being 5.18 francs to the dollar, and brokerage $\frac{1}{2}$%.

SPECIFIC GRAVITY.

489. Table of specific gravities of a few well-known substances.

Platinum	22.069	Ale (average)	1.035
Gold, pure, cast	19.258	Milk	1.030
Mercury, common	13.568	Sea-Water	1.028
Lead, cast	11.352	Vinegar	1.026
Silver, pure, cast	10.474	Wine, red port	.990
Copper, cast	8.788	Castor oil	.970
Brass, cast	8.395	Linseed oil	.940
Nickel, cast	8.279	Butter	.942
Iron, cast	7.207	Ice	.930
Loadstone	4.930	Wax	.897
Ruby, Oriental	4.283	Turpentine, oil of	.870
Garnet, precious	4.230	Beech	.852
Diamond, average	3.5[illegible]6	Ash	.845
Marble, Parian	2.837	Hickory	.838
Emerald of Peru	2.775	Brandy	.837
Pearl, Oriental	2.684	Alcohol, absolute	.796
Slate	2.672	Maple	.755
Stone, common	2.520	Walnut	.681
Clay	2.160	Elm	.600
Brick	2.000	Cypress	.598
Honey	1.456	Willow	.585
Opium	1.337	Cedar	.561
Lignum-vitae	1.333	Poplar	.383
Blood, human	1.045	Cork	.240

ARITHMETICAL PROGRESSION.

490. An **arithmetical progression** or series is a succession of numbers, each of which is greater or less than the preceding number by a constant difference.

(1) 2, 5, 8, 11, 14, etc., is an *increasing* arithmetical progression, in which the common difference is 3.

(2) 27, 23, 19, 15, 11, etc., is a *decreasing* arithmetical progression, in which the common difference is 4.

There are five elements in every arithmetical progression, viz.: the first term (a), the last term (l), the number of terms (n), the common difference (d), and the sum of all the terms (s). The first and last terms are the extremes. If any three of these elements are given the other two may be found.

491. ***To find the last term when the first term, common difference and number of terms are given.***

(1) If the following increasing progression,

5, 8, 11, 14, 17, etc.,

were extended until there were 15 terms in all, what would the last term be?

Operation.

$5 + (15 - 1)3 = 47$, Ans.

Explanation.—5, the first term, *increased* by *once* 3 gives the second term, by *twice* 3 gives the third term, by *three* times 3 gives the fourth term, etc. Hence, 3 increased by *fourteen* times 3 gives the fifteenth term.

Formula: $l = a + (n - 1) \times d.$

(2) If the following decreasing progression,

41, 37, 33, 29, 25, etc.,

were extended until there were 10 terms in all, what would the last term be?

Operation.

$41 - (10 - 1)4 = 5$, Ans.

Explanation.—41, the first term, decreased by *once* 4 (the common difference) gives the second term, by *twice* 4 gives the third term, by *three* times 4 gives the fourth term. Hence, 41 decreased by *nine* times 4 gives the tenth term.

Formula: $l = a - (n - 1) \times d.$

RULE.—*Multiply the common difference by a number less by* 1 *than the number of terms; add the product to the first term of an increasing series, subtract it from the first term of a decreasing series, and the result will be the last term.*

EXERCISE CXV.

3. Find the 30th term of the series 4, 9, 14, 19, etc.
4. Find the 20th term of the series 7, 10, 13, 16, etc.
5. Find the 10th term of the series 23, 21, 19, 17, etc.
6. Find the 15th term of the series 71, 68, 65, 62, etc.
7. Find the 12th term of the series $5\frac{1}{3}$, $8\frac{2}{3}$, 12, etc.
8. Find the 21st term of the series 103, $100\frac{1}{2}$, 98, etc.

9. Find the last term of an ascending series, the first term of which is 5, the common difference 3, and the number of terms 51.

10. The first term of a descending series is 200, the common difference 3, and the number of terms 47; what is the last?

492. *To find the first term when the last term, number of terms, and common difference are given.*

By reversing the terms of a series, the last term of an increasing series becomes the first term of a decreasing series, and the last term of a decreasing series becomes the first term of an increasing series. Hence,

RULE.—*Find the first term of an increasing series as if it were the last term of a decreasing series, and find the first term of a decreasing series as if it were the last term of an increasing series.*

11. Find the first term of an increasing arithmetical series, the last term being 191, the number of terms 45, and the ommon difference 4.

12. The last term of a decreasing arithmetical series is 7, the number of terms 31, and the common difference 5; what is the first term?

13. A man deposited in a savings bank \$7 for his son when the boy was one year old, and increased the deposit by \$10 at each subsequent birthday until his son was 21 years old; what was the last deposit?

493. ***To find the sum of all the terms when one of the extremes, the number of terms, and the common difference are given.***

(14) Find the sum of the eight terms of the following arithmetical progression:

3, 7, 11, 15, 19, 23, 27, 31.

Explanation.—The average of any two terms equally distant from the extremes, as 7 and 27, is $\frac{1}{2}$ of $(3 + 31)$, or 17. Hence, the average of all the terms is 17; therefore the sum of the eight terms is 8×17, or 136.

Operation.
$\frac{1}{2}$ of $(3 + 31) \times 8 = 136$, Ans.

Formula: $s = \left(\frac{a + l}{2}\right) \times n.$

RULE.—*Multiply half the sum of the extremes by the number of terms.*

NOTE.—If either of the extremes is not given, find it as in Art. 491, or 492.

15. If the first term is 5, the last term 62, and the number of terms 20, what is the sum of the terms?

16. Find the sum of the integers 1, 2, 3, 4, 5, etc., carried to 100 terms.

17. Find the sum of the odd numbers from 1 to 101 inclusive, of which there are 51.

Find the sum:

18. Of 20 terms of the series 2, 6, 10, 14, etc.

19. Of 37 terms of the series 165, 161, 157, etc.

20. Of 50 terms of the series $\frac{1}{3}$, $\frac{2}{3}$, 1, $1\frac{1}{3}$, etc.

21. Of 12 terms of the series 1, $1\frac{3}{4}$, $2\frac{1}{2}$, etc.

22. A boy saved \$.50 the first week, \$1.00 the second week, \$1.50 the third week, and so on; how much had he at the end of the year?

23. A falling body descends $16\frac{1}{2}$ ft. the first second, 3 times this distance the next, 5 times the next, and so on; (1) how far does it fall the 30th second? (2) How far does it fall altogether in 30 seconds?

24. Find the sum of \$800, \$800 plus its interest for 1 yr., \$800 plus its interest for 2 yr., and so on to \$800 plus its interest for 5 yr., rate of interest being 5%.

25. A laborer agreed to work for \$12 a month, payable at the end of each month; after working 12 months, and receiving no pay, how much is he entitled to, at 6% interest?

26. What is the annual interest of \$450 for 20 yr., at 8%?

GEOMETRICAL PROGRESSION.

494. A **geometrical progression** or series is a succession of numbers that increase or decrease by a constant multiplier, called the ratio.

(1) 3, 6, 12, 24, etc., is an increasing geometrical progression, in which the ratio is 2.

(2) 81, 27, 9, 3, etc., is a decreasing geometrical progression, in which the ratio is $\frac{1}{3}$.

There are five elements in every geometrical progression, viz: the first term (a), the last term (l), the number of terms (n), the ratio (d), and the sum of all the terms (s).

495. ***To find the last term when the first term, the ratio, and the number of terms are given.***

(1) If the following geometrical progression

3, 6, 12, 24, etc.,

is extended until there are 8 terms in all, what will the last be?

Explanation.—The first term multiplied by 2^1 (the ratio) gives the second term; multiplied by 2^2 gives the third term; multiplied by 2^3 gives the fourth term, etc. Hence, the first term multiplied by 2^{8-1} (the ratio raised to a power whose exponent is less by 1 than the number of terms) gives the eighth term.

Operation.

$3 \times 2^{8-1} = 384$, Ans.

Formula: $l = a \times r^{n-1}$.

RULE.—*Multiply the first term by the ratio raised to a power whose exponent is less by* 1 *than the number of terms; the product will be the last term.*

NOTES.—1. When the progression is decreasing the ratio is a proper fraction, and may be found by dividing the second term by the first.

2. When the last term is given instead of the first, the first may be found by reversing the series, and applying the above rule, remembering that reversing the series inverts the ratio.

EXERCISE CXVI.

Find the

2. 9th term of a geometrical progression whose first term is 5, and ratio 4.

3. 8th term of a geometrical progression whose first term is 28672, and ratio $\frac{1}{4}$.

4. 7th term of the series 5, 10, 20, etc.

5. 12th term of the series 64, 32, 16, etc.

6. 6th term of the series 2, 5, $12\frac{1}{2}$, etc.

7. 6th term of the series $3\frac{3}{8}$, $2\frac{1}{4}$, $1\frac{1}{2}$, etc.

8. A man sold 12 hogs; for the first he received 1 ct., for the second 2 cts., for the third 4 cts., etc.; what did he receive for the 12th hog?

9. What is the amount of $600 for 4 years at 5%, compounded annually?

Suggestion: Ratio = 1.05, number of terms = 5.

196. ***To find the sum of all the terms when one of the extremes, the ratio, and the number of terms are given.***

(9) Find the sum of 2, 6, 18, 54, 162.

Solution.

Let $s = 2 + 6 + 18 + 54 + 162.$

Multiply by 3, $3s = 6 + 18 + 54 + 162 + 3 \times 162.$

Subtract the first equation from the second,

$$(3 - 1) \times s = 3 \times 162 - 2$$

Divide by $(3 - 1)$ $s = \frac{3 \times 162 - 2}{3 - 1} = 242$, Ans.

Formula: $$s = \frac{r \times l - a}{r - 1}.$$

RULE.—*Multiply the last term by the ratio, subtract the first term from the product, and divide the remainder by the ratio less* 1.

10. The first term is 4, the ratio 3, and the last term 972; what is the sum of the terms?

11. A man bought 12 sheep, paying 2 cts. for the first, 6 cts. for the second, and so on; what did he pay for all?

Find the sum:

12. Of 9 terms of the series 1, 3, 9, etc.

13. Of 7 terms of the series 8, 20, 50, etc.

14. Of 10 terms of the series 32, 16, 8, etc.

15. The first term of a decreasing series is 972, the last term 4, and the ratio $\frac{1}{3}$; find the sum of the series.

16. How large a debt may be discharged in 11 months by paying 10 cts. the first month, 30 cts. the second month, 90 cts. the third month, and so on?

17. A man deposited in a savings bank $500 for his son when the boy was 15 years old, and $500 each subsequent birthday until his son was 21 years old; what amount did bank then owe the son, allowing 6% compound interest?

ANNUITY.

497. An **annuity** is a sum of money to be paid regularly at stated periods.

498. A **certain annuity** is an annuity that continues for a specified time.

499. A **contingent annuity** is an annuity, the payment of which is dependent upon some particular circumstance.

500. A **perpetual annuity** is an annuity that is to continue without specified limit.

An annuity is said to be *in possession* when there is a present claim upon it; *in reversion* or *deferred* when the claim upon it begins in the future.

501. The **present worth** of an annuity is the sum that in the given time and at the given rate amounts to the sum of all the payments, with the interest on each from the time it is due until the annuity ceases.

502. The **amount** or **final value** of an annuity is the sum of all the payments, with interest on each until the annuity ceases.

503. ***To find the amount of an annuity at simple interest.***

(1) What is the amount of an annuity of $600 for 5 years, at 5% simple interest?

Operation.

$$\frac{\$600 + \$720}{2} \times 5 = \$3300$$

Explanation.—From the time the first payment is due until the last payment is due is 4 yr.; that is, the first payment draws 4 years' interest; the second 3 years' interest; the third, 2; the fourth, 1, and the fifth draws no interest. These amounts form a decreasing arithmetical progression of 5 terms, of which the first is $600 plus its interest for 4 years, or $720, and the last $600, the common difference being $30, or the interest of $600 for 1 year. The sum of the series is the amount or final value of the annuity.

EXERCISE CXVII.

2. What is the amount of an annuity of $1800 for 10 yr., at 8%?

3. If a clerk saves $500 a year, and invests it at 6% simple interest, what will he be worth in 20 years?

4. What will the rent of a farm, at $1500 a year, amount to in 12 years, if the rent when received is invested at 5%?

NOTE.—When the periods and payments are equal, the balance due on notes or accounts, according to the mercantile rule (354), may be easily computed by the principles of annuities at simple interest, as in the following example:

5. A merchant bought goods, Jan. 1, to the amount of $1500, and paid $120 at the end of each month; how much did he owe at the beginning of the next year, at 5% interest?

504. ***To find the amount of an annuity at compound interest.***

(6) What is the final value of an annuity of $500 for 6 years, at 4% compound interest?

Operation.

$$\frac{\$500 \times 1.04^6 - \$500}{.04} = \$3316.49$$

Explanation. — From the time when the first payment is due until the last payment is due is 5 years; that is, the first payment draws compound interest for 5 years. The second payment draws compound interest for 4 years; the third, 3 years; the fourth, 2 years; the fifth, 1 year; and the sixth draws no interest. These amounts, taken in reverse order, form an increasing geometrical series of 6 terms, the first of which is $500, and the ratio 1.04. The sum is found as explained in geometrical progression.

7. What is the amount, at 6% compound interest, of an annuity of $800 for 8 years?

8. A man deposited $1000 a year for 6 years in a savings bank that paid 4% compound interest; what amount was due him?

9. If a person saves $500 a year and invests it at 5% compound interest what will he be worth in 20 years?

10. What is the amount, at 5% compound interest, of an annuity of $2000 for 10 years?

NOTE.—When the periods and payments are equal, the balance due on notes, according to the U. S. Rule (353), may be easily computed by the principles of annuities at compound interest, as in the next example:

11. A man gave his note for $10000, on which he made annual payments of $1500 each for 6 years; how much was then due, allowing 6% compound interest on principal and payments?

505. Amount of $1 annuity, at compound interest, from 1 year to 25, inclusive.

Yrs	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.
1	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000
2	2.030 000	2.035 000	2.040 000	2.050 000	2.060 000	2.070 000
3	3.090 900	3.106 225	3.121 600	3.152 500	3.183 600	3.214 900
4	4.183 627	4.214 943	4.246 464	4.310 125	4.374 616	4.439 943
5	5.309 136	5.362 466	5.416 323	5.525 631	5.637 093	5.750 739
6	6.468 410	6.550 152	6.632 975	6.801 913	6.975 319	7.153 291
7	7.662 462	7.779 408	7.898 294	8.142 008	8.393 838	8.654 021
8	8.892 336	9.051 687	9.214 226	9.549 109	9.897 468	10.259 803
9	10.159 106	10.368 496	10.582 795	11.026 564	11.491 316	11.977 989
10	11.463 879	11.731 393	12.006 107	12.577 893	13.180 795	13.816 448
11	12.807 796	13.141 992	13.486 351	14.206 787	14.971 643	15.783 599
12	14.192 030	14.601 962	15.025 805	15.917 127	16.869 941	17.888 451
13	15.617 790	16.113 030	16.626 838	17.712 983	18.882 138	20.140 643
14	17.086 324	17.676 986	18.291 911	19.598 632	21.015 066	22.550 488
15	18.598 914	19.295 681	20.023 588	21.578 564	23.275 970	25.129 022
16	20.156 881	20.971 030	21.824 531	23.657 492	25.670 528	27.888 054
17	21.761 588	22.705 016	23.697 512	25.840 366	28.212 880	30.840 217
18	23.414 435	24.499 691	25.645 413	28.132 385	30.905 653	33.999 033
19	25.116 868	26.357 180	27.671 229	30.539 004	33.759 992	37.378 965
20	26.870 374	28.279 682	29.778 079	33.065 954	36.785 591	40.995 492
21	28.676 486	30.269 471	31.969 202	35.719 252	39.992 727	44.865 177
22	30.536 780	32.328 902	34.247 970	38.505 214	43.392 290	49.005 739
23	32.452 884	34.460 414	36.617 889	41.430 475	46.995 828	53.436 141
24	34.426 470	36.666 528	39.082 604	44.501 999	50.815 577	58.176 671
25	36.459 264	38.949 857	41.645 908	47.727 099	54.864 512	63.249 030

506. Present worth of $1 annuity, at compound interest, from 1 year to 50, inclusive.

Yrs.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	8 per cent.	10 per cent
1	.961538	.952381	.943399	.934579	.925926	.909091
2	1.886095	1.859410	1.833393	1.808018	1.783265	1.735537
3	2.775091	2.723248	2.673012	2.624316	2.577097	2.486552
4	3.629895	3.545951	3.465106	3.387211	3.312127	3.169867
5	4.451822	4.329477	4.212364	4.100197	3.992710	3.791787
6	5.242137	5.075692	4.917324	4.766540	4.622880	4.355261
7	6.002055	5.786373	5.582381	5.389289	5.206370	4.868419
8	6.732745	6.463213	6.209794	5.971299	5.746639	5.334926
9	7.435332	7.107822	6.801692	6.515232	6.246888	5.759024
10	8.110896	7.721735	7.360087	7.033582	6.710081	6.144567
11	8.760477	8.306414	7.886875	7.498674	7.138964	6.495061
12	9.385074	8.863252	8.383844	7.942686	7.536078	6.813692
13	9.985648	9.393573	8.852683	8.357651	7.903776	7.103356
14	10.563123	9.898641	9.294984	8.745468	8.244237	7.366687
15	11.118387	10.379658	9.712249	9.107914	8.559479	7.606080
16	11.652296	10.837770	10.105895	9.446649	8.851369	7.833709
17	12.165669	11.274066	10.477260	9.763223	9.121638	8.021553
18	12.659297	11.689587	10.827603	10.059087	9.371887	8.201412
19	13.133939	12.085321	11.158116	10.335595	9.603599	8.364920
20	13.590326	12.462210	11.469921	10.594014	9.818147	8.513564
21	14.029160	12.821153	11.764077	10.835527	10.016803	8.648694
22	14.451115	13.163003	12.041582	11.061241	10.300744	8.771540
23	14.856842	13.488574	12.303379	11.272187	10.371059	8.883218
24	15.246993	13.798642	12.550358	11.469334	10.528758	8.984744
25	15.622080	14.093945	12.783356	11.653583	10.674776	9.077040
26	15.982769	14.375185	13.003166	11.825779	10.809978	9.160945
27	16.329586	14.643034	13.210534	11.986709	10.935165	9.237223
28	16.663063	14.898127	13.406164	12.137111	11.051078	9.306567
29	16.983715	15.141074	13.590721	12.277674	11.158406	9.369606
30	17.292033	15.372451	13.764831	12.409041	11.257783	9.426914
31	17.588494	15.992811	13.929086	12.531814	11.349799	9.479013
32	17.873552	15.802677	14.084043	12.646555	11.434999	9.526376
33	18.147646	16.002549	14.230230	12.753790	11.513888	9.569432
34	18.411198	16.192904	14.368141	12.854009	11.586934	9.608575
35	18.664613	16.374194	14.498246	12.947672	11.654568	9.644159
36	18.908282	16.546852	14.620987	13.035208	11.717193	9.676508
37	19.142579	16.711287	14.736780	13.117017	11.775179	9.705917
38	19.367864	16.867893	14.846019	13.193473	11.828869	9.732651
39	19.584485	17.017041	14.949075	13.264928	11.878582	9.756956
40	19.792774	17.159086	15.046297	13.331709	11.924613	9.779051
41	19.993052	17.294368	15.138016	13.394120	11.96735	9.799137
42	20.185627	17.423208	15.224543	13.452449	12.006699	9.817397
43	20.370795	17.545912	15.306173	13.506962	12.043240	9.833998
44	20.548841	17.662773	15.383182	13.557908	12.077074	9.849089
45	20.720040	17.774070	15.455832	13.605522	12.108402	9.862808
46	20.884654	17.880067	15.524370	13.650020	12.137409	9.875280
47	21.042936	17.981016	15.589028	13.691608	12.164267	9.886618
48	21.195131	18.077158	15.650027	13.730474	12.189136	9.896926
49	21.341472	18.168722	15.707572	13.766799	12.212163	9.906296
50	21.482185	18.255925	15.761861	13.800746	12.233485	9.914814

Calculations by the Table.

507. ***To find the present worth of an annuity, at compound interest.***

(1) What is the present worth of an annuity of $100 for 9 years, at 6%.

Operation.

100 × $6.801692 = $680.1692.

Explanation.—I find from the table that the present worth of an annuity of $1 for the given time and rate is $6.801692; hence, the present worth of an annuity of $100 is 100 × $6.801692, or $680.1692.

RULE.—*Multiply the present worth of an annuity of $1 for the given time and rate by the number of dollars in the given annuity.*

EXERCISE CXVIII.

2. Find the present worth of an annuity of $500 for 10 years, at 5%.

3. What is the present worth of an annuity of $1000, to continue 15 years, at 8%?

4. A gentleman wishes to purchase an annuity, which shall afford him, at 7% compound interest, $500 a year, for ten years; what sum must he deposit in the annuity office to produce it?

5. If a widow be entitled to $250 a year, payable semi-annually, from a fund, for 8 years, what is its value at present, at 8% compound interest?

508. ***To find the annuity when the present worth, time and rate are given.***

(6) The present value of an annuity for 4 years, at 6% compound interest, is $207.90; what is the annuity?

Operation.

207.90 ÷ 3.465 = 60

Explanation.—Since the present value of an annuity of $1 for the given time and rate is $3.465, an annuity whose present value is $207.90 is as many dollars as $3.465 is contained times in $207.90, or $60.

RULE.—*Divide the given present worth by the present worth of an annuity of* $1 *for the given time and rate, and the result will be the annuity required.*

NOTE.—When the amount of an annuity, the time and rate, are given, the *annuity* may be found *by dividing the given amount by the amount of* $1 *for the given time and rate.*

7. The present value of an annuity, to be continued 10 years, at 6 % compound interest, payable annually, is $3680.04; required the annuity.

8. How much a year should I pay to secure $15,000 at the end of 17 years, interest 7 %?

9. A man sells a farm for $6000; if this amount is paid in 11 equal annual payments, at 6% compound interest, what will be the value of each payment?

10. What amount must I deposit annually in a savings bank in order that it may amount to $6000 in 11 years, at 6% compound interest?

BUILDING ASSOCIATIONS.

509. **Building associations** have for their object the accumulation of a fund from which the members can obtain the means to purchase lots and build, buy or repair houses.

510. The shares are usually estimated at $100 each, and are paid for in monthly installments of $1 per share. When the accumulated payments are sufficient, the funds are offered at auction, and given to the member paying the largest bonus or discount. Interest on the loan thus made is paid at the same time as the monthly or periodical dues, and the loans are secured by mortgage on real estate. Generally, on all sums not paid when due, a fine of 5 or 10 per cent is imposed.

The association closes when the total amount received is sufficient to give each member the amount originally agreed upon.

"As promotive of habits of economy, and as affording means of profitable investments, these associations have been highly successful. The chief benefit, however, is derived from the increase in the value of the property purchased, and in the convenient form in which payments are made. Practically, these associations have given homes to hundreds who would otherwise never have owned them."

NOTE.—In the following problems, the shares are supposed to be \$100 each, the monthly payment on each share \$1, and the rate of interest 6%.

511. ***To find the cost of a share at simple interest.***

(1) What is the cost of a share in an association which has been in operation 5 years?

Explanation.—60 payments of \$1 each have been made, = \$60. The owner has lost the use of the first payment for 59 mo.; of the second, 58 mo.; of the third, 57 mo., etc., which loss = \$8.85. See Art. 503. Hence the entire cost to the owner is \$68.85.

Operation.

$59 \times \$.005 = \$\ \ .295$

$30 \times \$.295 = \$\ 8.85$

$60 \times \$1 \quad = \$60.$

$\$68.85$, Ans.

RULE—*Multiply the interest of the monthly payment for one month by the number of months, less* 1, *and this product by half the number of months, and to the result add the product of the monthly payment by the total number of payments.*

EXERCISE CXIX.

2. Find the cost of 10 shares for 66 months.

3. A owns 20 shares in a building association which has been in operation $3\frac{1}{2}$ years; what has A's investment cost him up to date?

512. ***To find the rate of discount at which stock may be sold without loss.***

4. At what rate of discount may the stock in example 2 be sold without loss?

Solution.—The nominal value of the shares is \$1000, and the cost \$767.25; the latter is 76.7 +% of the former; hence, the rate of discount is 100% — 76.7 +%, or 23.2 +%, Ans.

RULE.—*Find what per cent the cost of the shares is of their nominal value, and subtract the result from 100 per cent.*

5. Find the rate of discount at which the stock in example 3 may be sold without loss.

6. At what rate of discount may 30 shares be sold without loss, at the expiration of 82 months?

NOTE.—At 6% simple interest the cost of a share will be equal to its par value ($100), in about 82.98 months.

513. *To find the cost of a loan at simple interest.*

NOTE.—Only the approximate cost of a loan can be determined before the association closes; for, it is impossible to know the exact time the association will continue, owing to the variations of discounts, number of borrowers, etc. From 5 to 6 years, under the conditions stated, is the usual time.

(7) A member of an association who owns 12 shares, borrows $800, 68 months before the association closes; find the cost of the loan at the close.

Solution.—The monthly payments are $12 ($1 per share) plus $4 (1 month's interest of $800), or $16. That is, 68 payments of $16 each are made, of which the borrower loses the interest of the first for 67 mo.; of the second, for 66 mo., etc. Hence, the cost to the borrower is

(1) 68 payments, $16 each	$1088.
(2) Int. of $16 for 67 mo., 66 mo., etc	182.24
Cost of loan	$1270.24

8. A member who owns 20 shares secures a loan at a discount of 25% 61 months before the association closes; find the cost of his loan.

9. A held 30 shares and effected a loan at 35% discount 74 months prior to the closing of the association; how much had his loan cost him at the close?

10. An association was in operation 66 months; at the expiration of the 15th month a member who owned 6 shares

secured a loan at 20% discount; what was the cost of the loan at the close of the association?

11. A holds 20 shares and secures a loan at 70%, with which he builds a house; required the cost of the house at the end of 66 months.

NOTES.—1. The cost of a share or loan at compound interest may be found in accordance with Art. 504, or by the table on page 339.

2. When the time and rate are known, the present worth of a share or loan at compound interest, may be determined by the table on page 340.

EXTRACTION OF ANY ROOT.

514. When the index of the root to be extracted is a composite number, the root may be obtained by extracting successively the roots denoted by the factors of the given index.

Thus, the 4th root may be obtained by taking the square root of the square root; the 6th root, by taking the cube root of the square root; etc.

(1) $\sqrt[4]{256} = \sqrt{\sqrt{256}} = \sqrt{16} = 4.$

(2) $\sqrt[6]{15625} = \sqrt[3]{\sqrt{15625}} = \sqrt[3]{125} = 5.$

EXERCISE CXX.

1. What is the fourth root of 68719476736?
2. What is the sixth root of 11390625?
3. What is the ninth root of 262144?
4. What is the fourth root of 1.08?
5. What is the sixth root of 11?

HORNER'S METHOD.

515. Horner's method, named from its inventor, W. G. Horner, Esq., of Bath, England, is a simple and elegant process, and may be advantageously applied in extracting any root, whether the given number be a perfect power or not, and the given index a prime or composite number.

516. ***To extract any root of a number.***

(6) What is the cube root of 77308776?

Operation.

0	0	77,308,776
4	16	64
4	16	13308
4	32	10088
8	4800	3220776
4	244	3220776
120	5044	
2	248	
122	529200	
2	7596	
124	536796	
2		
1260		
6		
1266		

Explanation.

The greatest cube contained in the left-hand period is 64, whose root, 4, I write as the first figure of the answer. This figure 4 I write under the cipher of the first column; and adding it to the 0, obtain 4; this sum multiplied by the 4 gives 16, which I write under the next 0 to the right, and adding it to the 0, obtain 16. This sum I also multiply by the 4, write the product, in the last column under the left-hand period, 77, of the given number, subtract 64 from 77, and obtain 13, rem.

Again, I add the 4 to the last term, 4, of the 1st col., obtaining 8; and the result, 8, multiplied by the 4 gives 32, which I write under the last term, 16, of the 2d col , and, adding, obtain 48.

Again, I add the 4 to the last term, 8, of the 1st col. and obtain 12. I now annex one 0 to 12, two 0's to 48, and to the remainder, 13, annex the next three figures of the given number.

To obtain the next figure of the root, I take the last term of the 2d col., 4800, for a trial divisor, and the last term of the 3d col., 13308, for a dividend; and dividing, obtain 2.

I now proceed with the 2 just as I did with the 4. **Thus:**

I add 2 to the last term, 120, of the 1st col., multiply the result, 122, by 2, add the product, 244, to the last term of the 2d col. and obtain 5044, which sum I also multiply by 2, and subtract the product, 10088, from the last term of the 3d col., which leaves a rem. of 3220.

Again, I add the 2 to the last term, 122, of the 1st col., multiply the sum, 124, by 2, add the product, 248, to the last term of the 2d col.

Again, I add the 2 to the last term of the 1st col., and obtain 126.

I now annex one 0 to 126, two 0's to 5292, and to the last term of the 3d col. annex the next period of the given number.

To obtain the next figure of the root I divide, as before, the last term of the 3d col., 3220776, by the last term of the 2d col., and obtain 6.

I now proceed with the 6, as with the 4 and 2, and since there is no *remainder,* the required root is 426.

RULE.—*Commence as many columns as there are units in the exponent of the root to be extracted, by writing the given number as the head of the right-hand column, and a cipher as the head of each of the others.*

Separate the given number into as many periods as possible of as many figures each as the exponent of the root requires; and having found the nearest root of the left-hand period, write it as the first figure of the required root.

Write this figure in the first column, and, having added it to what stands above it, multiply the sum by the same figure, and write the product in the second column; add, in like manner, in the second column, and multiply the sum by the same figure, writing the product in the third column; and so proceed, writing the last product in the last column, and subtracting it from what stands above it.

Then add the same figure to the last term of the first column, multiply the sum by the same figure, and add the product to the last term of the second column; and so on, writing the last product in the last column but one. Repeat the process, stopping each time with one column farther to the left, till the last product shall fall in the second column.

Add the figure found for the root to the last term of the first column; annex one cipher to the last number in the first column, two ciphers to the last number in the second column, and so on; and to the last number in the last column bring down the next period for a dividend.

Take the last term of the column next to the last for a trial divisor, and see how often it is contained in the dividend, and write the result as the next figure of the root.

Add this figure to the last term of the first column, multiplying the sum by the same figure, add the product to the

second column, and so on; proceed as before, till all the periods have been brought down, or an answer sufficiently exact has been obtained.

NOTES.—1. When any dividend will not contain its corresponding trial divisor, write a cipher in the root, bring down to the dividend another period, annex an additional cipher to the last term of the first column, two additional ciphers to the last term of the second column, and so on; and use the same trial divisor as before, increased, however, by the additional ciphers.

2. When the given number does not have an exact root, periods of ciphers may be annexed.

7. What is the cube root of 92959677?

8. What is the cube root of 25?

9. What is the fifth root of 14348907?

10. What is the fourth root of 97.41?

11. A troy pound contains 5760 grains; what would be the scale if it were uniform in passing from gr. to pwt., from pwt. to oz., and from oz. to lb.?

DUODECIMALS.

517. **Duodecimals** is a system of numbers whose scale is 12. Its principal application is by artificers in estimating areas and volumes.

518. The foot is the principal unit. It is divided and subdivided according to the following

TABLE.

12 fourths ('''')	= 1 third,	written 1'''.
12 thirds	= 1 second,	" 1''.
12 seconds	= 1 inch, or prime,	" 1'.
12 primes	= 1 foot,	" 1 ft.

Multiplication of Duodecimals.

(1) Find the area of a table 6 ft. 10' by 3 ft. 8'.

Explanation.—Beginning at the right, I say $8' \times 10' = 80'' = 7'\,6''$, write 6'' to the right and carry 7'; $8' \times 6$ ft. $= 48', + 7' = 55' = 4$ sq. ft. 7', I next multiply 6 ft. 10' *by 3 ft.* and obtain 20 sq. ft. 6', *then add* the partial products.

Operation.

6 ft. 10'		
3 ft. 8'		
4 sq. ft.	7'	6''
20 sq. ft.	6'	
25 sq. ft.	1'	6''

EXERCISE CXXI.

2. Multiply 14 ft. 8′ by 1 ft. 2′.

3. Multiply 13 ft. 7′ 8″ by 6 ft. 5′.

4. Multiply the square of 5 ft. 10′ by 4 ft. 8′.

5. How many square feet in a stage 9 ft. 6′ by 3 ft. 5′?

6. A rectangular bin is 9 ft. 6′ long, 4 ft. 8′ wide, and 3 ft. 4′ deep; find its solid contents.

CALENDAR.

519. *To find the day of the week corresponding to any given date of the nineteenth century.*

RULE.—I. *Multiply the last two figures of the preceding year by* $1\frac{1}{4}$, *rejecting fractions, if any, and to the product add the day of the month, and* 1 *for January,* 4 *for February,* 4 *for March,* 0 *for April,* 2 *for May,* 5 *for June,* 0 *for July,* 3 *for August,* 6 *for September,* 1 *for October,* 4 *for November, and* 6 *for December.*

II. *Divide the sum by* 7, *and the day will be indicated by the remainder, thus: no remainder denotes Tuesday;* 1, *Wednesday;* 2, *Thursday;* 3, *Friday, etc.*

NOTE.—For dates after February in leap years, always increase the remainder by 1.

(1) On what day of the week was Nov. 7, 1861?

Solution: $60 \times 1\frac{1}{4} = 75$; $75 + 7 + 4 = 86$; $86 \div 7 = 12$ quo. and 2 rem. Hence, Thursday, Ans.

EXERCISE CXXII.

2. On what day of the week was June 16, 1844?

3. On what day of the week was April 19, 1889?

4. On what day of the week were you born?

5. On what day of the week was Gen. R. E. Lee born, Jan. 19, 1807?

MISCELLANEOUS PROBLEMS.

NOTE.—Each of the following exercises may be taken up immediately after completing the chapter having the same subject matter.

INTEGERS—I.

EXERCISE CXXIII.

520. 1. A farmer had $1768, of which he gave $175, $238.25 for 2 mules, and $369.50 for a carriage; how much had he left?

2. A gentleman gave $2700 to his three sons; to the first he gave $\frac{1}{3}$ of the whole, lacking $75; and to the second he gave $40 less than to the first; how much did the third receive?

3. A merchant in 4 days collected, respectively, $6384, $3752, $4564 and $1384; in the mean time he paid out $3564; find the difference between his receipts and expenses.

4. A man started on a journey of 7689 miles; the first month he traveled 189 miles farther than he did the second month, and the third month he went 561 miles farther than he did the first month; if he traveled 1250 miles the third month, how far had he then to go?

5. How many times can five hundred sixty-three be subtracted from two thousand eight hundred fifteen before nothing will be left?

6. What sum is that, from which if you take $42371 the remainder will lack $176.05 of being $19289?

7. What number added to the sum of 6389 and 6984 will make 18864?

8. An orchard contained 180 apple trees, 72 peach trees, and 56 pear trees, but during a storm $\frac{1}{6}$ of the apple, $\frac{1}{8}$ of the peach and $\frac{1}{7}$ of the pear trees were blown down; how many trees were left?

9. What is the average monthly wages of a clerk who receives \$36 a month for the first 3 months, \$48 a month for the next 4 months, and \$60 a month for the next 5 months?

10. Forty-five cows, at \$9 a head, are worth how much more than 8 acres of land, at \$32 per acre?

11. A grocer bought 21 barrels of flour, of which he sold 16 barrels at \$7, and the remainder at \$3, and gained \$22; what did the flour cost per barrel?

12. A farmer packed 204336 lb. of lint cotton in bales of 528 lb. each, and sent it to market in 43 equal loads; how many bales were there in each load?

13. A drover sold 324 mules; had the average price been \$12 more he would have received \$42768 for all; what was the average price?

14. A man sold 378 acres of land for \$2884.14; how much would he have received had the number of acres been 342 more, and the price per acre \$2.51 less?

15. A farmer has two fields containing 725 and 243 acres; the first field is worth \$9135, and both are worth \$18126; how much more per acre is the second worth than the first?

16. One of the divisors of 368109 is 63; find another integral divisor.

17. One of the divisors of 25542 is 43; find all the other integral divisors less than 43.

18. A man has 2 droves of cattle; the first drove is worth \$1564, at \$23 a head, and the second \$1688.40, at \$26.80 a head; find the average price per head of both droves.

19. All the prime divisors of a number are 2, 3, 7, 11, 23; name the only three integers between 65 and 80 that are also exact divisors of the number.

20. Find the value of $8 \div 28 \times 40 \times 8 \div 56$.

21. The product of 96, 22 and one other number is equal to the product of 704 and 9; what is the other number?

22. What is flour worth per barrel if 126 barrels of it will pay for 14 loads of corn, each containing 28 sacks of 3 bushels each, worth \$.75 a bushel?

23. Find the G. C. D. of 464320 and 18945.

24. The G. C. D. of 130 and another number is 26, and their L. C. M. is 910; find the other number.

25. Find the only four integers, less than 1000, which, when divided into 13078, will leave a remainder of 65.

26. A row of trees planted at equal distances from each other is 1891 ft. long; required the least possible number of trees in all if one of them is 427 ft. from one end of the row, and another 671 ft. from the other end.

27. Four wheels, whose circumferences are 27, 35, 63 and 81 inches, are connected by a system of gearing; how many revolutions from any given position must each wheel make before they all will be in the same position?

28. A man went from A to B, then from B to A, and then from A to B, at the rates of 36, 15 and 48 miles per day, respectively, and traveled the distance each time in an exact number of days; find the least possible distance from A to B.

29. A fox is 60 ft. ahead of a hound, but the hound runs 12 ft. while the fox runs 10 ft.; how far will the hound run before he overtakes the fox?

30. The product of 1807 by one number is equal to the product of 1105 by another number; find the least values of the two numbers.

FRACTIONS—II.

EXERCISE CXXIV.

521. 31. Find the sum of 3, $2\frac{1}{3}$, $\frac{2}{3}$ and $5\frac{2}{7}$.

32. *How many fourths in* $\frac{1}{3}$, $\frac{1}{6}$, $\frac{2}{8}$ and $\frac{2}{5}$?

33. What number added to the sum of $\frac{2}{3}$, $\frac{7}{8}$ and $4\frac{5}{12}$ will produce $41\frac{1}{12}$?

34. $68\frac{5}{9}$ plus a certain number is equal to $199\frac{184}{315} - 47\frac{3}{7}$; find the number.

35. A man started on a journey of $289\frac{211}{420}$ miles; after traveling $125\frac{17}{35}$ and $85\frac{5}{12}$ miles, how much farther had he to go?

36. A man cut $42\frac{7}{20}$ cords of wood, and after reserving $6\frac{3}{4}$ cords for his own use, sold the remainder at $\$3\frac{3}{4}$ per cord; what did he receive for it?

37. A bought $\frac{7}{15}$ of $48\frac{3}{4}$ yd. of velvet, at $\$3\frac{2}{3}$ a yard; what did it cost him?

38. What cost $38\frac{3}{4}$ tons of hay, at $\$15\frac{7}{8}$ a ton?

39. Multiply $\frac{5}{9}$ of $18\frac{1}{7}$ by $\frac{4}{5}$ of $24\frac{1}{2}$.

40. Divide $2\frac{2}{9}$ by $\frac{3}{4}$ of $3\frac{5}{9}$.

41. How many pounds of sugar can be bought for $\$6.21\frac{9}{16}$, at $9\frac{3}{4}$ cts. per pound?

42. Simplify $\dfrac{17\frac{5}{12} - 9\frac{3}{4} + 4\frac{5}{7}}{\frac{5}{9} \text{ of } 9\frac{2}{7}}$.

43. Divide $\left(\dfrac{4}{6\frac{4}{7}} - \frac{1}{7}\right)$ by $\frac{8}{11}$.

44. Reduce $\left(1 + \dfrac{1 + \frac{1}{5}}{5}\right) \div \left(1 + \dfrac{5}{1 + \frac{1}{5}}\right)$ to a decimal.

45. Add $\dfrac{\frac{1}{5} \text{ of } 2\frac{1}{7}}{.5 + \frac{3}{8}}$ to $\dfrac{.06 + .3\frac{1}{3}}{3\frac{1}{5} - 2\frac{1}{2}}$.

46. B is $\frac{1}{4}$ older than A, and $\frac{1}{6}$ younger than C, and C is 12 years older than A; what are their ages?

47. A and B together have $550, and $\frac{2}{3}$ of A's money equals $\frac{4}{5}$ of B's; how much has each?

48. After paying out $\frac{1}{4}$ and $\frac{1}{5}$ of my money, I had $16 more than I had spent; what had I at first?

49. Four years since my age was $\frac{9}{11}$ of what it will be 4 years from now; how old am I?

50. A, B and C together receive $94 for doing a work which A alone could do in 6 days, B in 8 days, and C in 10 days; how much is each entitled to?

51. A and B each receive $1.50 per day; A works 5 days out of 7, and B 3 days out of 5; after a certain time both together receive $276; how much would each have received had they worked regularly?

52. A traveler who goes at the rate of 22 miles in 4 hours. is followed after 5 hours by another, who goes at the rate of 38 miles in 6 hours; in how many hours will the second overtake the first?

53. A man went to town and back in $5\frac{5}{6}$ hours; in going he traveled at the rate of 8 miles per hour, and in returning at the rate of 6 miles per hour; how far was it to town?

54. In still water A and B can row a boat at the rates of $5\frac{1}{3}$ and $4\frac{5}{6}$ miles per hour, respectively; how long will it take both together to row a boat 19 miles up a river which flows at the rate of $3\frac{1}{6}$ miles per hour.

55. A crew, which can pull at the rate of 14 miles down stream, finds that it takes 4 times as long to come up the river as to go down; at what rate per hour does the stream flow, and at what rate could the crew row in still water?

56. $\frac{3}{4}$ of A's age is equal to $\frac{4}{7}$ of B's, $\frac{5}{8}$ of B's is equal to $\frac{7}{10}$ of C's, and C is 18 years older than A; what is the age of each?

57. A speculator bought two farms at an average price of $15.345 per acre; he sold one for $300.625 at $16.25 per acre, and the other for $615.15625, at $15.875 per acre; *how much* did he gain?

58. A farm was bought at $42.75 per acre; .6 of it was sold at $54.50 per acre, $.6\frac{2}{3}$ of the remainder, at $65.6 per acre, and the remainder, 60.4 acres, at $75.75 per acre; what was the gain?

59. A man can go around an island in an exact number of minutes by traveling at the rate of 6.75 chains in 2 min., or 17.1 ch. in 4 min., or 26.4375 ch. in 5 min.; required the least possible distance around the island.

60. At what times between 5 and 6 o'clock is the minute hand of a watch perpendicular to the hour hand?

DENOMINATE NUMBERS—III.

EXERCISE CXXV.

522. **61.** Reduce $2\frac{1}{4}$ ℈ to the fraction of a pound.

62. Reduce $.00694^{\circ}$ to seconds.

63. Find the cost of 27 T. 18 cwt. 3 qr. 15 ℔. 12 oz. of fertilizer, at $24.10 a ton.

64. A field is 168 rd. long and $86\frac{1}{2}$ rd. wide; what is it worth at $12.80 an acre?

65. Find the sum of 2 yd. 2 ft. $9\frac{3}{4}$ in., 16.54 in., 2.376 ft., 16.19 yd., and $\frac{3}{4}$ yd. $1\frac{5}{6}$ ft. $\frac{7}{8}$ in.

66. A boy sold $\frac{3}{8}$ bu. $\frac{2}{5}$ pk. $\frac{7}{9}$ qt. $\frac{2}{3}$ pt. of berries at 5 cts. per pint; what did he receive for them?

67. How much more will it cost to build a fence 144.86 rd. long than one $\frac{2}{5}$ mi. long, at $2 per rod?

68. If $\frac{5}{24}$ of a troy pound of tobacco is worth 12 cts., what is it worth per ℔. avoirdupois?

69. The complement of an angle is the difference between the angle and a quadrant; find the complement of $72^{\circ}\ 23'\ 26.25''$.

70. A boy had £20; after spending £9 18 s. $6\frac{1}{2}$ d. he invested the remainder in oranges at 30d. per dozen; how many oranges did he buy?

71. How many loads of 16 cwt. 1 qr. 7.88 lb. each will 8 T. 19 cwt. 2 qr. 11.68 lb. of cotton make?

72. If £16185 14 s. $\frac{3}{4}$ d. be divided equally among 75 persons, what will each receive?

73. A farmer sold 10 loads of corn each containing 26 bu. 2 pk. 7 qt. .37 pt., at 25 cts a peck; what did he receive for all?

74. What is the longitude of a place which is midway between Mobile, Ala., and Vienna, Austria?

75. When it is exactly 12 o'clock M. on a ship, the ship's chronometer set to Greenwich time points to 4 hr. 43 min. 12 sec. P. M.; what is the ship's longitude?

76. If it is 12 o'clock M. at Washington, D. C., what time is it at a point midway between Portland, Me., and San Francisco, Cal.?

77. A bin which will hold 750 hektoliters of wheat will hold how many bushels?

78. What will it cost to plaster the walls and ceiling of a room $22\frac{1}{2}$ ft long $16\frac{1}{2}$ ft. wide, and 12 ft. high, at $.40 per sq. yard?

79. What is the gain on 1200 steres of wood bought at $.55 a stere, and sold at $3.50 a cord?

80. A cask containing 250 l. of syrup was bought at 15 cts. a liter, and sold at $.68 a gallon; how much was gained?

81. What is the gain on 12000 m. of silk imported at a cost of $1.75 a meter, and sold at $1.95 a yard?

82. How many yards of carpeting $\frac{3}{4}$ of a yard wide will be required for a room $18\frac{1}{2}$ ft. long and 15 ft. 9 in. wide, if the strips run lengthwise, and there be a waste of $\frac{1}{8}$ of a yard in matching figures?

83. The outside dimensions of a box, exclusive of its top, are 6 ft., $2\frac{1}{4}$ ft. and $2\frac{2}{3}$ ft.; if it is made of plank $1\frac{1}{2}$ in. thick, how many board feet of lumber were required to construct it?

84. A's land is bounded as follows: Beginning at a certain stake, thence N. 14 ch., thence E. 20 ch., thence N. 10 ch., thence E. 12 ch., thence S. 30 ch., thence W. 8 ch., thence S. 18 ch., thence W. 19 ch., thence N. 24 ch., thence W. to the point of starting, 5 ch.; how many acres has he in all?

85. A cellar is 10.5 m. long, 7.6 m. wide and 1.8 m. deep, and it is desired to make it 2.5 m. longer, 4.4 dm. wider and 25 cm. deeper; what will be the cost of making this change, at 25 cts. per cu. meter.

86. A druggist bought medicine to the amount of 198 ℔. avoirdupois, sold it at 5 ct. an oz. apothecaries' weight, and realized a profit of $125.565; what did it cost per ℔. avoirdupois?

87. How many times will a wheel turn over in rolling 195 yd. if an arc of 13° 20′ of its circumference is 1 ft. 1 in.?

88. A boy has a plank 10 ft. 10 in. long, 12 in. wide and 1 in. thick, with which he makes a topless box, by sawing it into 5 pieces, 3 ft., 3 ft., 3 ft., 11 in., and 11 in. long; how many bushels will the box hold, allowing .8 bu. to the cu. ft?

89. A gentleman having been asked the time, replied: "By my watch it is the 10th hour 24th minute and 36th second, but my watch is 40 minutes 20 seconds too fast;" required the correct time.

90. What will it cost to enclose a garden that is $12\frac{2}{3}$ rods long, and $9\frac{1}{4}$ rods wide, with a stone wall 4 feet high and $3\frac{1}{2}$ feet thick, at $.62$\frac{1}{2}$ a perch? (A perch = $24\frac{3}{4}$ cu. ft.)

PERCENTAGE—IV.

EXERCISE CXXVI.

523. 91. For what must I sell a horse, which cost me $125, to gain $6\frac{3}{5}$%?

92. A man sold land which cost $1450 so as to gain $210.25; what per cent did he make?

93. A farmer bought 560 sheep, which he sold at $5 a head after the flock had increased $6\frac{3}{7}$%; how much did he receive for all?

94. A barrel of vinegar lost 20% by leakage, and was sold for 40% above cost price; what was the gain per cent.

95. How much more money has A than B if $2\frac{7}{9}$% of A's, or $3\frac{1}{3}$% of B's, is $5?

96. A is 18 years old; if he were $11\frac{2}{3}$% older his age would be $42\frac{6}{7}$% of B's age; how old is B?

97. Sold a house for $952.82, at a loss of 12%; had I sold it at a profit of 12% what would I have received?

98. F sold G a lot at a gain of $13\frac{1}{3}$%, and G sold it to H at a profit of 24%; if H paid $3952.50 for the lot how much did it cost F?

99. A distiller pays a tax of 90 cts. a gallon, and makes $9\frac{8}{13}$% by selling whisky at $1.14 a gallon; what is the price of corn per bushel, allowing 1 bushel to make $2\frac{1}{2}$ gallons of whisky?

100. A father gave 15% of his money to his son, 25% of the remainder to his daughter, and had $15300 left; how much had he at first?

101. Sent $2650 to my agent to be invested in land; how much land did he buy at $5 per acre, his commission being 6%?

102. An agent sold land at 5% commission, and invested the net proceeds in wheat, at 2% commission; his whole commission was $630; for what did he sell the land?

103. An agent collected a debt for $8\frac{1}{2}\%$ commission, and invested $\frac{2}{3}$ of the net proceeds, at $1\frac{2}{3}\%$ commission, in 20 mules, at $150 each; what was his entire commission?

104. What sum of money placed at 7% interest April 1, 1882, will amount to $82.24 September 10, 1883?

105. The interest of a certain sum of money is $180, and the true discount of the same sum for the same time and rate is $150; find the sum.

106. A man directed in his will that enough bonds yielding a semi-annual dividend of $1\frac{3}{4}\%$ should be bought to pay his wife an annual income of $3500; what did they cost at $94\frac{1}{8}$, brokerage $\frac{1}{8}\%$?

107. If interest be compounded annually, to what will $148.50 amount in 4 yr. 7 mo. 12 da., at 7%?

108. When the exact interest of $14965 for 186 days is $381.30, what is the rate per cent.?

109. The duty on 8500 yards of broadcloth, invoiced at $\$3\frac{3}{4}$ a yard, is $10200; find the rate of ad valorem duty on the cloth, specific duty being 45 cts. a yard.

110. A house that cost $8400 rents for $823.20 a year; the insurance is $\frac{3}{8}\%$, the repairs $\frac{1}{2}\%$, and the tax $\frac{3}{4}\%$, annually; what rate of interest does it pay?

111. The simple interest of a certain sum is $23.128; in 1 year it will be $64.60; how long before it will be $42.4816?

112. A holds 6 notes of $500 each, due without interest in 1, 2, 3, 4, 5 and 6 years, respectively; to what will all together amount in 10 years, at 6% simple interest?

113. $\frac{3}{7}$ of a vessel is insured at $2\frac{1}{2}\%$, $\frac{5}{14}$ of it at $3\frac{4}{5}\%$, the rest at $4\frac{2}{3}\%$, and the entire premium is $7200; what is the value of the vessel?

114. A certain article of consumption is subject to a duty of 6 cents a pound, but in consequence of a reduction of the

duty the consumption increased $\frac{1}{4}$, but the revenue fell $\frac{1}{5}$; what was the duty per pound after the reduction?

115. Find the cost in New Orleans of a 60 day draft on Boston for \$4000, at $1\frac{1}{2}$% premium and 6% interest.

116. A grocer in Shreveport paid \$3200 for a 30 day draft on St. Louis in payment for 600 barrels of flour; what did the flour cost per barrel, exchange being at $1\frac{3}{4}$% discount and interest 8%?

117. For what sum must a note be drawn, dated Aug. 10, 1889, payable in 90 da., that when discounted at 7% at a bank, Sept. 15, the proceeds may be \$640?

118. A owes \$100; if he pays annually the interest at 6% for 24 years, what balance will he then owe, by the mercantile rule?

119. A speculator sold 88 shares of 6% railroad stock at a premium of $1\frac{3}{8}$%, and invested the proceeds in 7% city bonds at a discount of $1\frac{1}{8}$%, paying $\frac{1}{8}$% brokerage in each transaction; by what amount was his annual income increased?

120. A brewery is worth 4% less than a tannery, and the tannery 16% more than a boat; the owner of the boat traded it for 75% of the brewery, and lost \$103; what is the tannery worth?

121. A man bought a farm for \$8000, to be paid in 4 equal annual payments, interest to be computed at 6% by the mercantile rule; that is, the debt and each payment are to draw simple interest from their respective dates to the time of final settlement; required the annual payment.

122. In the last example, if the interest be computed by the U. S. rule; that is, if the debt and each payment draw compound interest from their respective dates to the time of al settlement, what would be the annual payment?

PROPORTION AND AVERAGES—V.

EXERCISE CXXVII.

524. 123. If $\frac{7}{8}$ of a yard of silk cost \$2.10, what will $\frac{3}{5}$ of a yard cost?

124. If $6\frac{1}{4}$ tons of hay cost \$58.75, how many tons can be bought for \$173.90?

125. If 12 men can build a wall in 35 days, how long will it take 21 men to build it?

126. In an examination A made $54\frac{1}{6}\%$, B made $62\frac{1}{2}\%$, and B answered 75 questions; how many questions did A answer?

127. The use of \$1200 for 2 yr. 5 mo. 5 da. is equivalent to the use of \$1750 for what time?

128. If telegraph posts are 77 yards apart, at what rate per hour does a train travel which passes 18 posts per minute?

129. If a deduction of $37\frac{1}{2}$ cts. is allowed on cloth priced at \$$3.62\frac{1}{2}$ a yard, what deduction should be allowed on flour priced at \$$6.13\frac{6}{13}$ a barrel?

130. If a deduction of \$35 is made on a bill of \$240 for payment 7 months before due, how much should \$342 be discounted for payment 8 months before due?

131. Find the cost of 21 yd. of muslin, $1\frac{1}{7}$ yd. wide, if $15\frac{1}{5}$ yd. of the same quality, $1\frac{1}{4}$ yd. wide, cost \$$2\frac{3}{8}$?

132. By working 8 hr. a day 25 men can do a certain work in 24 da.: how many hours a day must 30 men work for 16 da. to do the same work?

133. If 20 masons build a wall 70 ft. long, $1\frac{1}{2}$ ft. thick, and $13\frac{1}{3}$ ft. high, in $9\frac{3}{5}$ da. of $8\frac{3}{4}$ hr. each, in how many days, of 12 hr. each, will 50 masons build a wall 500 ft. long, $2\frac{2}{3}$ ft. thick, and 24 ft. high?

134. If a block of stone 5 ft. long, 3 ft. 9 in. broad, and 2 ft. 6 in. deep, weighs 5625 lb., how much would it weigh

if it were 7 ft. 6 in. longer. 2 ft. 9 in. broader, and 5 ft. 9 in. deeper?

185. If $500 be divided among 3 persons in the proportion of 3, 4 and $5\frac{1}{2}$, how much will each receive?

186. Divide $3479 into 4 parts so that the second part will be $\frac{1}{2}$ more than the first, the third $\frac{1}{3}$ more than the second, and the fourth $\frac{1}{4}$ more than the third.

187. A and B are partners; $\frac{5}{7}$ of A's capital is equal to $\frac{3}{4}$ of B's, and their gain in business is $287; what is each partner's share?

188. A and B are partners for 1 year; during the first 5 months $\frac{3}{5}$ of A's capital is equal to $\frac{4}{5}$ of B's, and for the remaining time A's capital is $\frac{2}{7}$ larger and B's $\frac{3}{8}$ smaller; apportion an entire profit of $89.60.

189. A piece of cork weighs in air 4 ounces, and a piece of iron 24 ounces. In water the two together weigh 6 ounces, and the iron alone weighs $18\frac{3}{5}$ ounces; find the specific gravity of the cork.

140. A, B and C hire a pasture for 50 days, in which A puts 60, B 50, and C 40 head of cattle. After 10 days A sells $\frac{1}{3}$ of his to B; 20 days after that B sells $\frac{2}{7}$ of his to C, and 10 days after that C sells $\frac{3}{10}$ of his to A; what per cent of the entire rent should each pay?

141. A grocer has candies worth 12, 15 and 20 cts. a pound; how much of each should he take to form a mixture of 27 pounds, worth 18 cts. a pound?

142. I owe $800, payable in 7 months; if I pay $300 at the end of 4 months, how long from that time may I keep the balance as an equivalent?

143. A man owes $300 due in 5 mo., $700 due in 3 mo., and $200 due in 8 mo.; if he pays $\frac{1}{2}$ of the whole in 2 mo., when ought the other half to be paid?

144. A owes $600, of which *a part* is due in 2 mo., *a part* in 5 mo., the remaining *part* in 8 mo., and the average term of credit is 6 mo.; find the amounts of the three parts.

145. A owes B, Jan. 1, 1871, $1800; of which $700 is payable in 6 mo., $300 in 4 mo., and $800 in 18 mo.; when can the whole be paid without gain or loss of interest to either party?

146. Find the date on which a note in settlement of the following account shall be made payable:

JOHN JONES.

To H. E. LEWIS, *Dr.*

1867.				
Mar.	3	To Mdse @ 3 mos., as per bill rendered,	250	00
Apr.	4	" " 30 days, " " "	100	00
"	16	" " 60 " " " "	300	00
May	1	" " 60 " " " "	420	00
			1070	00

147. Forty-eight stones are placed in a straight line and extend a distance of 175 ft.; some of them are 3 ft., some 5 ft., and others 8 ft. in length; how many of each kind are there?

148. A hare makes 3 leaps to the dog's 2, but 3 of the dog's leaps are equal to 7 of the hare's. If the hare has a start of 60 of her own leaps, how many leaps must the hound make to overtake the hare?

149. A, B, and C engage to hoe an acre of corn for $5. A alone can hoe it in 48 hours; B, in 36 hours; and C, in 24 hours. A works alone 10 hours, then A and B work together 6 hours, when C begins and all work together till the job is finished. How much of the $5 ought each to receive?

150. A father left \$19275 to his four sons, A, B, C, D, whose ages were 9, 13, 15, 17 years, respectively, to be so divided that the respective parts, at 5% simple interest, should amount to the same sum when they became 21 years of age; what was the share of each?

151. A father willed to his sons, A, B, C, whose ages were 15, 17, 19 years, respectively, \$73482, to be divided into three shares, which, at 10% compound interest, would amount to the same sum when they became 21 years of age; what was the share of each?

HIGHER OPERATIONS—VI.

EXERCISE CXXVIII.

525. **152.** Extract the square root of 16499844.

153. Extract the cube root of 5088.448.

154. Find the square root of 3.15 to 6 places of decimals.

155. Find $\sqrt[3]{3}$ to 6 places of decimals.

156. Find $\sqrt{\frac{5}{14}} \times \sqrt{\frac{2}{15}}$, to 3 decimal places.

157. Find $\sqrt[3]{3\frac{1}{4} + 6\frac{5}{7} + 8\frac{9}{15} + 65\frac{8}{9}}$. , to 2 decimal places.

158. The amount of \$700 for 2 yr., interest compounded annually, is \$786.52; find the rate.

159. The compound interest of \$800 for 3 yr. is \$126.10; what is the rate?

160. What will it cost, at \$$1\frac{1}{2}$ per rod, to fence a square field whose area is equivalent to that of a rectangular field 64 rd. long and 49 rd. wide?

161. The width of a field is $\frac{3}{4}$ of its length, and its area is $10\frac{4}{5}$ acres; what are its dimensions?

162. If each side of a square field were 8 rd. shorter the area would be less by $4\frac{2}{5}$ acres; what is its area?

163. The volume of a cube whose edge is 36 inches is equal to that of a rectangular block whose dimensions are in the ratio of 3, 2, $1\frac{1}{3}$; find the dimensions of the block.

164. How many iron balls 2 in. in diameter will weigh as much as an iron ball 10 in. in diameter?

165. Find the cost of a circular field whose circumference is 125.664 rods, at $5 per acre.

166. A cylinder, 8 inches long, contains 3767 cu. in.; what is the diameter of its base?

167. The length of a rectangular field is 91 rods, and its diagonal is 109 rods; if it were 4 rods wider how much longer should it be to contain 48 acres?

168. If a pipe 6 inches in diameter will discharge a quantity of water in 4 hours, in what time will 3 pipes 4 inches in diameter discharge twice the quantity?

169. Find the number of cubic feet in a room whose dimensions are in the ratio of $5\frac{1}{4}$, 4, 3, the distance from a lower to the opposite upper corner being 29 feet.

170. If the minute hand of a clock is 5 inches long, how many inches does its point pass over in 35 minutes?

171. A cubic foot of water weighs $62\frac{1}{2}$ lb.; how many tons of water will a cistern contain that is 8 ft. 8 in. long, 6 ft. 4 in. wide, and 6 ft. 9 in. deep?

172. A ladder is standing by a wall of equal height, and if the bottom of the ladder be pulled out 10 ft. from the wall it will throw the top of the ladder 2 ft. below the top of the wall; find the length of the ladder

173. A ladder 53 ft. long just reaches a window on one side of a street, and when turned about its foot, just reaches a wall on the other side. If the two positions of the ladder are perpendicular to each other, and the window 28 ft. high, what is the width of the street?

174. Of 5 similar triangles the area of the first is equal to the sum of the areas of the other four; if the altitudes of the latter are 2, 4, 5 and 6 in. respectively, what is the altitude of the fifth?

175. If it requires 500 boards to fence an acre of land, what is the area of a square field which contains as many acres as there are boards in the fence that encloses it?

176. If an iron ball whose diameter is 15 in. be dropped into an upright cylindrical tank whose diameter is 10 ft., how much will the surface of the water be raised?

177. I have an inch board 5 ft. long, 17 in. wide at one end, and 7 in. at the other; how far from the larger end must it be cut straight across so that the two parts shall be equal?

178. The base of a triangular field is 15 ch., and its altitude 12 ch.; how far from the vertical angle must two lines be run parallel to the base so as to divide the area into three equal parts?

179. In the last example, if the land at the vertex is worthless and increases uniformly in value towards the base, how far from the vertex must the two lines be run so as to divide the field into three parts having equal values?

180. A conical piece of maple wood, whose altitude is 10 inches, is floating on the water with its vertex downward; how much of its altitude is above the surface of the water?

ANSWERS.

NOTE.—Answers to problems intended for mental or oral work are not given.

Page 22. Ex. IV. 3. 360. **4.** 285. **5.** 364. **6.** 371. **7.** 334. **8.** 283. **9.** 10965. **10.** 10154. **11.** 2517. **12.** 2312424. **13.** 1617129. **14.** 159. **15.** 1413. **16.** 182611. **17.** 135317. **18.** 154625. **19.** 64536. **20.** 1320. **21.** 6885. **22.** 9027. **23.** 108130. **24.** 1280340. **25.** \$274.92.

Page 23. 26. \$2485.91. **27.** \$10408.86. **28.** \$280.10. **31.** 512. **32.** 493. **33.** 484. **34.** 603. **35.** 508. **36.** 33314.

Page 24. 37. \$5385.15. **38.** \$9464.40. **39.** 7912 mi. **40.** 48637000. **41.** 34884848. **42.** 3602990 sq. mi.

Page 28. Ex. V. 2. 13. **3.** 25. **4.** 18. **5.** 16. **6.** 39. **7.** 17. **8.** 31. **9.** 231. **10.** 162. **11.** 22. **12.** 162. **13.** 88. **14.** 186. **15.** 2396. **16.** 719. **17.** 1575. **18.** 4803. **19.** 7076. **20.** 519. **21.** 1223. **22.** 1888. **23.** 2729. **24.** 2985. **25.** 4542. **26.** 56441. **27.** 15536. **28.** 436428. **29.** \$240.51. **30.** \$74.25. **31.** \$184.35. **32.** \$3428.52.

Page 29. 33. \$7096.75. **34.** \$472.25. **35.** \$107.75. **36.** \$9999.15. **37.** 8214. **38.** 7407. **39.** 9011. **40.** 10479. **41.** 8668. **42.** 1447888. **43.** \$76.44. **44.** \$360.65. **45.** \$1423.20. **46.** \$28.35. **47.** 1576. **48.** \$365.88. **49.** 578. **50.** \$4754.35. **51.** 1868. **52.** \$1372.75. **53.** 3001. **54.** \$3998.25. **55.** 1026. **56** \$156.35. **57.** 10746. **58.** \$25 35. **59.** 284. **60.** 48. **61.** 1732. **62.** 1777. **63.** \$97.75.

Page 30. **64.** $9452 50. **65.** $1149.20. **66.** 245 mi.. **67.** 993044. **68.** 4 times. **69.** 68. **70.** 270. **71.** $8281.85. **72.** 11102 ft.

Page 31. **Ex. VI.** **2.** 560. **4.** 2254. **6.** 220. **8.** 311. **10.** 282. **12.** 173. **14.** 169. **16.** 165. **18.** 121. **20.** 130. **22.** 283. **24.** 338. **26.** $1901.25. **28.** $69.15. **30.** 567, 370, 646.

Page 32. **32.** 6786. **34.** 52812. **36.** $9187. **38.** $23975.

Page 36. **Ex. VII.** **2.** 3025. **3.** 6902. **4.** 470120. **5.** 2922. **6.** 18304. **7.** 450674. **8.** 2925. **9.** 60688. **10.** 590244. **11.** 6336. **12.** 34821. **13.** 7541469. **14.** 3882. **15.** 43825. **16.** 4484139. **17.** 752. **18.** 1988. **19.** 1904. **20.** 2142.

Page 37. **22.** 1260. **23.** 8640. **24.** 170500. **25.** 1632. **26.** 14476. **27.** 1036800. **28.** 6256. **29.** 52325. **30.** 1556856. **31.** 2923. **32.** 59940. **33.** 5619501. **34.** 26645. **35.** 119109094835. **36.** 21053. **37.** 594731545. **38.** 24336. **39.** 475065601536. **40.** 4888. **41.** 213255462816. **42.** 59784. **43.** 114879044530. **44.** $5440. **45.** $276.66. **46.** $10658. **47.** $15547.68. **48.** $167491.80. **49.** $573806.25. **50.** $3996.75. **51.** $107.52. **52.** $470.08.

Page 38. **53.** 57810. **54.** $151272. **55.** 91289796 mi. **57.** 6246. **58.** 18648. **59.** 28518. **60.** 28350. **61.** 47817. **62.** 92016. **63.** 8008. **64.** 5845. **65.** 1665.

Page 39. **67.** 4700. **68.** 863000. **69.** 5040. **70.** 90250000. **71.** 1300. **72.** 137000. **73.** 17000. **75.** 840000. **76.** 4380000. **77.** 140700000000. **78.** $6000000. **79.** 342000 ft. **80.** 52290000.

Page 40. **82.** 3393. **83.** 4361875. **84.** 58114. **85.** 640065987. **86.** 3366. **87.** 479040. **88.** 1640967768.

Page 41. **90.** 1674918. **91.** 407736000. **92.** 268056. **93.** 4006007016. **94.** 7062272. **95.** 28063298070. **97.** 7296. **98.** 86462. **99.** 5175. **100.** 95524. **101.** 2450. **102.** 169955.

Page 46. Ex. VIII. **2.** $1251\frac{1}{3}$. **3.** $1620\frac{2}{5}$. **4.** $2446\frac{2}{6}$. **5.** 3936. **6.** 486. **7.** $10801\frac{6}{7}$. **8.** 1377. **9.** 6253. **10.** 11916. **11.** 746. **12.** $30722\frac{1}{3}$. **13.** $2100\frac{3}{9}$. **14.** 881. **15.** $653\frac{1}{7}$. **16.** 1247. **17.** $734\frac{6}{8}$. **18.** 7458. **19.** 4747. **20.** 344. **21.** 3225. **22.** $8024.

Page 47. **23.** 2875. **24.** 379. **25.** $696.

Page 48. **27.** 35. **28.** 786. **29.** 87. **30.** 408. **31.** 375. **32.** 507. **33.** 537. **34.** 9008. **35.** 347. **36.** 752. **37.** 19785. **38.** 987654. **39.** $153227\frac{44}{52}$. **40.** $8650\frac{111}{135}$. **41.** $226447\frac{174}{387}$. **42.** $1335652\frac{61}{225}$. **43.** $194283161\frac{1134}{1263}$. **44.** 961. **45.** 38 bu.

Page 49. **46.** $638\frac{17}{39}$ da. **47.** $\$136\frac{360}{365}$. **48.** $17. **49.** $57\frac{164}{348}$. **50.** $1133\frac{20}{60}$ mi. **51.** 545. **52.** $85\frac{148}{365}$. **53.** 326. **54.** 81864 min. **55.** 205307 hr.

Page 50. **57.** 144. **58.** 217. **59.** 408. **60.** 224. **62.** $73\frac{43}{63}$. **63.** $205\frac{13}{32}$. **64** $36\frac{83}{108}$. **65.** $124\frac{64}{75}$.

Page 51. **67.** $47\frac{35}{100}$. **68.** $8\frac{9123}{10000}$. **69.** $6\frac{887}{1000}$. **70.** $87\frac{34}{1000}$. **72.** $7\frac{16}{80}$. **73.** $11\frac{1431}{6400}$. **74.** $18\frac{242}{3500}$. **75.** $52\frac{9741}{17000}$. **76.** $12. **77.** $13. **78.** 230 cts.

Page 52. **80.** $75\frac{4}{5}$. **81.** $4\frac{713}{4125}$. **82.** $14\frac{22}{25}$. **83.** $764\frac{4076}{5625}$. **84.** $12\frac{17}{75}$. **85.** $59\frac{7882}{9375}$. **86.** $6\frac{74}{125}$. **87.** $409\frac{8326}{13875}$. **88.** $45\frac{11}{16}$. **89.** $27193\frac{243}{320}$. **90.** 19. **91.** 16. **92.** 28. **93.** $146\frac{1375}{2750}$.

Page 53. Ex. IX. **1.** 10. **2.** 13. **3.** 3. **4.** 1. **5.** 14. **6.** 4. **7.** 17. **8.** 2. **9.** 25.

Page 54. **10.** 6. **11.** 2. **12.** 6. **13.** 1. **14.** $2\frac{27}{54}$. **15.** 16. **16.** 16. **17.** 4. **18.** 1. **19.** 36. **20.** $\frac{21}{20}$. **21.** $7\frac{17}{18}$. **28.** $2\frac{17}{30}$. **24.** $\frac{4}{15}$. **25.** $81\frac{324}{700}$.

Page 55. **26.** 13. **27.** 3. **28.** 6. **29.** 4. **30.** 12. **31.** 2. **32.** 34. **33.** 32. **34.** 4. **35.** 47. **37.** 299. **39.** $126\frac{1}{3}$.

Page 56. **41.** 439. **42.** \$428.85 +. **43.** \$57.37. **44.** \$371.48 +. **46.** \$2342.81 +. **48.** \$31. **50.** \$123.

Page 57. **52.** 109. **54.** 30. **56.** 2486. **58.** \$9. **60.** Gain \$578.

Page 58. **62.** 6 cts. **64.** 151353.

Page 63. **Ex. X.** **2.** 2, 5, 5. **3.** 2, 2, 2, 2, 3, 3. **4.** 2, 2, 2, 3, 3, 13. **5.** 3, 5, 5. **6.** 2, 3, 5, 5. **7.** 3, 5, 7, 11. **8.** 2, 2, 2, 2, 2, 2. **9.** 3, 3, 3, 7. **10.** 2, 5, 7, 7, 11, 13. **11.** 2, 2, 3, 5. **12.** 3, 3, 5, 5. **13.** 3, 5, 5, 7, 7, 7. **14.** 2, 2, 2, 2, 2, 3. **15** 2, 2, 3, 5, 7. **16.** 5, 5, 11, 11, 13. **17.** 7, 13. **18.** 5, 5, 5, 5. **19.** 2, 2, 2, 2, 2, 2, 2, 2, 2, 7, 11.

Page 64. **Ex. XI.** **2.** 5. **3.** 36. **4.** 7. **5.** 32. **6.** 3. **7.** 54. **8.** 11. **9.** 35. **10.** 6. **11.** 78. **12.** 25. **13.** 11. **14.** 12. **15.** 136. **16.** 15. **17.** 91.

Page 65. **19.** 159. **20.** 19. **21.** 31. **22.** 7. **23.** 37. **24.** 13. **26.** 425.

Page 66. **28.** 11 ft. **30.** 31 bu., 139.

Page 67. **Ex. XII.** **3.** 168. **5.** 6384. **7.** 6300.

Page 68. **9.** 51051. **11.** 6300. **13.** 146261. **15.** 24255. **17.** 533610. **19.** 262080. **21.** 1921506000. **23.** 2520. **25.** 232560. **27.** \$1008. **29.** 2835 ft. **31.** 924 in.

Page 69. **33.** 6, 5, 4 pr. hour.

Page 70. **Ex. XIII.** **4.** 8. **6.** 5. **8.** $11\frac{2}{3}$. **9.** 14. **11.** 32. **13.** $9\frac{2}{7}$.

Page 71. **15.** 195. **17.** 14. **19.** \$84. **21.** 33. **28** $7\frac{1}{2}$ cts.

Page 74. Ex. XIV. **2.** $\frac{2}{3}$. **3.** $\frac{7}{8}$. **4.** $\frac{7}{13}$. **5.** $\frac{7}{11}$. **6.** $\frac{4}{5}$. **7.** $\frac{7}{10}$. **8.** $\frac{7}{9}$. **9.** $\frac{27}{32}$. **10.** $\frac{3}{4}$. **11.** $\frac{1}{4}$. **12.** $\frac{5}{8}$. **13.** $\frac{3}{4}$.

Page 75. Ex. XV. 2. $\frac{12}{6}$, $\frac{12}{6}$. **3.** $\frac{48}{24}$, $\frac{48}{24}$, $\frac{48}{24}$. **4.** $\frac{4}{6}$, $\frac{3}{6}$. 5. $\frac{60}{90}$, $\frac{72}{90}$, $\frac{15}{90}$. **6.** $\frac{18}{42}$, $\frac{35}{42}$. **7.** $\frac{40}{160}$, $\frac{96}{160}$, $\frac{100}{160}$. **8.** $\frac{35}{56}$. $\frac{24}{56}$. **9.** $\frac{280}{336}$, $\frac{288}{336}$, $\frac{294}{336}$. **10.** $\frac{8}{12}$, $\frac{9}{12}$. **11.** $\frac{18}{30}$, $\frac{25}{30}$. **12.** $\frac{50}{100}$, $\frac{60}{100}$, $\frac{70}{100}$.

Page 75. Ex. XVI. **2.** $\frac{8}{15}$. **3.** $\frac{23}{30}$. **4.** $1\frac{5}{12}$. **5.** 8. **6.** $5\frac{1}{6}$. **7.** $4\frac{1}{12}$. **8.** $1\frac{11}{12}$. **9.** $2\frac{7}{40}$.

Page 76. **10.** $11\frac{4}{15}$. **11.** $7\frac{4}{15}$. **12.** $\$20\frac{7}{20}$. **13.** $\$1\frac{11}{18}$. **14.** $8\frac{5}{12}$ mi.

Page 76. Ex. XVII. **2.** 2. **3.** $\frac{17}{63}$. **4.** $\frac{4}{15}$. **5.** $2\frac{6}{11}$. **6.** $\frac{17}{42}$. **7.** $\frac{5}{8}$. **8.** $1\frac{7}{20}$. **9.** $\frac{7}{40}$. **10.** $\frac{1}{30}$. **11.** $\frac{11}{84}$. **12.** $\$\frac{5}{12}$. **13.** $\$\frac{17}{20}$. **14.** $\$\frac{4}{7}$. **15.** $\$1\frac{29}{36}$.

Page 77. **16.** $\frac{9}{20}$ cts. **17.** $\frac{1}{6}$ mi. **18.** $\$14\frac{23}{42}$.

Page 77. Ex. XVIII. 2. $3\frac{1}{2}$. **3.** $1\frac{1}{2}$. **4.** $2\frac{1}{2}$. **5.** $2\frac{1}{7}$. **6.** $2\frac{3}{5}$. **7.** $1\frac{1}{2}$. **8.** $1\frac{1}{5}$. **9.** $5\frac{12}{13}$. **10.** $\$2\frac{1}{7}$. **11.** $9. **12.** $\$31\frac{1}{2}$.

Page 78. Ex. XIX. **15.** $\frac{10}{27}$. **16.** $\frac{25}{84}$. **17.** $\frac{18}{35}$. **18.** 2. **19.** $\frac{5}{22}$. **20.** $\frac{1}{11}$. **21.** $\frac{10}{21}$. **22.** $\frac{3}{4}$. **23.** 1. **24.** 33 cts. **25.** $\$29\frac{1}{4}$. **26.** $\$11\frac{2}{3}$. **27.** $\$274\frac{2}{7}$. **28.** 16 cts. **29.** $\$1\frac{9}{16}$. **30.** $\$\frac{27}{40}$. **31.** $\$10\frac{5}{6}$. **32.** $12. **33.** $13\frac{4}{5}$ cd.

Page 79. Ex. XX. **2.** $\frac{3}{4}$. **3.** $\frac{4}{35}$. **4.** $\frac{2}{7}$. **5.** $\frac{5}{66}$. **6.** $\frac{7}{123}$. **7.** $\frac{7}{36}$. **8.** $\frac{3}{5}$. **9.** $\frac{15}{584}$. **10.** $\$1\frac{3}{10}$. **11.** $\$5\frac{1}{6}$. **12.** $\$1\frac{5}{12}$. **15.** 9, 10, $14\frac{2}{5}$.

Page 80. **16.** 16, $17\frac{1}{7}$, $30\frac{5}{9}$. **17.** $2\frac{2}{15}$, $\frac{15}{49}$, $\frac{7}{22}$. **18.** $\frac{6}{7}$, $\frac{8}{15}$. $1\frac{4}{5}$. **21.** $3\frac{1}{3}$. $\frac{14}{45}$. $\frac{3}{14}$. **22.** $1\frac{2}{7}$, $\frac{7}{45}$. $7\frac{1}{2}$. **23.** $\frac{25}{27}$. **24.** $\$\frac{2}{7}$. **25.** $\$\frac{3}{14}$. **26.** $\$\frac{3}{40}$. **27.** $\$\frac{9}{77}$. **28.** 10. **29.** 10. **30.** 6. **31.** 36 **32.** $4\frac{1}{2}$. **33.** 273 A.

Page 85. Ex. XXI. 2. $\frac{2}{3}$. 3. $\frac{7}{54}$. 4. $\frac{3}{4}$. 5. $\frac{13}{21}$. 6. $\frac{1}{7}$. 7. $\frac{1}{7}$. 8. $\frac{19}{31}$. 9. $\frac{17}{31}$. 10. $\frac{5}{8}$. 11. $\frac{81}{422}$. 12. $\frac{5}{11}$. 13. $\frac{19}{49}$. 14. $\frac{6}{7}$. 15. $\frac{4}{5}$. 16. $\frac{2}{3}$. 17. $\frac{7}{40}$. 18. $\frac{8}{11}$. 19. $\frac{3}{5}$. 20. $\frac{3}{7}$. 21. $\frac{11}{21}$.

Page 86. Ex. XXII. 3. $\frac{7}{2}$. 4. $\frac{428}{25}$. 5. $\frac{465}{4}$. 6. $\frac{16039}{20}$. 7. $\frac{23}{3}$. 8. $\frac{341}{14}$. 9. $\frac{7107}{61}$. 10. $\frac{3886}{125}$. 11. $\frac{66}{7}$. 12. $\frac{1093}{8}$. 13. $\frac{20000}{199}$. 14. $\frac{14087}{3}$. 15. $\frac{57}{5}$. 16. $\frac{4591}{22}$. 17. $\frac{9554}{97}$. 18. $\frac{127037}{1728}$. 19. $\frac{130}{7}$. 20. $\frac{5213}{10}$. 21. $\frac{69691}{97}$. 22. $\frac{99053}{1000}$.

Page 86. Ex. XXIII. 3. 7. 4. $8\frac{1}{6}$. 5. 81. 6. $14\frac{1}{4}$. 7. $6\frac{1}{8}$. 8. $6\frac{1}{3}$. 9. $120\frac{1}{13}$. 10. $28\frac{4}{5}$. 11. $18\frac{3}{4}$. 12. 20. 13. $347\frac{7}{8}$. 14. $45\frac{1}{15}$. 15. $16\frac{1}{3}$. 16. $18\frac{3}{5}$. 17. 848. 18. $163\frac{3}{4}$.

Page 87. Ex. XXIV. 2. $\frac{9}{12}$. 3. $\frac{25}{45}$. 4. $\frac{144}{156}$. 5. $\frac{16}{20}$. 6. $\frac{28}{60}$. 7. $\frac{55}{187}$. 8. $\frac{21}{24}$. 9. $\frac{25}{70}$. 10. $\frac{207}{252}$. 11. $\frac{9}{12}, \frac{8}{12}, \frac{6}{12}$. 12. $\frac{15}{18}, \frac{6}{18}, \frac{14}{18}$. 13. $\frac{50}{60}, \frac{45}{60}, \frac{35}{60}, \frac{44}{60}$. 14. $\frac{9}{12}, \frac{18}{12}, \frac{28}{12}$. 15. $\frac{36}{18}, \frac{60}{18}, \frac{10}{18}, \frac{21}{18}$.

Page 88. Ex. XXV. 2. $\frac{9}{12}, \frac{10}{12}$. 3. $\frac{9}{30}, \frac{14}{30}, \frac{6}{30}$. 4. $\frac{8}{12}, \frac{6}{12}, \frac{10}{12}, \frac{9}{12}$. 5. $\frac{4}{8}, \frac{3}{8}$. 6. $\frac{15}{24}, \frac{14}{24}, \frac{16}{24}$. 7. $\frac{48}{60}, \frac{18}{60}, \frac{25}{60}, \frac{15}{60}$. 8. $\frac{27}{63}, \frac{42}{63}$. 9. $\frac{27}{36}, \frac{20}{36}, \frac{14}{36}$. 10. $\frac{96}{168}, \frac{60}{168}, \frac{64}{168}, \frac{105}{168}$.

Page 89. Ex. XXVI. 4. $2\frac{1}{24}$. 5. $2\frac{19}{42}$. 7. $2\frac{1}{20}$. 8. $2\frac{1}{35}$. 10. $1\frac{2}{3}$. 11. $1\frac{17}{24}$. 13. $1\frac{61}{90}$. 14. $1\frac{1}{24}$. 17. $6\frac{1}{3}$. 19. $9\frac{115}{252}$. 21. $18\frac{311}{360}$. 23. $26\frac{8}{9}$. 25. $\$1\frac{1}{4}$. 27. $2\frac{8}{9}$ mi.

Page 90. 29. $\$160\frac{19}{24}$. 31. $\$119\frac{131}{240}$. 33. $128\frac{17}{40}$ mi. 35. $241\frac{1}{60}$ A. 37. $1152\frac{13}{20}$ mi. 39. $\$1459\frac{1}{15}$.

Page 91. 41. All. 43. $\$1319\frac{1}{8}$.

Page 92. Ex. XXVII. 3. $\frac{19}{56}$. 5. $\frac{49}{120}$. 7. $3\frac{5}{6}$. 9. $\frac{79}{224}$. 11. $\frac{79}{180}$. 13. $\frac{77}{216}$. 22. $10\frac{1}{2}$. 23. $39\frac{31}{63}$. 24. $86\frac{81}{82}$. 25. $17\frac{59}{72}$. 26. $11\frac{41}{336}$. 27. $1\frac{119}{120}$. 29. $\$5\frac{3}{20}$. 31. $8\frac{8}{15}$ lb.

Page 93. **33.** $\frac{41}{126}$. **35.** $270\frac{1}{20}$ bu. **37.** $\$364\frac{1}{5}$, $\$623\frac{1}{20}$. **39.** $143\frac{5}{32}$ gal. **41.** $183\frac{17}{60}$ A. **43.** $100\frac{488}{500}$ mi.

Page 94. **45.** Less by $\frac{1}{8}$. **47.** $3\frac{25}{84}$.

Page 95. **Ex. XXVIII.** **4.** 48. **5.** $4\frac{1}{13}$. **7.** $54\frac{1}{6}$. **8.** $9\frac{9}{14}$. **10.** 55. **11.** $6\frac{1}{4}$. **13.** $23\frac{7}{16}$. **14.** $9\frac{3}{8}$. **16.** $53\frac{2}{3}$. **17.** $3\frac{2}{3}$. **19.** $66\frac{1}{2}$. **20.** $44\frac{1}{5}$.

Page 96. **28.** $\frac{5}{12}$. **29.** $1\frac{1}{2}$. **30.** $\frac{3}{5}$. **31.** $430\frac{4}{5}$. **32.** $\frac{2}{7}$. **33.** $85\frac{1}{2}$. **34.** 1. **35.** $4\frac{3}{8}$ **36.** $\frac{5}{6}$. **37.** 1. **38.** $85\frac{1}{3}$. **39.** $\frac{1}{4}$. **40.** $2\frac{3}{16}$. **41.** 24. **42.** 2. **43.** $44\frac{37}{216}$. **44.** 24. **45.** $25\frac{17}{245}$. **46.** 7. **47.** $26\frac{8}{35}$. **49.** $6\frac{29}{36}$. **51.** $61\frac{49}{80}$. **53.** 64. **55.** \$12. **57.** $\$13\frac{4}{5}$. **59.** $\$351\frac{9}{16}$. **61.** $277\frac{7}{9}$ cts.

Page 97. **63.** $10\frac{5}{6}$ mi. **65.** $\frac{14}{135}$. **67.** $\frac{5}{21}$. **69.** $\$4\frac{1}{5}$. **71.** $58\frac{1}{8}$ min. past 12. **73.** $\$478\frac{17}{30}$. **74.** $574\frac{1}{5}$ mi. **75.** \$1149.17.

Page 98. **Ex. XXIX.** **4.** $\frac{5}{24}$. **6.** $\frac{1}{6}$. **8.** $\frac{3}{56}$. **10.** $13\frac{5}{7}$. **12.** $21\frac{9}{11}$. **14.** $8\frac{3}{4}$.

Page 99. **17.** $\$\frac{10}{39}$. **19.** $7\frac{11}{20}$ yd. **21.** $24\frac{1}{28}$ mi. **24.** $2\frac{2}{9}$. **26.** $5\frac{17}{75}$. **28.** 20. **30.** $\frac{41}{371}$. **32.** $3\frac{12}{55}$. **34.** $1\frac{2}{3}$. **36.** $13\frac{13}{15}$. **38.** $\frac{8}{15}$. **40.** $\frac{153}{160}$. **42.** $\frac{8}{41}$.

Page 100. **44.** $23\frac{3}{4}$. **45.** $\frac{2}{3}$. **46.** 18. **47.** $\frac{90}{391}$. **48.** $1\frac{11}{25}$. **49.** $4\frac{1}{3}$. **50.** $\frac{9}{91}$. **51.** $\frac{11}{224}$. **52.** $\frac{3807}{5500}$. **54.** $\frac{3}{22}$. **55.** $\frac{4}{7}$. **56.** $4\frac{1}{3}$.

Page 101. **57.** $3\frac{1}{2}$. **59.** $7\frac{1}{2}$. **61.** $2\frac{1}{8}$. **63.** $10\frac{2}{5}$. **65.** $19\frac{1}{2}$ yr. **67.** \$45. **69.** $21\frac{7}{9}$ cts. **71.** $32\frac{9}{10}$.

Page 102. **72.** $\$46.10\frac{343}{466}$. **73.** 9. **74.** $8\frac{17}{25}$. **75.** 8 mo.

Page 103. **Ex. XXX.** **3.** $\frac{7}{24}$. **5.** $2\frac{29}{48}$. **7.** $4\frac{25}{42}$. **9.** $3\frac{3}{8}$. **11.** $3\frac{3}{8}$ bu., 51 no.

Page 104. **Ex. XXXI.** **3.** $15\frac{3}{4}$. **5.** $5343\frac{1}{4}$. **7.** $52950\frac{6}{7}$. **9.** 3150. **11.** $21\frac{1}{4}$ yr. **12.** $37\frac{1}{2}$ mi. **13.** $\$127\frac{1}{2}$.

Page 105. Ex. XXXII. **3.** $\frac{1}{3}$. **4.** $\frac{1}{12}$. **5.** $\frac{4}{5}$. **6.** $\frac{25}{41}$. **7.** $\frac{2}{3}$. **8.** $\frac{1}{5}$. **9.** $\frac{7}{40}$. **10.** $\frac{7}{23}$. **11.** $\frac{1}{2}$. **12.** $\frac{4}{9}$. **13.** $\frac{4}{21}$. **14.** $\frac{7}{240}$. **16.** $\frac{19}{350}$. **18.** A $\frac{7}{15}$, B $\frac{121}{360}$, C $\frac{1}{6}$, D $\frac{11}{360}$.

Page 106. **21.** $170\frac{5}{11}$. **23.** $37\frac{1}{3}$. **25.** $5\frac{2}{5}$. **27.** \$$39\frac{47}{52}$. **29.** 30 min. past 7. **31.** $\frac{4}{15}$. **33.** \$$10\frac{58}{63}$. **34.** \$47.85.

Page 107. Ex. XXXIII. **3.** \$473.60. **5.** \$$76.56\frac{1}{4}$. **7.** 4500 bu. **9.** $37\frac{3}{5}$ A.

Page 108. **12.** \$403.20. **14.** \$1.92. **16.** \$$3\frac{1}{5}$. **18.** $121\frac{209}{400}$ lb. **21.** 70 A. **23.** 6 A.

Page 109. **25.** $25\frac{7}{8}$ lb. **28.** 140. **30.** $225\frac{18}{25}$ lb. **33.** $10\frac{2}{3}$ bar.

Page 110. **35.** 16 da. **37.** 9. **40.** \$220. **42.** 8 da.

Page 111. **45.** $4\frac{4}{17}$ da. **47.** $2\frac{32}{59}$ da. **49.** $2\frac{46}{67}$ da. **51.** 30. **53.** A 60 da., B 15 da.

Page 112. **56.** 10°. **57.** 149°. **58.** 30°. **59.** 167°. **60.** 55°. **61.** 230°. **62.** —10°. **63.** 14°. **64.** —$24\frac{4}{9}$°. **65.** $26\frac{3}{5}$°. **67.** $\frac{35}{224}$. **69.** 237 bu.

Page 113. **71.** \$1.10. **73.** $\frac{3}{4}$ of a cord. **75.** \$$\frac{7}{8}$. **77.** $15\frac{3}{5}$ cts. **79.** \$1128. **81.** 2040 gal.

Page 114. **83.** \$$1002\frac{66}{67}$. **85.** \$$9\frac{5}{8}$. **87.** \$$3\frac{1}{3}$. **89.** 65. **91.** A \$55.50, B \$19.50. **93.** 60.

Page 115. **95.** \$$446\frac{13}{16}$. **97.** \$$14\frac{2}{3}$, **99.** \$$1305\frac{7}{9}$ cost. **101.** $7\frac{17}{49}$. **103.** 6. **104.** A $\frac{7}{24}$, B $\frac{5}{24}$, C $\frac{1}{24}$. **105.** A 8, B 12, C 24.

Page 116. **106.** 630 p., 315 a., 252 or. **108.** 5. **110.** [illegible]$3\frac{1}{4}$. **112.** $5422\frac{1}{2}$. **113.** \$13000. **114.** 960. **115.** \$$28\frac{3}{4}$.

[P]age 117. **116.** \$14, \$10. **117.** A 69, B 51, C 60.

Page 120. Ex. XXXIV. **2.** .31. **3.** .035. **4.** .124. **5.** .000305. **6.** 25.6. **7.** 31.08. **8.** 425.00517. **9.** 43025.0064. **10.** 356.000049697. **11.** 5000000.0000012. **12.** 3007.0067-42006. **13.** 05.00011. **14.** 49.6005. **15.** .016. **16.** .207. **17.** .034025. **18.** 5.03. **19.** 24.039. **20.** 408.0307. **21.** 17.001. **22.** 4091.0020. **23.** 3.0000003.

Page 120. Ex. XXXV. **2.** 6 hundredths. **3.** 10 and 10 thousandths. **4.** 251 and 3 tenths. **5.** 5 thous'ths. **6.** 12 and 12 thous'ths. **7.** 64 and 37 thous'ths. **8.** 42 thous'ths. **9.** 365 and 7 thous'ths. **10.** 444 and 444 thous'ths. **11.** 507 thous'ths. **12.** 40 and 40 ten-thous'ths. **13.** 161616 mill'ths. **14.** 4716 ten-thous'ths. **15.** 3 and 33331 hundred-thous'ths. **16.** 1 and 10001 mill'ths. **17.** 837 hundred-thous'ths. **18.** 85 and 62471 hundred-thous'ths. **19.** 385 and 95 ten-thous'ths.

Page 122 Ex. XXXVI. **3.** $\frac{1}{8}$. **4.** $\frac{1}{32}$. **6.** $\frac{7}{8}$. **7.** $\frac{1}{64}$. **9.** $\frac{64}{125}$. **10.** $\frac{79}{10000}$.

Page 122. 12. $\frac{5}{8}$. **13.** $\frac{81}{1600}$. **14.** $\frac{12}{13}$. **15.** $\frac{15}{16}$. **16.** $\frac{33}{70}$. **17.** $\frac{15}{1120}$. 18. \$$6\frac{2}{5}$. **19.** \$$5\frac{3}{4}$. **20.** \$$7\frac{3}{8}$. **21.** \$$3\frac{3}{5}$. **22.** \$$6\frac{1}{3}$. **23.** \$$2\frac{5}{176}$.

Page 123. Ex. XXXVII. **3.** .56. **4.** .044. **6.** .1875. **7.** .056. **9.** .46875. **10.** .31875. **12.** $.33\frac{1}{3}$. **13.** $45.71\frac{3}{7}$.

Page 124. Ex. XXXVIII. **3.** 38.885. **4.** 118.093. **5.** 72.949. **6.** 533.5796. **7.** 64.521.

Page 125. 8. 6913.5477. **9.** 2236.579495. **10.** 88.9429. **11.** 199.074. **12.** 70.672. **13.** 77.752. **14.** 21.753. **15.** 4.10001. **16.** 199.9327. **17.** 999999.999999. **18.** 47.575. **19.** \$52.70. **20.** \$3.625. **21.** 5.000005 mi. **22.** 115.353 A. **23.** 1.04308°.

Page 126. **24.** $58.3625. **25.** 361.2175. **26.** Fell 1.1115°.

Page 126. Ex. XXXIX. **2.** 38.962. **3.** .09639. **4.** 272.25. **5.** .00056704. **6.** 243.525. **7.** .438496. **8.** $25.4125. **9.** $.77625. **10.** $1.020075.

Page 127. **11.** 261.3765 mi. **12.** .43910073. **13.** 598.050 lb. **14.** $392.51265625. **15.** 1.365545. **16.** 41.759375 moist., 42.776125 vol. mat., 41.904625 carb., 18.809875 ash.

Page 128. **19.** 165.6995. **20.** .094.

Page 129. **21.** 149.51 **22.** 1.522. **23.** $361.61. **24.** 22.64063. **25.** 29.4928 lb. tot. sol., 23.703 lb. suc., 1.74 lb. glu.

Page 130. Ex. XL. **2.** 4.5 **3.** 43. **4.** .48. **5.** 375. **6.** .758. **7.** .0716 **8.** 32000. **9.** 123400. **10.** .53. **11.** .000129. **12.** 13.861 +. **13.** .3183 +. **14.** $1.25. **15.** $28\frac{3}{4}$ hhd. **16.** $3\frac{1}{5}$ A. **17.** $1326. **18.** $1.125. **19.** 15820. **20.** 125 lb.

Page 131. **22.** 1.562. **23.** 13.861. **24.** 426.1043. **25.** $.062. **26.** 332.94. **27.** 141421. **28.** 69.163 mi.

Page 132. Ex. XLI. **3.** $$12\frac{1}{3}$. **5.** $$12\frac{1}{6}$.

Page 133. **7.** $$3\frac{9}{16}$. **9.** $$17\frac{1}{2}$. **11.** $$3\frac{3}{4}$. **14.** $$113\frac{1}{3}$, $$506\frac{1}{4}$. **17.** 71, $53\frac{1}{4}$, $35\frac{1}{2}$. **19.** 104, 78. **21.** 6.

Page 134. **23.** $342\frac{2}{5}$. **25.** $45.72. **26.** $2.883, $.86+. **27.** 326.40. **28.** $1415.75. **29.** 205.40. **30.** $234.95. **31.** $$14.78\frac{3}{4}$.

Page 135. **33.** $49.20. **34.** $579.375. **35.** $27.945 ***36.*** *$$22.07\frac{1}{2}$.*

Page 136. Ex. XLII. **1.** \$7.86$\frac{1}{4}$. **2.** \$4.895. **3.** \$21.37.

Page 137. **4.** \$241.27. **5.** \$158.26. **6.** \$100.30$\frac{1}{4}$. **7.** \$35.225. **8.** \$192.095 +.

Page 138. Ex. XLIII. **1.** $\frac{13}{160}$. **2.** .371875. **3.** 1.09375. **4.** 3.200000, .237500, 13.200625. **5.** 63.5475. **6.** 320. **7.** 399.514. **8.** 1031.25. **9.** \$28.63. **10.** \$138.80. **11.** \$6963.84. **12.** \$17.20 +. **13.** \$19.689. **14.** \$.21+. **15.** \$117.70. **16.** \$99.35. **17.** \$6876. **18.** \$7.75. **19.** \$1.25.

Page 139. **20.** 3$\frac{1}{8}$ yr. **21.** \$108.75. **22.** \$36. **23.** 47.3085, 45.9515. **24.** \$5000. **25.** .375. **26.** 112.5. **27.** 4.4 da. **28.** 7$\frac{1}{3}$ hr.

Page. 158. Ex. XLIV. **3.** 353 in. **5.** 811 sq. rd. **7.** 190284 in. **9.** 61 pt. **11.** 1147 gr. **13.** 9292 oz. **15.** 1120 scr. **17.** 456 scr. **19.** 2178000 sec. **21.** 19043″. **22.** 937865 in. **23.** \$364.65. **24.** 297. **25.** 30500 mm. **26.** 420300 cl. **27.** 5000305 sq. dm. **28.** 4015000125 cu. cm. **29.** 5478. **30.** \$1039.18$\frac{1}{4}$. **31.** \$1802.50. **32.** 2030090400 sec.

Page 159. **33.** 528000. **34.** \$330. **35.** \$2597.82. **36.** \$368.145.

Page 160. Ex. XLV. **3.** £5 11 s. 11 d. 3 far. **5.** 2 rd. 2 yd. 11 in. **7.** 1 sq. mi. 620 A. 7 sq. ch. **9.** 156 gal. 1 qt. 1 pt. 3 gi. **11.** 29 bu. 2 pk. 1 qt. 1 pt. **13.** 8 ℔. 11 pwt. 20 gr. **15.** 3 ℔. 10 ℥. 5 ʒ. 1 ℈. **17.** 1 wk. 12 hr. 6 min. 1 sec. **19.** 4 S. 9° 15′ 1″. **21.** 26 cd. 12 cu. ft. **22.** 74 ℔. 11 pwt. 4 gr. **23.** 263 T. 2 cwt. 95 ℔. 4 oz. **24.** 313 sq. rd. 49.75 sq. ft. **25.** 9 mi. 880 ft. **26.** 40 l. 3 dl. 3 cl.

5 ml. **27.** 37 sq. Hm. 4 sq. Dm. **28.** .0321 Hg. **29.** 45 bu. 3 pk. 4 qt. **30.** 24 ℔. 4 oz. 6 pwt. **31.** 32 A. 6 sq. rd. **32.** 150 rd. 2 ft. 4 in. **33.** 20 da. 20 hr.

Page 161. Ex. XLVI. **3.** $\frac{7}{10}$. **5.** $\frac{3}{50}$. **7.** $\frac{7}{100}$.

Page 161. Ex. XLVII. **3.** $\frac{3}{104}$. **5.** $\frac{2341}{504}$. **7.** $\frac{1}{1314000}$. **9.** $\frac{1}{1375}$.

Page 162. Ex. XLVIII. **4.** 3 da. 2 hr. 40 min. **6.** 7 oz. 4 pwt. **8.** 1 pk. 5 qt. $1\frac{3}{7}$ pt. **10.** 15 hr. 34.56 sec. **12.** 11 sq. mi. 256 A. 7.68 sq. ch.

Page 163. Ex. XLIX. **2.** $\frac{31}{32}$. **3.** .6093 + bu. **4.** .69791$\frac{2}{3}$ ℔. **5.** $\frac{5}{88}$ mi. **6.** 39.6 mo., 3.3 yr. **7.** 91.2 mo., 7.6 yr. **8.** $26\frac{1}{2}$ qt., 3.3125 pk., .8281 + bu. **9.** 154.8 pwt., 7.74 oz., .645 ℔. **11.** $\frac{4}{9}$. **12.** $\frac{6}{13}$. **13.** $\frac{127}{384}$ **14.** .12648 +. **15.** $\frac{52}{123}$, $\frac{265}{492}$, $\frac{19}{492}$.

Page 164. Ex. L. **3.** 18.88 +. **5.** 210.6 +. **7.** 49.70 +. **9.** 430560. **11.** 3.08 +. **13.** 31392. **15.** 340.5. **17.** 120.56 +. **18.** \$4194.24 +. **19.** \$425.62 +. **20.** \$5.94 +. **21.** \$132.27 +. **22.** \$33.10 +. **23.** 103.02 + gal. **24.** 77 +.

Page 165. Ex. LI. **3.** 15.24 +. **5.** 17.78 +. **7.** 241.40 +. **9.** 4046.8 +. **11.** 199.06 +. **13.** 11.92 +. **15.** 1.13 +. **18.** $\frac{1309}{3937}$. **19.** 91.75 +. **20.** \$23.65. **21.** 764.43. **22.** \$28.64 +.

Page 166. Ex. LII. **2.** £27 5 s. 3 d. **3.** £23 11 s. 3 d. **4.** 62 ℔. 10 oz. 4 pwt. 16 gr. **5.** 125 bu. **6.** 1 ℔. 2 ℥. 6 ʒ. 1 ℈. 14 gr. **7.** 7 yr. 8 mo. 4 da.

Page 167. **9.** 17 s. 6 d. **10.** 2 rd. 4 yd. 2 ft. $1\frac{1}{2}$ in. **11.** 3 T. 12 cwt. 30 lb. 12 oz. **12.** 137 rd. 2 ft. 9 in. **13.** 6 da. 17 hr. 45 min. **14.** 7 wk. 1 da. 19 hr. **15.** 7 S. 18° 5′. **16.** 9 hr. 30 min. 36 sec. **17.** 83.885 m. **18.** 8.31151.

Page 167. Ex. LIII. **2.** 22 bu. 2 pk. 6 qt. **3.** 7 mi. 88 rd. $14\frac{1}{8}$ ft. **4.** 2 T. 17 cwt. 53 lb. 12 oz. **5.** 4 da. 4 hr. 25 min. **6.** £24 17 s. 4 d. 2 far. **7.** 36 A. 29 sq. rd. 4 sq. yd. 3 sq. ft. 36 sq. in. **8.** 30 bu. 1 pk. 7 qt. 1 pt.

Page 168. **9.** 6 rd. 1 yd. 2 ft. 8 in. **10.** 0. **11.** 3 pk. 3 qt. 1 pt. **12.** $2833295\frac{7}{12}$ cu. m. **13.** 8206.675 g. **15.** 8 yr. 4 mo. 3 da. **16.** 2 yr. 4 mo. 27 da. **17.** 23 yr. 3 mo. 21 da. **18.** 2 yr. 10 mo. 16 da. **20.** 83 yr. 3 mo. 2 da. **21.** Jan. 19, 1807.

Page 169. Ex. LIV. **2.** 47 bu. 2 pk. 7 qt. **3.** 526 rd. 4 ft. 9 in. **4.** 39 lb. 11 oz. 10 pwt. **5.** 26 bu. 4 qt. **6.** 195 gal. 3 qt. 1 gi. **7.** 236 mi. 150 rd. 3 yd. 1 ft. 6 in.

Page 169. Ex. LV. **2.** £2 5 s. 3 d. **3.** 5 lb. 7 ℥. 3 ʒ. 2 ℈. **4.** 6 cd 18 cu. ft. 1400 cu. in. **5.** 6 sq. rd. 18 sq. yd.

Page 170. **6.** 8 oz. 17 pwt. 6 gr. **7.** 11 mi. 220 rd. 1 ft. 6 in. **8.** 234.34 m. **9.** 6.24 l. **10.** 350.8 mg.

Page 170. Ex. LVI. **1.** 19° 19″; 45° 26′ 9″; 96° 56′ 45″; 76° 20′ 46″; 103° 3′; 100° 2′ 30″; 114° 4′ 12″.

Page 171. **2.** 24° 42′ 33″. **3.** 93° 31′ 35″. **4.** 13° 48′ 21″. **5.** 781.9 + mi.

Page 172. Ex. LVII. **2.** 12° 20′. **3.** 16° 34′ 15″. **4.** 71° 30′. **5.** 51° 21′ 30″. **6.** 49° 48′ 30″ W. **7.** 126° 34′ 44″ W. **8.** 47° 20′ 52″ E. **9.** 19° 21′ W. **10.** 95° 48′ 45″ W. **11.** 83° 3′ W.

Page 172. Ex. LVIII. **2.** 2 hr. 3 min. 56.6 sec. **3.** 56 min. 42 + sec. **4.** 5 hr. 37 min. 46 sec.

Page 173. **5.** 1 hr. 11 min. 16 sec. **6.** 4 hr. 45 min. 57 + sec. P. M. **7.** 5 hr. 15 min. 49 + sec. P. M. **8.** 8 hr. 25 min. 6 + sec. P. M. **9.** 8 min. 7 + sec. past 5 o'clock A. M.

Page 173. Ex. LIX. **2.** \$16.35. **4.** \$5.70. **6.** \$173.80. **8.** \$1.78. **10.** 65.

Page 174. **12.** 3 hr. **14.** \$2.56. **16.** 95 rings, 15 gr. rem. **18.** 177 rd. 4 yd. 0 ft. 10 in. **20.** 6.2491 + lb. **22.** 260. **24.** 50 times. **26.** 127 rd. 3 ft. 3.6 in. **28.** £3 2 s. 4 d.

Page 175. **30.** $\frac{491}{1920}$. **32.** \$286.20. **34.** \$8.27. **36.** \$2.73 +. **37.** \$1256.25. **38.** 35 gal. 2 qt. 1 pt. **39.** 32.80 + cts. **40.** \$301.75.* **41.** \$33.72 +. **42.** \$206250. \$4077.27. **43.** 5.4 + cts. **44.** \$180, \$152.16. **45.** 23° 48′.

Page 176. **46.** 174° 11′ 26″ W. **47.** 194 lb. 5 oz. 6 pwt 16 gr. **48.** 1 oz. 13 pwt. **49.** 3 lb. 2 ℥. 4 ʒ. **50.** 60 gal. 3 qt. **51.** Lose 2⅔ sec.

Page 177. Ex. LX. **3.** 9030 sq. ft.

Page 178. **5.** 14976 sq. rd. **7.** 13⅓ A. **9.** \$1980. **11.** 145⅓ yd. **13.** 11.6973 Ha. **14.** \$9498⅓. **15.** 60. **16.** \$4212. **17.** \$17.34. **18.** 3456. **19.** 4480.

Page 179. **20.** 14800. **21.** \$280.80. **23.** 40⅔ yd. **24.** 57⅓ yd., 55⅗ yd. **25.** \$371.72.

Page 180. **26.** \$3. **28.** \$30.88. **29.** \$24. **30.** \$16.24.

Page 182. **32.** 160 A. **33.** 160 A. **34.** 120 A. **35.** 360 A. **36.** 440 A. **37.** 320 A. **38.** 200 A. **39.** A. 440

A.; B, 360 A.; C, 320 A.; viz.: N. $\frac{1}{2}$ of S. E. $\frac{1}{4}$ and S. E. $\frac{1}{4}$ of N. E. $\frac{1}{4}$ of Sec. 21, and W. $\frac{1}{2}$ of N. W. $\frac{1}{4}$, and N. $\frac{1}{2}$ and S. W. $\frac{1}{4}$ of S. W. $\frac{1}{4}$ of Sec. 22.

Page 183. Ex. LXI. **3.** 238 cu. ft. **5.** 14 cu. yd. 2 cu. ft. 432 cu. in. **7.** 6 cu. yd. 4 cu. ft. 648 cu. in. **9.** 175$\frac{25}{32}$ cu. ft. **11.** 8 ft. 3 in. **12.** \$180. **13.** 288$\frac{3}{4}$. **14.** \$3197.70.

Page 184. **15.** 5$\frac{1}{2}$ in. **16.** 180 cu. m. **17.** 16$\frac{7}{8}$ cd. **18.** \$2.95. **19.** 10$\frac{2}{3}$ ft. **20.** 31$\frac{1}{2}$ s. **21.** 21.01. **22.** 6480. **23.** 5100. **24.** \$180.234.

Page 185. **26.** 30 gal. **27.** 196$\frac{7}{8}$ gal. **28.** 72 da. **29.** \$12$\frac{1}{2}$. **30.** 6 in.

Page 186. **32.** 3$\frac{3}{5}$ bu. **33.** 72 bu. **34.** 20 bu. **35.** 44$\frac{4}{5}$ bu. **36.** \$107.44. **37.** 11$\frac{1}{5}$ in. **39.** 12$\frac{1}{2}$. **40.** 13$\frac{1}{8}$. **41.** 7$\frac{1}{2}$. **42.** 200. **43.** 600. **44.** \$9. **45.** \$46.75.

Page 187. **46.** \$29.70. **47.** \$26.99 +.

Page 187. Ex. LXII. **2.** 2880. **4.** \$35.91. **6.** 67$\frac{1}{2}$ yd. **8.** \$1800. **10.** \$67.86.

Page 188. **12.** \$185. **14.** \$170$\frac{5}{8}$. **16.** \$76.13 +. **18.** 16$\frac{1}{2}$ ft. **20.** 48787.8768 lb. **22.** \$7.29. **24.** 5520 lb.

Page 189. **26.** 21 bu. **28.** 120 A. each; viz.: First, S. $\frac{1}{2}$ and N. W. $\frac{1}{4}$ of S. W. $\frac{1}{4}$; second, W. $\frac{1}{2}$ and N. E. $\frac{1}{4}$ of N. W. $\frac{1}{4}$; third, E. $\frac{1}{2}$ and N. W. $\frac{1}{4}$ of N. E. $\frac{1}{4}$; fourth, S. W. $\frac{1}{4}$ of N. E. $\frac{1}{4}$, S. E. $\frac{1}{4}$ of N. W. $\frac{1}{4}$ and N. E. $\frac{1}{4}$ of S. W. $\frac{1}{4}$. **30.** 18 rd. **32.** 7540 cu. ft.

Page 192. Ex. LXIII. **5.** \$271.25. **7.** 364 sheep. **9.** 90 da. **11.** 13$\frac{1}{2}$%. **13.** $\frac{1}{28}$. **15.** \$737.56 +. **17.** \$127.75. **19.** \$513.

Page 193. **21.** $5.859. **23.** $\frac{13}{125}$. **25.** $5486.25. **27.** 55300 rd. **29.** $44455. **31.** $313.60, bd.; $145.92, cl.; $8, svts.; $201.60, ex.; $610.88, saved. **32.** 51.48 ℔. ph. acid, 113.88 ℔. nit., 26.52 ℔. pot.

Page 194. **Ex. LXIV.** **5.** $350. **7.** 160. **9.** 144.

Page 195. **11.** 2567.34 ℔. **13.** $18\frac{1}{4}$ cts. **15.** $17535. **17.** 240 A. **19.** 5450 men. **21.** 650 bu. **23.** $5280.

Page 196. **25.** $62\frac{1}{2}$ gal. **27.** 900 ℔. **28.** $750. **29.** 180. **30.** 720 mi.

Page 197. **Ex. LXV.** **5.** $16\frac{2}{3}$%. **6.** 100%. **8.** 35%. **9.** 25%. **11.** 52%. **12.** $66\frac{2}{3}$. **14.** 7%. **16.** 88%, 12%. **18.** 9%. **20.** $22\frac{2}{3}$, 20, $26\frac{2}{3}$, $\frac{1}{5}$ and $29\frac{13}{15}$%.

Page 198. **22.** $87\frac{1}{2}$%. **24.** 8%. **26.** 8%. **28.** 5%.

Page 199. **Ex. LXVI.** **4.** $1711. **6.** 90 da. **8.** $1952.75. **10.** $8858.75. **12.** $9682.40. **14.** $357.3048.

Page 200. **Ex. LXVII.** **4.** 750. **6.** .6.

Page 201. **8.** $360. **10.** $578. **12.** $\$279\frac{59}{179}$. **14.** $90. **16.** $2025. **18.** 210000. **20.** 180 w., 250 c. **22.** 8%. **24.** $7\frac{11}{27}$%.

Page 202. **25.** $469. **26.** $223.11. **27.** $19\frac{36}{41}$%.

Page 203. **Ex. LXVIII.** **2.** $380.19. **3.** $3366. **4.** $344.25. **5.** $411.15.

Page 204. **7.** 10.23%. **8.** 34%. **9.** $6. **10.** $\$29\frac{1}{3}$.

Page 204. **Ex. LXIX.** **2.** $753.75. **4.** $18870. **6.** *$151.31.*

Page 205. 8. $25800. 10. $1.75. 12. $450. 14. $43\frac{3}{4}$%. 16. 30%. 18. (1). 16, $16\frac{2}{3}$, and 25 per cent. (2). 40 + %.

Page 206. 20. $3.50. 22. $282.35. 24. $7.50. 25. Made $4. 27. $55.10. 29. $56100.

Page 207. 31. $$12\frac{59}{68}$%. 33. $14.28. 35. $30\frac{10}{23}$%. 37. $15\frac{15}{19}$%. 39. $97.82 +. 41. $.663. 43. $4.735 +.

Page 208. 45. $488.376. 47. H, $3.808; G, $1.353, T, $.851. 49. $11\frac{1}{9}$%. 51. $1500. 53. $1\frac{288}{530}$% rise.

Page 209. 55. $37\frac{1}{2}$%. 56. $5. 57. $200.

Page 209. Ex. LXX. 2. $37.50.

Page 210. 3. $372.60. 4. $513. 5. $662.97. 6. $50. 7. $$2925.22\frac{3}{4}$. 8. $1500. 9. $15764 10. $150. 11. $17040. 12. $500. 13. $2550.

Page 211. 14. $150. 15. 7 cts. 17. 80. 18. 364. 19. 208.

Page 212. Ex. LXXI. 3. $$84.37\frac{1}{2}$. 5. $2650. 7. $34.74.

Page 213. 10. $396000. 12. $32160. 14. $37000. 15. $1\frac{1}{2}$%. 17. $1\frac{27}{74}$%. 19. $1\frac{3}{4}$%. 21. $\frac{25}{64}$%. 23. $1000.

Page 214. Ex. LXXII. 1. $3792. 2. $88.40. 3. $4\frac{4}{5}$%. 4. $1\frac{1}{5}$%. 5. 3 mills, $15.

Page 215. 6. $70[illegible]0. 7. $1410. 8. $1\frac{3}{4}$%, $3760. 9. $1050. 10. $2200000. 11. 8 mills. 12. $43.30. 13. $72.616.

Page 217. Ex. LXXIII. 2. $425. 3. $882. 4. $1389. 5. $602.94 +. 6. $2555.25. 7. $794.375. 8. $430.50 +. 9. 35%. 10. $1.54. 11. $$8\frac{3}{4}$.

Page 218. Ex. LXXIV. 1. 319.5 lb. 2. 649.344 lb. 3. 281.46 + lb.

Page 219. 4. 26.17 + lb. 5. 62.08 lb., 12.8 lb., 8.96 lb. 6. \$23.75. 7. \$4.879. 8. 650 lb. 9. 1100 lb. 10. $4\frac{4}{5}$ T. 11. 1100 lb. 12. $11\frac{1}{2}$%. 13. $66\frac{2}{3}$%.

Page 220. 14. $14\frac{1}{2}$%, $12\frac{1}{2}$%, $86\frac{6}{25}$%. 15. $4\frac{4}{5}$%, $22\frac{9}{10}$%. 16. $6\frac{57}{113}$%, $3\frac{13}{113}$%, $1\frac{90}{113}$%. 17. \$245.91.

Page 222. Ex. LXXV. 5. \$15.27, \$14.84. 7. \$1221.58, \$161.09. 9. \$18.71, \$25.72. 11. \$.32 +, \$1.44. 12. \$101.43. 13. \$185.47. 14. \$182.47.

Page 223. 15. \$601. 16. \$848.97. 17. \$637.42. 19. \$523.13. 20. \$19.85. 21. \$254.62. 22. \$323.05. 23. \$240.70.

Page 224. 24. \$2040.23. 25. \$1744.05. 26. \$765.67. 27. $\$1052.13\frac{1}{3}$. 28. \$227.60. 30. \$124.58. 31. \$240.70. 32. \$40.77. 33. \$27.50.

Page 225. 36. \$100.048. 37. \$4.69 +. 38. \$1.26. 39. \$9.80. 40. \$6. 41. $\$9\frac{1}{3}$. 42. \$5.03. 43. \$6.14.

Page 226. 45. \$101. 46. \$442.55. 47. \$3.11. 48. \$32.07. 49. \$2619.78. 50. \$158.09. 51. \$410.84. 52. \$132.40.

Page 226. Ex. LXXVI. 2. \$2.64. 3. \$2.85. 4. \$4.54. 5. \$122.66.

Page 227. Ex. LXXVII. 3. \$369.60. 5. \$672.24. 7. \$2180. 9. \$2500.

Page 228. Ex. LXXVIII. 3. 6%. 5. 30% +. 7. $\frac{3}{5}$%. 8. 8%. 9. 100%, 200%, etc. 10. 100%, 200%, etc. 11. 50%, $33\frac{1}{3}$%, 25%, 20%, 10%.

Page 229. **12.** 100%, $66\frac{2}{3}$%, 50%, 40%, 20%. **13.** $4\frac{1}{2}$%.

Page 229. Ex. LXXIX. **3.** 3 yr. 8 mo. 12 da. **5.** 2 yr. 7 mo. 18 da. **6.** 3 yr. 11 mo +. **7.** 3 yr. 5 mo. 18 da. **8.** 50 yr., 25 yr., etc. **9.** 50 yr., $33\frac{1}{3}$ yr., etc.

Page 230. **10.** $66\frac{2}{3}$ yr., $28\frac{4}{7}$ yr., 20 yr., 10 yr. **11.** 1 yr. 7 mo. 6 da.

Page 230. Ex. LXXX. **2.** $856. **3.** $252. **4.** $17146.75.

Page 231. **5** $1419.16.

Page 232. Ex. LXXXI. **1.** $600.67. **2.** $5132.13. **3.** $722.56. **4.** $1055.43.

Page 234. Ex. LXXXII. **2.** $695.25. **3.** $1574.65. **4.** $1194.26. **5.** $672.14. **6.** $210.806 +.

Page 235. **8.** $46.54. **9.** $149.21.

Page 236. Ex. LXXXIII. **2.** $389.568. **3.** $161.783. **4.** $386.43. **5.** $102.04. **6.** $203.05. **7.** $85.43.

Page 237. **8.** $2082.25. **9.** $895.52.

Page 238. Ex. LXXXIV. **2.** $6135.75. **3.** $1200. **4.** $102.39. **5.** 25049.22. **6.** $50.18. **7.** $40.50. **8.** $7714.29. **9.** $4595.50.

Page 240. Ex. LXXXV. **2.** $1929.62 pro. **3.** $1367.44 pro. **4.** $7295 pro. **5.** $244.187 pro. **6.** $79.909 pro. **7.** $1228.54 pro. **8.** $6.94. **9.** $336.86. **10.** $56.62 +. **11.** $726.34. **12.** $1871.14.

Page 241. **14.** $800. **15.** $4473.16. **16.** $518.45. **17.** $770. **18.** $1021.80 +.

Page 242. **Ex. LXXXVI.** 2. \$51312.50. 3. \$3726.

Page 243. 4. \$16582.50. 5. \$6975. 6. \$247.50. 7. \$43.75. 9. 50. 10. 60. 11. 120. 12. \$540. 13. \$67.

Page 244. 15. \$150850. 16. \$7750. 17. \$78750. 18. \$17580. 19. \$84. 21. $5\frac{3}{5}$%. 22. $7\frac{1}{2}$%. 23. 8% stock is better by 2%.

Page 245. 24. $5\frac{5}{7}$%. 25. $\frac{5}{12}$%. 27. $33\frac{1}{3}$% prem. 28. $37\frac{1}{2}$% dis. 29. $40\frac{1}{4}$% dis. 30. $59\frac{7}{8}$% prem. 31. 10%.

Page 247. **Ex. LXXXVII.** 3. \$663. 4. \$3223.80. 5. \$7368.34. 6. \$4309.87. 7. \$392.58. 8. \$644.15. 9. \$7150.09 +. 10. \$12645.24. 11. \$314.613.

Page 248. 14. \$740.66. 15. \$2581.30. 16. \$1250. 17. \$800. 18. \$4160.16. 19. \$400.

Page 250. **Ex. LXXXVIII.** 2. $2\frac{6}{7}$. 4. $\frac{7}{11}$. 6. $23\frac{19}{27}$. 8. $\frac{5}{28}$. 10. $1\frac{63}{3937}$. 12. $\frac{10500}{10567}$. 14. $\frac{1925}{1929}$. 16. $4\frac{4}{241}$ 18. $\frac{1}{8}$.

Page 251. 19. $1\frac{1}{2}$. 20. $\frac{8}{9}$. 21. $\frac{10}{13}$. 22. $\frac{5}{7}$. 23. $2\frac{29}{35}$. 24. $\frac{3}{4}$. 25. $\frac{2}{3}$. 26. $\frac{1}{56}$. 27. $1\frac{7}{12}$. 28. $\frac{29}{90}$. 29. $\frac{1}{12}$. 30 $1\frac{1}{24}$. 31. $\frac{5}{33}$. 32. $\frac{6}{25}$. 33. $\frac{1}{6}$. 34. $1\frac{3}{5}$. 35. $\frac{2}{5}$. 36. 12. 37. $2\frac{3}{16}$. 38. $\frac{9}{40}$. 39. $\frac{1}{15}$. 40. $\frac{3}{20}$. 41. $\frac{33}{7}$.

Page 253. **Ex. LXXXIX.** 4. 80. 6. 102. 8. \$12.40 10. .258048 yd. 12. \$$\frac{55}{72}$. 14. \$$25\frac{1}{2}$. 16. 36. 19. \$2362.50. 21. 38 yd.

Page 254. 23. \$141.75. 25. 27 m. 27. 45. 29. 200 da. 31. $55\frac{1}{2}$ lb. 32. $18\frac{3}{4}$. 34. $83\frac{1}{2}$ ft.

Page 255. 36. \$2.20. 38. \$35. 40. B, \$18.37 +; *C, \$7.14 +*. 41. \$104.65. 42. \$1800.

Page 256. Ex. XC. **3.** 45. **4.** 30. **5.** 12. **6.** 84. **7.** 3. **8.** 4.

Page 257. **10.** $180. **11.** 4 da. **12.** 10 men. **13.** $120. **14.** $23.765. **15.** $59\frac{5}{7}$ A. **16.** 25 da. **17.** 36 da. **18.** 14 m. **19.** 25 lb. **20.** 18 da. **21.** 36 men.

Page 258. **22.** 675. **23.** 64 da.

Page 259. Ex. XCI. **3.** 28 cts. **4.** 15 min. **5.** 49 da. **6.** $43\frac{1}{5}$ ft. **7.** 100 pl. **8.** $3.84. **9.** $\$251\frac{1}{4}$. **10.** £3 12 s. 6 d. **11.** $70.

Page 260. **12.** $1\frac{1}{4}$ ft. **13.** 1 yr. 8 mo. **14.** $55.11. **15.** 4 yr. 11 mo. 6 da. **16.** $7. **17.** $560. **18.** $12.152.

Page 261. Ex. XCII. **4.** 5. **5.** $1\frac{1}{3}$. **6.** $7\frac{2}{9}$. **7.** 221.184 cu. in. **8.** 117.504 cu. in. **9.** 25.92 cu. in. **10.** $46\frac{7}{18}$ lb.

Page 262. **11.** $188\frac{3}{4}$ lb. **12.** $5319\frac{3}{8}$ lb. **13.** $8\frac{3}{5}$ lb. +. **14.** .617. **15.** .681. **16.** .993. **17.** 9.238 cu. in. **18.** 15 boys.

Page 263. Ex. XCIII. **3.** 96, 132. **5.** 77.7, 74. **7.** 175, 275, 375. **9.** 195 bu., 280 bu., 470 bu. **11.** 12, 24, 72. **12.** $\$88\frac{4}{11}$, $\$113\frac{5}{11}$, $\$98\frac{2}{11}$.

Page 265. Ex. XCIV. **2.** $60, $100. **3.** $78, $86.50, $90.50. **4.** $225, $315. **5.** $10.50, $13.44, $15.06, $19.20. **6.** A, $2; B, $1.60. **7.** A, $\$2828\frac{4}{7}$; B, $\$3651\frac{3}{7}$.

Page 266. Ex. XCV. **2.** A. $53.75; B, $146.25. **3.** A, $24.19 +; B, $51.61 +; C, $24.19 +. **4.** $128, $112. **5.** $137.70, $218.70. **6.** A, $47.25; B, $43.80. **7.** F, $2238; G, $1620; H, $1760.

Page 267. **8.** $2270, $2150. **9.** A, $690, $720. **10.** $10000. **11.** $\frac{3}{4}$. **12.** $\frac{1}{4}$. **13.** A, $1350; B, $1200; C, $5000.

Page 268. Ex. XCVI. 2. $[illegible]. 3. $[illegible]. 4. [illegible] ct. 5. 4[illegible] mi.

Page 269. 6. 51[illegible]%.

Page 270. Ex. XCVII. 2. 4. 1. 2. [illegible]. 3. 2. 2. [illegible]. 4. [illegible]. 4. 2. 1. [illegible]. 2. 5. 4. 1. 2. [illegible]. 7. 1. 2. [illegible].

Page 271. 8. 1. 1. [illegible]. 9. [illegible]. 24. 10. [illegible]. 14. 11. 26. [illegible]. [illegible]. 10.

Page 272. 14. 12. [illegible]. 4. 15. 15. 14. 14. [illegible]. 16. 30, 60, 75. 17. 6 gr., 24 gr., 18 gr., 48 gr. 18. [illegible]. 19. $250. $250. $500. 20. $[illegible]. $[illegible]. $[illegible].

Page 274. Ex. XCVIII. 3. [illegible] mo. 4. [illegible] mo. 5. 12[illegible] mo. 6. 78 da.; April 2, 18[illegible]. 7. Aug. 2[illegible], '[illegible]. 9. Jan. 2[illegible], '[illegible]. 10. 76 da.

Page 275. 11. April 30, next year. 12. Aug. [illegible]. 13. 6[illegible] mo. after Jan. 15. 15. Aug. 15, '[illegible]. 16. May 3[illegible], '[illegible]. 17. May 23, '[illegible].

Page 276. 18. July 11.

Page 278. Ex. XCIX. 2. Sept. 2[illegible], '[illegible]. 3. $[illegible]. 4. Oct. 24, '[illegible]. 5. $54.7[illegible].

Page 279. 6. Nov. 2[illegible], '[illegible]. 7. $[illegible].52.

Page 281. Ex. C. 3. 625; 1225; 2116; 20.25; .0064; .[illegible]; [illegible]; [illegible]; [illegible]; 11[illegible]; 5.0625. 4. 3375; 91125; [illegible]; 1[illegible]; .125; .[illegible]; .[illegible]; [illegible]; [illegible]; [illegible]; [illegible]. 5. [illegible]. 6. 15625. 7. 92.3521. 8. [illegible]. 9. .[illegible]. 10. [illegible]. 11. [illegible]. 12. [illegible]. 13. 1024. 14. [illegible].

Page 285. Ex. CI. **3.** 15. **4.** 35. **5.** 43. **6.** 55. **7.** 42. **8.** 56. **9.** 96. **10.** 75. **11.** 52.

Page 286. **12.** 125. **13.** 225. **14.** 308. **15.** 109. **16.** 530. **17.** 99.9. **18.** 1.53. **19.** .245. **20.** .064. **21.** $\frac{12}{17}$. **22.** $\frac{33}{65}$. **23.** $\frac{48}{241}$. **24.** $3\frac{1}{3}$. **25.** $6\frac{1}{2}$. **26.** $7\frac{1}{4}$. **27.** 1.4142. **28.** 1.7320. **29.** 2.2360. **30.** 2.6457. **31.** .6123. **32.** 2.5298. **33.** 2.3299. **34.** 1.7748. **35.** 14. **36.** 55. **37.** 6.8. **38.** 2.39. **39.** 145 yd. **40.** 416 yd. **41.** 34 and 102 yd. **42.** $360. **43.** 43.3 in. **44.** $26\frac{2}{3}$, 15, and $9\frac{3}{5}$ in. **45.** 8.63 sec. **46.** 62 yd.

Page 291. Ex. CII. **3.** 32. **4.** 27. **5.** 42. **6.** 7.5. **7.** 54. **8.** .172. **9.** 1.94. **10.** .035. **11.** 364. **12.** .0806. **13.** $\frac{23}{25}$. **14.** $\frac{125}{237}$. **15.** $4\frac{1}{3}$. **16.** $1\frac{4}{5}$. **17.** 7.4 **18.** 1.259. **19.** 1.442. **20.** 1.709. **21.** .873. **22.** .310. **23.** 3.382. **24.** 4 ft. **25.** 42 in. **26.** $507. **27.** 42, 28, and 14 ft.

Page 292. **28.** 2.884 ft. **29.** 8 ft. 6.44 in. **30.** 18 in. **31.** 174.96 sq. in.

Page 294. Ex. CIII. **2.** 35 ft. **3.** 51 ft. **4.** 145 yd. **5.** 26 ft. **6.** 24 ft. **7.** 55.9 + ft. **8.** 83.21 + mi. **9.** 28.28 + ft. **11.** 360 sq. rd. **12.** $4735\frac{1}{2}$ sq. yd.

Page 295. **13.** 6 A. **14.** 6 A. **15.** $90. **16.** 250 sq. ft. **17.** $3\frac{1}{3}$ in. **18.** 65 rd. **20.** 96 sq. ft. **21.** 62.35 sq. ft. **22.** 507.84 sq. ft. **23.** $510. **24.** 16 ft.

Page 296. Ex. CIV. **2.** 96 sq. rd. **3.** $328\frac{1}{8}$ A. **4.** 15 ft. **5.** $$257\frac{5}{8}$.

Page 297. **7.** 10 sq. yd. **8.** 70.3 A. **9.** $2\frac{1}{2}$ sq. ft. **10.** 400 A., 240 A.

Page 298. **13.** 45 A. **14.** 57.415 A. **15.** 2 A. **16.** 1 A. 3.9 *sq. rd.* **17.** $3162.

Page 299. Ex. CV. **2.** 565.488 ft. **3.** 1727.88 in. **4.** 20.849 + in. **5.** 79.577 + ft. **6.** 14.1372 ft. **7.** 560.2 + yd. **9.** 20106.24 sq. yd. **10.** 30 ft.

Page 300. **11.** $1413.72. **12.** 7.97 rd. **13.** 179.3 rd. **14.** 2.73 + A.

Page 301. Ex. CVI. **3.** (1) 192 sq. ft.; (2) 222 sq. ft. **4.** (1) 180 sq. in.; (2) 192 sq. in. **5.** 450 sq. in. **6.** 1694.89 + sq. in.

Page 302. **7.** 30 ft. **8.** 12 ft. **11.** 10.825 cu. ft. **12.** 188.496 cu. ft. **13.** 7068$\frac{3}{5}$ gal. **14.** 56$\frac{1}{4}$ lb. **15.** 2150.42 + cu. in. **16.** 800 cu. yd.

Page 303. **17.** 2$\frac{17}{64}$ lb. **18.** 3$\frac{15}{16}$ in. **19.** 21.3 + lb. **20.** 19.17 + in.

Page 304. Ex. CVII. **3.** 161 sq. in. **4.** 112$\frac{1}{2}$ sq. ft. **5.** (1) 360 sq. yd., (2) 422.35 + sq. yd.

Page 305. **6.** (1) 188.496 sq. in., (2) 301.593 sq. in. **7.** 38.6 + sq. ft. **10.** 300.79 + cu. in. **11.** 848.7 + cu. in. **12.** 119 + lb. **13.** 195$\frac{5}{16}$ lb. **14.** 109.28 + cu. ft.

Page 306. Ex. CVIII. **2.** 28.2744 cu. ft. **3.** 19.635 sq. ft. **4.** 196663355.75 sq. mi. **5.** $6.82. **7.** **33.5104** cu. ft. **8.** 136.4 + lb. **9.** 4.09 + lb. **10.** 125000 ft.

Page 307. Ex. CIX. **3.** 22$\frac{1}{2}$ lb. **4.** 4.24 + ft. **5.** 5 hr.

Page 308. **6.** 18 in. **7.** $7.20. **8.** .31 + in. in diameter. **9.** 9.19 + ch. **10.** 13 in.

Page 309. **13.** 7\frac{1}{18}$. **14.** $7.776. **15.** 20 ft. **16.** 20 *ft.* ***17.*** *207.72* lb. **18.** $46.29 +. **19.** 9.53 + ft. **20.** 17$\frac{1}{2}$ *in.* ***21.*** *6* in. **22.** 4.76 + in.

Page 310. **23.** Diam. .696 + in.; thickness, .029 + in. **24.** 5.44 + in., 4.76 + in., 3.77 + in.

Pages 311 to 315. Ex. CX. It is unnecessary to give the answers to these problems: the student may verify the results by the ordinary methods.

Page 316. Ex. CXI. **2.** $\frac{7}{9}$; $\frac{13}{99}$; $\frac{352}{999}$. **3.** $\frac{5}{33}$; $\frac{16}{111}$; $\frac{14}{37}$; $\frac{192}{1111}$. **4.** $\frac{5}{11}$; $\frac{59}{111}$; $\frac{365}{1111}$; $\frac{3}{7}$.

Page 317. **6.** $\frac{57}{110}$; $\frac{179}{1100}$; $\frac{279}{1100}$; $\frac{38533}{166500}$. **7.** $\frac{41}{18}$; $\frac{127}{30}$; $\frac{163}{90}$; $\frac{1223}{990}$. **8.** $2\frac{56}{165}$; $5\frac{67}{90}$; $8\frac{139}{1100}$; $\frac{359}{1100}$; 19. **10.** $.46\dot{3}4634\dot{6}$, $.58\dot{1}5815\dot{8}$, $.63\dot{2}4242\dot{4}$. **11.** $.327\dot{3}2732\dot{7}$, $.635\dot{3}5353\dot{5}$, $.176\dot{3}3333\dot{3}$. **12.** $.32\dot{1}4214214214\dot{2}$, $.75\dot{4}34343434343\dot{3}$, $.52\dot{3}9523952395\dot{2}$. **13.** $16.151\dot{5}1515\dot{1}$, $37.124\dot{8}2482\dot{4}$, $.003\dot{1}7171\dot{7}$.

Page 318. **14.** $103.2\dot{5}9122\dot{7}$. **15.** $12.\dot{4}$. **16.** $33.13\dot{3}\dot{4}$. **17.** $4.1766\dot{3}4561\dot{8}$. **18.** $40.079\dot{3}6072\dot{4}$. **19.** $2.0\dot{8}$. **20.** $5.04\dot{6}\dot{2}$. **21.** $2.\dot{4}0595\dot{1}$. **22.** $1.7\dot{8}3629\dot{0}$. **23.** $90.\dot{5}037\dot{7}6$. **24.** $11.2\dot{3}0867\dot{2}$.

Page 319. **25.** $530.\dot{8}1044\dot{6}$. **26.** $90.5\dot{2}0374\dot{9}$. **27.** $19.0\dot{7}\dot{2}$. **28.** $.0\dot{7}06\dot{7}$.

Page 322. Ex. CXII. **1.** 2 yr. 6 mo. 16 da.; 2 yr. 6 mo. 18 da.; 2 yr. 199 da.; 929 da. **2.** 4 yr. 8 mo. 13 da.; 4 yr. 8 mo. 12 da.; 4 yr. 257 da.; 1718 da.

Page 325. Ex. CXIII. **2.** \$1325.822. **3.** \$640.405. **4.** 857.25. **5.** \$397.96. **7.** \$3000 —. **8.** \$400 +.

Page 326. **9.** \$5000. **11.** 7 yr. **12.** 6%. **13.** 10 yr 7 mo. 15 da. **14.** 7% +. **15.** 11 yr. 10 mo. 21 da +.

Page 329. Ex. CXIV. **5.** £1337 8 s. 11 + d. **6.** *7740 fr.* *7. \$195.26.* 8. \$405.30. **9.** \$769. **10.** £240.

Page 330. **11.** $974.1136. **12.** $1465.952. **13.** $4212.81. **14.** $4966.80.

Page 332. **Ex. CXV.** **3.** 149. **4.** 64. **5.** 5. **6.** 29. **7.** 42. **8.** 53. **9.** 155. **10.** 62. **11.** 15.

Page 333. **12.** 157. **13.** $207. **15.** 670. **16.** 5050. **17.** 2601. **18.** 800. **19.** 3441.

Page 334. **20.** 425. **21.** $61\frac{1}{2}$. **22.** $689. **23.** (1) $973\frac{1}{2}$ ft.; (2) 14850 ft. **24.** $5400. **25.** $147.96. **26.** $1267.20.

Page 335. **Ex. CXVI.** **2.** 327680. **3.** $1\frac{3}{4}$. **4.** 320. **5.** $\frac{1}{32}$. **6.** $195\frac{5}{16}$. **7.** $\frac{4}{9}$. **8.** $20.48. **9.** $729.303.

Page 336. **10.** 1456. **11.** $5314.40. **12.** 9841. **13.** $3249\frac{7}{8}$. **14.** $63\frac{15}{16}$. **15.** 1456. **16.** $8857.30. **17.** $4196.91.

Page 338. **Ex. CXVII.** **2.** $24480. **3.** $15700. **4.** $22950. **5.** $102. **7.** $7917.97. **8.** $6632.98. **9.** $16532.98.

Page 339. **10.** $25155.80. **11.** $3722.22.

Page 341. **Ex. CXVIII.** **2.** $3860.8675. **3.** $8559.479. **4.** $3516.791. **5.** $2913.074.

Page 342. **7.** $500. **8.** $486.37 +. **9.** $760.75 +. **10.** $400.75 +

Page 343. **Ex. CXIX.** **2.** $767.25. **3.** $926.10.

Page 344. **5.** 53.6 + %. **6.** 1.3 + %. **8.** $1929.125. **9.** $3478.32 +. **10.** $598.05.

Page 345. **11.** $2071.575.

Page 345. **Ex. CXX.** **1.** 512. **2.** 15. **3.** 4. **4.** *1.019* +. **5.** 1.49 +.

Page 348. **7.** 453. **8.** 2.924 +. **9.** 27. **10.** 3.141 +. **11.** 17.92 +.

Page 349. Ex. CXXI. **2.** 17 sq. ft. 1′ 4″. **3.** 87 sq. ft. 6′ 2″ 4.‴ **4.** 158 cu. ft. 9′ 6″ 8″. **5.** 32 sq. ft. 5′ 6″. **6.** 147 cu. ft. 9′ 4″.

Page 349. Ex. CXXII. **2.** Sunday. **3.** Friday. **5.** Monday.

Page 350. Ex. CXXIII. **1.** $985.25. **2.** $1090. **3.** $12520. **4.** 5250 mi. **5.** 5 times. **6.** $61483.95. **7.** 5491. **8.** 256.

Page 351. **9.** $50. **10.** $149. **11.** $5. **12.** 9. **13.** $120. **14.** $3686.40. **15.** $24.40. **16.** 5843. **17.** 2, 3, 9, 11, 6, 18, 22, 27, 33. **18.** $24.82 +. **19.** 66, 69, 77. **20.** $1\frac{31}{49}$.

Page 352. **21.** 3. **22.** $7. **23.** 5. **24.** 182. **25.** 77, 91, 143, 169. **26.** 32. **27.** 105, 81, 45, 35. **28.** 720 mi. **29.** 360 ft. **30.** 85, 139.

Page 352. Ex. CXXIV. **31.** $11\frac{19}{42}$. **32.** 21.

Page 353. **33.** $35\frac{1}{8}$. **34.** $83\frac{3}{5}$. **35.** $78\frac{3}{5}$ mi. **36.** $\$133\frac{1}{2}$. **37.** $\$83\frac{5}{12}$. **38.** $\$615\frac{5}{32}$. **39.** $197\frac{5}{9}$. **40.** $\frac{5}{6}$. **41.** $63\frac{3}{4}$. **42.** $2\frac{2}{5}$. **43.** $\frac{825}{1288}$. **44.** .24. **45.** $1\frac{38}{735}$. **46.** 24, 30 and 36 yr. **47.** $300, $250. **48.** $160.

Page 354. **49.** 40 yr. **50.** A $40, B $30, C $24. **51.** $210. **52.** 33 hr. **53.** 20 mi. **54.** $2\frac{8}{11}$ hr. **55.** $8\frac{3}{4}$ mi., $5\frac{1}{4}$ mi. **56.** A 32, B 42, C 50. **57.** $37.28.

Page 355. **58.** $7947.13. **59.** 3013.875 ch. **60.** $10\frac{10}{11}$ *min.*, $43\frac{7}{11}$ *min.*

Page 355. Ex. CXXV. **61.** $\frac{1}{128}$ lb. **62.** 25″. **63.** \$673.48 +. **64.** \$1162.56. **65.** 21 yd. 2 ft. 3.517 in. **66.** \$1.63$\frac{1}{8}$. **67.** \$33.72. **68.** \$.70. **69.** 17° 36′ 33.75″. **70.** 967.

Page 356. **71.** 11. **72.** £215 16 s. 2$\frac{1}{4}$ d. **73.** \$267.24+. **74.** 35° 51′ 14″ W. **75.** 70° 48′ W. **76.** 10 hr. 42 min. 38 sec. + A. M. **77.** 2128$\frac{1}{8}$ bu. **78.** \$58.10. **79.** \$498.80 gain. **80.** \$7.41 gain. **81.** \$4590.50 gain. **82.** 43$\frac{17}{18}$ yd.

Page 357. **83.** 81$\frac{1}{4}$. **84.** 107$\frac{1}{5}$ A. **85.** \$17.65 +. **86.** 9$\frac{1}{2}$ cts. **87.** 20. **88.** 2$\frac{7}{90}$ bu. **89.** 43 min. 16 sec. past 8 o'clock. **90.** \$260.64.

Page 358. **91.** \$133.25. **92.** 14$\frac{1}{2}$%. **93.** \$2980. **94.** 12%. **95.** \$30. **96.** 46.9 yr. **97.** \$12.68. **98.** \$2812.50. **99.** 35 cts. **100.** \$24000. **101.** 500 A. **102.** \$9180.

Page 359. **103.** \$475. **104.** \$74.70. **105.** \$900. **106.** \$94250. **107.** \$203.05. **108.** 5%. **109.** 20%. **110.** 7$\frac{19}{20}$%. **111.** 5 mo. 18 da. **112.** \$4170. **113.** \$210000. **114.** 2$\frac{2}{3}$ cts.

Page 360. **115.** \$4018. **116.** \$5.469 +. **117.** \$647.17. **118.** \$.64. **119.** \$102. **120.** \$696. **121.** \$2275.22$\frac{102}{109}$. **122.** \$2308.73.

Page 361. Ex. CXXVII. **123.** \$1.44. **124.** 18$\frac{1}{2}$ T. **125.** 20 da. **126.** 65. **127.** 1 yr. 8 mo. **128.** 47$\frac{1}{4}$ mi. **129.** \$.63$\frac{6}{13}$. **130.** \$57. **131.** \$3. **132.** 10 hr. **133.** 64 da. **134.** 80437$\frac{1}{2}$ lb.

Page 362. **135.** \$120, \$160, \$220. **136.** \$497, \$745.50. \$994, \$1242.50. **137.** A, \$147; B, \$140. **138.** \$56.70, \$32.90. **139.** .240. **140.** A 31$\frac{11}{15}$%, B 38$\frac{2}{3}$%, C 29$\frac{3}{5}$%. **141.** 18 lb. at 20 cts., 6 lb. at 15 cts., 3 lb. at 12 cts. **142.** 4 mo. **143.** 6$\frac{2}{3}$ mo.

Page 363. **144.** \$100, \$200, \$300. **145.** Dec. 1, 1871. **146.** June 14. **147.** 34, 13, 1; or 37, 8, 3; or 40, 3, 5. **148.** 72. **149.** A \$2.24$\frac{14}{39}$, B \$1.60$\frac{10}{39}$, C \$1.15$\frac{15}{39}$.

Page 364. **150.** A \$4095, B \$4680, C \$5040, D \$5460. **151.** \$20000, \$24200, \$29282.

Page 364. Ex. CXXVIII. **152.** 4062. **153.** 17.2. **154.** 1.774821 +. **155.** \$1.442249 +. **156.** .218. **157.** \$4.38 +. **158.** 6%. **159.** 5%. **160.** \$336. **161.** 48 rd., 36 rd. **162.** 14$\frac{2}{5}$. **163.** 54 in., 36 in., 27 in.

Page 365. **164.** 125. **165.** \$39.27. **166.** 24.48 + in. **167.** 29 rd. **168.** 6 hr. **169.** 4032 cu. ft. **170.** 18.326 in. **171.** 11 T. 11 cwt. 56 lb. 4 oz. **172.** 26 ft. **173.** 73 ft. **174.** 9 in.

Page 366. **175.** 250000. **176.** $\frac{5}{32}$ in. **177.** 2 ft. **178.** 6.99 + ch, 9.79 + ch. **179.** 8.31 + ch., 10.48 + ch. **180.** .894 + in.

Richard

Toulmon

Born June 16th 1596